Risk Arbitrage in Takeovers*

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Abstract. The paper studies the role of risk arbitrage in takeover contests. We show that arbitrageurs have an incentive to accumulate non-trivial stakes in a company which is the target of a takeover. For each arbitrageur, the knowledge of his own presence (and that he will tender a positive fraction of his shares) is an informational advantage which guarantees that there is a scope for trade with the other shareholders. In equilibrium, the number of arbitrageurs buying shares and the number of shares they buy are determined endogenously. The paper also presents a range of empirical implications, including the relationship between trading volume, takeover premium, liquidity of the shares, the probability that the takeover will succeed and the number of risk arbitrageurs investing in one particular deal.

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1. Introduction

It is well known that risk arbitrageurs play an important role in the market for corporate control. After a tender offer, the trading volume increases dramatically in large part because of risk arbitrageurs activity.[1] They take long positions in the target stock, in the hope that the takeover will go through. They are also usually hedged by taking short positions in the acquirer's stock.

Risk arbitrage used to be a very inconspicuous activity, but in the mid-70s the emergence of Ivan Boesky and the increasing volume of corporate takeover deals contributed to make it more visible.[2] Attracted by the high rewards, many firms started new arbitrage departments and more people became involved in this activity. As a consequence of the large volume of new arbitrage capital, in more recent years spreads narrowed and the share price, after a takeover announcement, rises much more rapidly. The arbitrage community has often come to control, in total, 30 to 40 per cent of the stock and therefore it has become the single most important element in making many deals happening. Risk arbitrageurs are perceived as a crucial element in determining the success of a takeover. They are typically perceived as favoring the acquirer since they are more likely to tender.[3]

In this paper we study the role that risk arbitrageurs play in takeover contests. We abstract from differences in attitude towards risk and focus on the explanation most commonly given: difference in information. It is often argued that arbitrageurs have better information about the chance of a successful takeover and purchase shares as long as their forecast of the "correct" security price exceeds the current market price.

[1]Numerous case studies reveal that the increased trading volume is largely due to arbitrage activities. See Harvard Business School case 9-282-065: *Note on Hostile Takeover Bid Defense Strategies.* For specific examples see Harvard Business School (HBS) case 9-285-053: *Gulf Oil Corp—Takeover,* HBS case 9-285-018: *The Diamond Shamrock Tender for Natomas (A)* and D. Commons: *Tender Offer.* On the other hand, it is common knowledge among financial arbitragers that a takeover bid represents one of the best opportunities for them to operate, see Ivan Boesky, *Merger Mania — Arbitrage: Wall Street's Best Kept Money-making Secret.*

[2]See Welles (1981).

[3]See Grinblatt and Titman (1998).

We argue that it is not necessary to assume that risk arbitrageurs have specific knowledge on the takeover fight. Instead, the information advantage can arise endogenously from the choice of a risk arbitrageur to enter the contest. The intuition is quite simple: if the presence of risk arbitrageurs increases the probability of a takeover, then the fact that one risk arbitrageur bought shares is per se relevant to the value of these shares. Therefore, an arbitrageur who has bought shares has an informational advantage: he knows he bought shares. After all, risk arbitrageurs are often quoted saying that a crucial part of their activities is trying to predict what other arbitrageurs will do.[4]

We model the decision of risk arbitrageurs to enter the contest and the way they accumulate shares. The number of arbitrageurs who choose to take positions, the number of shares they buy and the price they pay are determined endogenously in equilibrium.

We start from a company with diffuse ownership, with no large shareholders who can facilitate the takeover. After a bidder has made a tender offer, arbitrageurs decide whether to buy shares. If they succeed in accumulating non-trivial stakes, they become temporary large shareholders. Unlike small shareholders, they tend to sell their shares to the bidder and therefore facilitate the takeover. For this to happen, however, it is necessary to show how they can be successful in accumulating these positions without driving the price up so much that they end up losing money. In other words, we ask ourselves how arbitrageurs can afford to pay a price which is high enough to persuade small shareholders to give up their shares.

The value of the shares depends on the probability that the takeover will take place, and therefore it should be higher the larger is the number of risk arbitrageurs in the market (since they are more likely to tender). Both small shareholders and risk arbitrageurs do not know how many arbitrageurs have entered the contest and update their beliefs looking at the trading volume. However, a risk arbitrageur always has an informational advantage over the small shareholders: he knows that at least he is

[4]In general, risk arbitrageurs talk to a small subset of other arbitrageurs about whether or not they are involved in a specific deal. In the conclusions, we briefly discuss this possibility.

buying shares. This informational advantage guarantees that he is willing to pay a price which is high enough to persuade the small shareholders to sell their shares.

As the trading volume increases, small shareholders think it is more likely that some arbitrageurs are buying shares. Consequently, the share price increases. We show that there exists a range of trading volumes at which risk arbitrageurs buy shares from small shareholders.

As long as the expected profits are strictly positive, more arbitrageurs will choose to buy shares. If too many arbitrageurs are trying to buy shares, however, the price will rise too much and they will not buy any shares. We show that there exists a symmetric equilibrium where each arbitrageur randomizes between entering or not and the takeover has a positive probability to succeed.

While the press has often depicted risk arbitrageurs in an unfavorable way, this paper shows that they can actually increase welfare, facilitating takeovers which increase the value of a company. Moreover, small shareholders, who sell their shares to risk arbitrageurs, do not lose money but they actually appropriate the ex ante surplus, which would be lost if the takeover did not succeed.

In a similar spirit, Kyle and Vila (1991) studies a case in which the bidder buys shares before announcing the takeover. Because of noise trading, the bidder succeeds in hiding at least partially his presence. In our paper, since we focus on post-announcement trading, risk arbitrageurs do not have any initial private information: the informational advantage arises endogenously when they start buying. Moreover, since there is more than one arbitrageur with the same informational advantage, we have to check that they do not compete away their rent.

Larcker and Lys (1987) offers a careful empirical study of risk arbitrageurs in takeovers. Their hypothesis is that risk arbitrageurs are better informed than the market about the takeover success rate. They find that firms purchased by arbitrageurs have an actual success rate higher than the average probability of success implied by market prices. As a result, they can generate substantive positive returns on their portfolio positions. This is compatible with the results of our model, although we argue that the explanation could be different: the risk arbitrageurs may not know ex ante which takeover attempt are more likely to be successful, but it is their presence which increases the probability of success. Larcker and Lys (1987) also shows that the amount of shares arbitrageurs buy is not significantly correlated with their return rate, which is consistent with our result that the limit to the number of shares bought comes from the need of risk arbitrageurs to hide their presence.

Jindra and Walkling (1999) show how the percentage difference between the offer price and the the market price of the shares after the takeover announcement depends on variables which affect the probability of success of a takeover, which is consistent on how the share price is determined in our equilibrium.

Focusing on the role of risk arbitrageurs allows us to explain certain empirical patterns during takeover activity and derive testable implications. A widely observed phenomenon is that, after the takeover announcement, both the stock price and the transaction volume of the target rise tremendously relative to their pre-announcement levels.[5] We find a positive relationship between trading volume and the probability that the takeover is successful. We also find that the more liquid is the target stock, the better risk arbitrageurs can hide their trade and, as a consequence, the higher are their returns if they decide to invest in the deal and their presence in the deal. Similarly, a higher takeover premium increases arbitrageurs interim profits and in equilibrium we should expect a higher number of arbitrageurs investing in the deal. We present the trade-off facing a bidder when choosing the takeover premium and show that it is never optimal either to offer a very low premium or to pay out all the merger surplus.

An implication of this paper is that the well known free-riding problem of Grossman and Hart (1980) is mitigated.[6] The reason is that risk arbitrageurs have the role

[5]Based on a sample of mergers before the 1980, Jensen and Ruback (1983) found in a comprehensive survey that the average jump in share price of the target firm ranges from 17% to 35%. For the 1980's merger wave, Bradley, Desai and Kim (1988) found similar results.

[6]Several other papers have shown how this problem can be mitigated because of various reasons. See, among others, Bagnoli and Lipman (1988), Giammarino and Heinkel (1986), Harrington and Prokop (1993), Holmstrom and Nalebuff (1992), Bebchuk (1989) and Yilmaz (1997). Hirshleifer

of large shareholders, as in Shleifer and Vishny (1986) and Hirshleifer and Titman (1990). The contribution of this paper is to show that, even if at the time of the tender offer there are no large shareholders, there exist equilibria where arbitrageurs enter and buy shares, becoming in this way large shareholders.

We assume that small shareholders take the probability of a takeover for given in order to simplify the analysis. In fact, in this way the behavior of small shareholders is straightforward and we can focus on risk arbitrageurs. However, the intuition remains the same with more strategic players (as long as the probability of the takeover, at the announcement of the tender offer, is less than one): an individual who owns a larger stake will suffer less of the free-riding problem and tender a larger fraction of his shares, thereby facilitating the takeover. The rest of the analysis would then be as above.

Finally, we focus on the case in which risk arbitrageurs buy shares only after the takeover announcement. In the data set of Larcker and Lys (1987), only in three cases the transaction date was prior to the first tender offer. In general, risk arbitrageurs do not attempt to forecast acquisition candidates, but rather to resolve the uncertainty surrounding an announced proposal. The model can be extended to the case in which risk arbitrageurs can take positions also prior to the takeover announcement, speculating on the probability that the announcement will indeed happen. As long as their presence increases the chances of success of a (possible) takeover, risk arbitrageurs have an informational advantage and the same result holds.

Following the introduction, the model is described in detail. Section 3 studies the tendering strategies of risk arbitrageurs, once they have taken position in the target shares. Section 4 studies the choice of risk arbitrageurs to buy shares and Section 5 their decision to enter the contest. Section 6 considers several empirical implications. Finally, the conclusions summarize the results and discuss possible extensions.

(1995) gives an overview of the subject.

2. The Model

In order to focus on the role of arbitrageurs, we assume that at the beginning small shareholders control 100% of the outstanding shares (as in Grossman and Hart (1980)). The model does not consider situations where there are large shareholders, but they could be easily incorporated into the model. The crucial feature is that small shareholders free ride at least partially, so that when the tender offer is made the probability of a successful takeover is less than 1.

At time 0, a bidder announces a cash tender offer of P_T for all shares. If more than 50% shares are tendered, the bidder purchases them all at the price P_T, otherwise all tendered shares are returned.

We assume that P_0 is the initial share price and both P_0 and P_T are observable to all. So is the value improvement per share that the bidder can bring to the firm, ΔP. Naturally, we assume that

$$P_0 + \Delta P \geq P_T \geq P_0.$$

The bidding price is between the *status quo* share price and the potential improved value of the share. In addition, we assume that if the takeover bid proves to be a failure, the stock price goes back to P_0.[7]

At time 1, arbitrageurs decide whether to enter and speculate. At time 2, stock trading takes place — arbitrageurs take positions, hiding among small investors. At the beginning of time 3 arbitrageurs reveal their position, so that their presence and their holdings is common knowledge. Finally, at time 3 all shareholders decide how many shares to tender and the outcome of the takeover is determined.

Let us now look at the players.

The Small Shareholders

[7]The implied assumption is that the occurrence of this takeover bid does not change the probability of new takeover bids and their success. If the stock price falls to a different value than P_0, a similar analysis can still be performed.

A small shareholder controls so few shares that he believes that his decision to tender his shares will have no effect on either the trading price or the outcome of the takeover. We assume that the small shareholders are risk neutral, so that there are no differences in risk preferences between small shareholders and risk arbitrageurs.

The Arbitrageurs

There are N potential arbitrageurs, who can choose to take a position. The arbitrageurs are assumed to be risk neutral. After the takeover announcement, each of them has to decide whether to arbitrage or not in the stock of the target firm. If an arbitrageur A_i decides to arbitrage (i.e. decides to "enter the contest"), he must bear a cost c. Such cost can be interpreted as the cost of collecting information or as the opportunity cost of other investment opportunities, given that risk arbitrageurs, as argued in Shleifer and Vishny (1997), do not have unlimited financial resources. If he decides to enter, the arbitrageur buys a portion δ_i of the total outstanding shares of the firm, where δ_i is endogenously determined. Legally, there is an upper limit $\bar{\delta}$ (which in US is 5%) so that if $\delta_i > \bar{\delta}$, the risk arbitrageur has to declare the amount of shares he owns to comply with Section 13D of the Security Exchange Act.

Let us call n the number of arbitrageurs who enter. In equilibrium, the entry decision of each arbitrageur is endogenized. For the moment, let us call $G(n)$ the distribution of n (with density $g(n)$), which will be endogenously derived in equilibrium. For technical tractability, n will be treated as a real (continuous) number and, likewise, $g(n)$ as a continuous function.

After the trading session is closed, each arbitrageur who purchased shares reveals the fraction of equity he holds. He then makes a decision regarding the portion of shares to be tendered. For A_i, let us define the portion as $\gamma_i \in [0, 1]$.

Noise Traders and the Total Trade Volume

Shares can be bought also by small investors, who exist due to external reasons (such as diversification of their investment portfolio)[8]. Since in this paper we focus

[8] They can also be program or package traders whose decision to trade is based on information uncorrelated to the takeover process.

on symmetric equilibria, we need to assume the presence of noise traders in order to guarantee that the equilibrium in the trading game is not perfectly revealing. However, if we looked instead at the asymmetric equilibria, the presence of noise traders would not be necessary, since the uncertainty about the number of arbitrageurs n would be enough to guarantee that the equilibrium is not fully revealing.

The trading volume from noise traders, ω, is random and independent of both the share price and the demand of arbitrageurs. By definition, the volume ω is non-negative and it is common knowledge that it is distributed uniformly on the interval $[0, 1]$. [9]

Let y be the total trading volume of the shares of the firm, then

$$y = \omega + \sum_{i=1}^{n} \delta_i.$$

The number of arbitrageurs n who entered is unknown to both risk arbitrageurs and small shareholders, but everybody knows that n is distributed according to $G(n)$ (which will be determined in equilibrium) and can observe the trading volume y.

We use the concept of Perfect Bayesian equilibrium and focus on the symmetric equilibria, where each arbitrageur buys and tenders the same proportions, δ and γ, of shares. To determine the equilibria, we solve the game backwards. We start from the tendering game: after n arbitrageurs entered and bought shares, we determine their optimal tendering strategy, which depends on how many other arbitrageurs are around. Then, given their tendering strategy, we look at the trading game. We find the rational expectations equilibria and whether there exists an equilibrium where the risk arbitrageurs buy shares. Finally, we look at the choice of the arbitrageurs whether to enter or not.

[9] We are assuming that noise traders and risk arbitrageurs cannot short sell the shares of the target firm. In reality, risk arbitrageurs usually short sell the bidder's shares in order to hedge. Since risk arbitrageurs are risk neutral in this paper, they have no reason to hedge. In the conclusions, we discuss the possibility to let risk arbitrageurs short sell the target's shares.

3. The Tendering Game

The tendering game is played among the arbitrageurs. Small shareholders stay out of the picture, since they take the probability of a takeover for given and therefore, by the Grossman-Hart (1980) argument, they never tender their shares.

At the beginning of period 3, n arbitrageurs have entered. Each arbitrageur A_i reveals he bought δ_i shares and observes exactly how many other arbitrageurs entered and how many shares they bought. Then, the arbitrageur chooses how many shares to tender, given n and given the strategies of the other risk arbitrageurs.[10]

If the fraction of equity in the hands of the risk arbitrageurs is less than 50%—that is, if $\sum_{i=1}^{n} \delta_i < 0.5$—then there exist multiple equilibria, since the tendering strategy of the risk arbitrageurs is irrelevant: the takeover will fail in any case and the price of the shares will return to P_0.[11]

If instead $\sum_{i=1}^{n} \delta_i \geq 0.5$, in equilibrium the risk arbitrageurs tender exactly 50% of the shares. There are multiple equilibria: in all of them each arbitrageur tenders a fraction γ_i such that $\sum_{i=1}^{n} \gamma_i \delta_i = 0.5$. The unique symmetric equilibrium is given in the following proposition.

Proposition 1: *If $\sum_{i=1}^{n} \delta_i \geq 0.5$, then for each n there exists a unique symmetric*

[10] It is not necessary for the result of the paper that arbitrageurs reveal their presence. In a previous version of this paper, Cornelli and Li (1998), arbitrageurs did not reveal their presence. They chose the fraction of shares to tender, given their own beliefs on how many other arbitrageurs had bought shares, based on the observed volume y. The resulting equilibria were the same as in this version. Assuming that risk arbitrageurs disclose their presence has two main advantages. First of all, it simplifies the derivation of empirical implications in Section 6. Second, in the previous version a takeover could fail even if enough risk arbitrageurs were around, just because they did not tender enough shares. This feature may not be very realistic, since the bidder could repeat its bid until the offer is successful. Any model that allows renegotiation or repeated bids will not have that feature.

[11] We rule out the possibility of collusion between arbitrageurs and acquirer. If the acquirer colludes with a few arbitrageurs, he becomes a large shareholder in the sense of Shleifer and Vishny (1986), although this can be illegal. The rest of the arbitrageurs still have to play a tendering game. Thus, the takeover can be successful with less than 50% shares tendered. In this sense, the model can be expanded to cover collusion.

equilibrium where each risk arbitrageur tenders a fraction γ of its shares such that

$$\gamma(n) = \frac{0.5}{\sum_{i=1}^{n} \delta_i}$$

If $\delta_i = \delta$ for all risk arbitrageurs then

$$\gamma(n) = \frac{0.5}{n\delta}$$

Proof: The best response of a risk arbitrageur A_i, who owns δ_i shares, given that there are $n-1$ other arbitrageurs with δ shares who are going to tender a fraction γ of their shares, is given by

$$\gamma_i = \frac{0.5 - \gamma\delta(n-1)}{\delta_i}$$

if $0 \leq \gamma_i \leq 1$ (otherwise $\gamma_i = 0$). From this it follows immediately that the unique symmetric equilibrium is the one given in the proposition.

¶Q.E.D.

Notice that $\gamma(n)$ is a decreasing function of both n and δ: the more arbitrageurs entered (or the more shares each of them owns), the less each of them has to contribute for the takeover to be successful.

4. The accumulation of shares

Given the symmetric equilibrium of the last subgame, in which risk arbitrageurs tender a positive fraction of their shares, we now look at how risk arbitrageurs buy shares. The players in this stage are the small shareholders—who own all the shares at the beginning of the game and may choose to sell them—the risk arbitrageurs and the noise traders, who buy shares. We want to show that although the price increases when arbitrageurs buy shares, they do succeed in hiding, at least partially, their presence, so that it is profitable for them to buy shares.

We use the concept of rational expectation equilibrium. For each realization of the random variables, n and ω, we characterize an equilibrium where the total trading volume is $y = n\delta + \omega$, and the share price is P. The equilibrium is such that (1) given the volume y, risk arbitrageurs and small shareholders are maximizing their utility; (2) the beliefs τ and τ_i^a are consistent with the players' strategies.

4.1. The Post-announcement Share Price. The price is determined as follows. Since we are assuming that the small shareholders own 100% of the shares, if the total demand of shares (by risk arbitrageurs and noise traders) is less than 100%, then the market price P equals the reservation price of the small shareholders, so that each small shareholder is indifferent between selling the share and holding it (and waiting to see if the takeover takes place):

$$P = \tau(P_0 + \Delta P) + (1 - \tau)P_0 = P_0 + \tau\Delta P \tag{1}$$

where τ is the probability of success of the takeover bid, *as perceived by small shareholders.*

The reservation price V^a of a risk arbitrageur is given by

$$V^a = \tau_i^a[\gamma^e P_T + (1 - \gamma^e)(P_0 + \Delta P)] + (1 - \tau_i^a)P_0 \tag{2}$$

where γ^e is the number of shares the arbitrageur expects to tender and τ_i^a is the success rate *calculated by the arbitrageur A_i*, who has bought δ_i shares. Note that if $\tau_i^a = \tau$ then $V^a < P$ and there is no trade. However, if τ_i^a is sufficiently larger than τ, then there is room for trade between arbitrageurs and small shareholders.

If instead the demand is above 100%, then P in (1) does not clear the market and the competition between risk arbitrageurs in order to obtain the shares will drive the price up to their reservation price, V^a.

Both τ and τ_i^a are endogenous and depend on the transaction volume y, which conveys new information about the number of arbitrageurs and their positions. We have therefore first to look at the updating process of both arbitrageurs and small shareholders, after observing y, and derive their *posterior* probability that the takeover will be successful.

4.2. Updated Beliefs on the Probability of Success of the Takeover. Let us start by computing the probability τ_i^a that the takeover will succeed, as perceived by an arbitrageur A_i who bought δ_i shares and observed a volume y, given that all other arbitrageurs have bought δ shares.

Since small shareholders do not tender, when $y < 0.5$ it is clear that not enough shares are in the hands of arbitrageurs and ${\tau^a}_i = 0$. If $y \geq 0.5$ and no arbitrageur has declared to have $\bar{\delta}$ or more shares[12],

$$\begin{aligned} {\tau^a}_i = {\tau^a}_i(y, \delta, \delta_i) &= Prob\ [\ (n-1)\delta + \delta_i \geq 0.5\ \ |\ y - \delta_i\ ] \\ &= Prob\ [\ n - 1 \geq \frac{0.5 - \delta_i}{\delta}\ \ |\ y - \delta_i\ ] \end{aligned}$$

where $y - \delta_i$ is the arbitrageur's observation of the total transaction volume, excluding his own. A_i has to compute the conditional probability distribution of the number of arbitrageurs other than himself. Let $d\ (.)$ denote the density, then

$$\begin{aligned} d\ (n-1 = s\ \ |\ y - \delta_i\ ) &= \frac{d\ (n-1 = s, \omega + (n-1)\delta = y - \delta_i\ )}{d\ (\omega + (n-1)\delta = y - \delta_i\ )} \\ &= \frac{d\ (n-1 = s, \omega = y - s\delta - \delta_i\ )}{d\ [\omega + (n-1)\delta = y - \delta_i\ ]}. \end{aligned}$$

Under the assumption that, ex ante, n and ω are independent and that the coming of arbitrageurs is mutually independent (which will be shown to be true in equilibrium), it follows

$$d\ (n-1 = s, \omega = y - s\delta - \delta_i) = g(s+1)f(y - s\delta - \delta_i)$$

where $f(\cdot)$ is the density of the noise traders distribution and

$$Prob\ [\omega + (n-1)\delta = y - \delta_i\ ] = \int_0^{\frac{y-\delta_i}{\delta}} g(t+1)f[y - t\delta - \delta_i]dt$$

[12] If some other arbitrageurs have filed 13D, A_i will take that into account in computing τ_i^a.

Therefore, we have

$$Prob\ [\ n-1 \geq \frac{0.5-\delta_i}{\delta},\ \omega+(n-1)\delta = y-\delta_i\ ] = \int_{\frac{0.5-\delta_i}{\delta}}^{\frac{y-\delta_i}{\delta}} g(s+1)f[y-\delta_i-s\delta]ds \quad (3)$$

and consequently, given that noise traders are uniformly distributed,

$$\tau^a{}_i = \frac{\int_{\frac{0.5-\delta_i}{\delta}}^{\frac{y-\delta_i}{\delta}} g(s+1)ds}{\int_0^{\frac{y-\delta_i}{\delta}} g(t+1)dt} \quad (4)$$

In a symmetric equilibrium, $\delta_i = \delta$ and

$$\tau^a = \frac{\int_{\frac{0.5}{\delta}}^{\frac{y}{\delta}} g(s)ds}{\int_1^{\frac{y}{\delta}} g(t)dt} \quad (5)$$

The updating of the small shareholders is different, since they may think that perhaps all volume y is due to noise traders. If we repeat the calculation we did for τ_i^a for the case of the small shareholders, the *posterior* probability of success of the takeover (conditional on y) is[13]

$$\tau = \frac{\int_{\frac{0.5}{\delta}}^{\frac{y}{\delta}} g(s)ds}{\int_0^{\frac{y}{\delta}} g(t)dt}. \quad (6)$$

Even if in general we cannot compare τ and τ_i^a, in the symmetric case it is easy to see that $\tau^a > \tau$. However, if one arbitrageur A_i files 13D and declares $\bar{\delta}$, then $\tau = \tau_i^a$ always.[14]

Therefore, the probability of success of takeover as assessed by the risk arbitrageurs can be higher than the probability assessed by the small shareholders. If the difference

[13]Assuming no arbitrageur filed 13D.

[14]Even if the arbitrageur i is planning to buy more than $\bar{\delta}$ shares, the small shareholders are perfectly able to compute his optimal δ_i, once they know he is buying shares, and therefore there is no asymmetry of information.

is sufficiently high, $V^a(y) > P(y)$ and the arbitrageurs are willing to buy shares at the price P, which makes the small shareholders indifferent.

4.3. The equilibrium. Since both τ and τ_i^a depend on y, in equilibrium the beliefs have to be consistent with the strategies. To derive the equilibrium, we proceed in the following way: for given beliefs τ and τ^a, we derive the optimal choice of δ_i. Given this strategy, we then find the beliefs which are consistent in equilibrium.

In what follows, we show that, given the beliefs, the optimal choice for a risk arbitrageur is always either to buy no shares at all, or to buy up to $\bar{\delta}$ shares. There always exists an equilibrium where arbitrageurs buy no shares at all and the takeover will never succeed (since one arbitrageur alone, by deviating and buying $\bar{\delta}$, cannot make the takeover succeed). We want to find out whether there exist an equilibrium where arbitrageurs buy $\bar{\delta}$ shares and the takeover has a positive probability to be successful. In the next proposition we characterize the symmetric equilibria. The intuition is immediately after.

PROPOSITION 2: *Given that n arbitrageurs entered the contest and the noise trade is ω, then*

a) *If $n\bar{\delta}+\omega < 0.5$, there is a unique equilibrium where risk arbitrageurs buy no shares and the trading volume is $y = \omega < 0.5$. The takeover fails.*

b) *If $0.5 \leq n\bar{\delta} + \omega \leq 1$, then in equilibrium risk arbitrageurs buy either $\bar{\delta}$ or 0 shares. If P_T is not too low, then at least for values of y close to 50% there exists an equilibrium where risk arbitrageurs buy $\bar{\delta}$ shares and the trading volume is $y = n\bar{\delta}+\omega$. The probability that the takeover is successful is strictly positive.*

c) *If $n\bar{\delta} + \omega > 1$ risk arbitrageurs buy no shares and the takeover fails.*

PROOF: See Appendix I.

The intuition of the proposition and the logic of the proof are the following. If the trading volume is less than 50%, everybody knows that the takeover is going to fail and the share price is P_0. In equilibrium risk arbitrageurs are indifferent between buying and not buying shares at the price P_0. Since they are usually not interested in long term positions, we assume that in equilibrium they buy no shares at all. The expectation that the takeover will fail is therefore correct. This gives point (a) of the proposition.

If the trading volume is larger than 50%, the takeover could be successful. We have therefore to check that in equilibrium risk arbitrageurs do buy shares. To find the equilibrium we first show that, given the beliefs τ and τ_i^a, a risk arbitrageur always wants to buy either no shares at all or as many shares as possible without revealing his presence ($\bar{\delta}$). Moreover, a risk arbitrageur will never want to buy more than $\bar{\delta}$ shares: if he does, he will have to declare his transaction, $\tau = \tau_i^a$ and his expected profits become negative.[15] Therefore the arbitrageur will never buy more than $\bar{\delta}$ shares.[16]

Once we have restricted the choice to 0 or $\bar{\delta}$ shares, we can find out whether for some values of n and ω there exists an equilibrium where arbitrageurs buy $\bar{\delta}$ shares. In order to do this, we compute the expected profits of a risk arbitrageur buying $\bar{\delta}$ shares, given n, ω and the beliefs τ and τ_i^a corresponding to $y = n\bar{\delta} + \omega$. If, when buying $\bar{\delta}$ shares, risk arbitrageurs obtain positive expected profits, this is an equilibrium. If instead the expected profits are negative, then the only equilibrium corresponding to that n and ω is the one in which risk arbitrageurs buy no shares at all.

[15] Holderness and Sheehan (1985) study the price reaction to announcements that some "corporate raiders" had acquired stock in a specific firm. They show that there are positive abnormal returns at the announcement that the investors bought stock in a firm which was target of a reorganization. Unfortunately, they do not distinguish between the case in which the investor was the acquirer and the case in which a third party was the acquirer (although both cases are in the sample).

[16] In the reality, there is a delay between when $\bar{\delta}$ shares are bought and when 13D is filed. Consequently, risk arbitrageurs—although in general try to avoid it—may buy more than 5%. In this event, Larcker and Lys (1987) show that arbitrageurs buy most of the shares at one date, after which they reveal themselves. This cannot happen in this model, where all trade is happening in a one-shot period. However, one may see the limit $\bar{\delta}$ as the amount of shares arbitrageurs can buy in one day, disguising themselves among small trades. The 5% limit guarantees that after that day they will have to reveal themselves and therefore they will not buy any more shares. The finding in Larcker and Lys (1987) that the number of shares bought is not related with expected returns seems to confirm that there is an exogenous limit to the amount of shares an arbitrageur can buy without revealing his presence. The present model is therefore trying to capture these features without explicitly modelling trade over time.

We show in Appendix I that for low values of volume y, profits are either negative or positive depending on P_T. For P_T sufficiently high (but bounded away from $P_0 + \Delta P$) the profits are positive and increasing and it is therefore an equilibrium to buy $\bar{\delta}$ shares. As y increases, the probability of success of the takeover increases, but this has two effects on the profits of the arbitrageurs. First of all, more arbitrageurs are likely to be in position and this promises a greater chance of success of the takeover. On the other hand, also the price at which arbitrageurs can buy shares increases. For high levels of volume, the informational advantage of the risk arbitrageur becomes thinner, because more arbitrageurs are expected to be present, so the price of the shares increases faster than their benefit. In general, the net effect of an increase in y on the expected (interim) profits $\pi(y)$ can be either positive or negative. In the Appendix we show that they are in general non-monotonic and could become negative. In Figure 1 we show two possible configurations of the (interim) expected profits of risk arbitrageurs if they bought $\bar{\delta}$ shares: if $y < 0.5$ the profits are 0. Starting from $y = 0.5$ profits are positive and increasing (so this would be a case where P_T is sufficiently high). As y increases, profits first increase, but then begin to decrease and eventually may become negative. We define $\bar{y}$ as the level of volume at which expected profits are equal to 0. If profits never become negative, $\bar{y}$ is equal to one. All the interval where profits are positive corresponds to values of n and ω for which there exists an equilibrium where risk arbitrageurs buy $\bar{\delta}$ shares. In Appendix I we also show that if P_T is sufficiently high expected profits are always positive for the entire range $0.5 \leq y \leq 1$. This concludes case (b) of Proposition 3.

Finally, if $n\bar{\delta} + \omega > 1$, when risk arbitrageurs demand $\bar{\delta}$ shares the demand exceeds supply and the price rises up to their reservation price. In equilibrium, risk arbitrageurs buy no shares and the takeover fails. This is case (c) of Proposition 3.

We have therefore shown that if P_T is not too low there exist trading volumes at which it is an equilibrium for the arbitrageurs to buy shares.[17]

[17]Since arbitrageurs can only buy shares without being recognized up to 5%, we have considered this limit small enough for risk arbitrageurs to be price takers and therefore used the concept of noisy rational expectation equilibria. Alternatively, if one thinks arbitrageurs are not price takers, one could modify the trading game as in Kyle (1989), where risk arbitrageurs submit demand functions

5. The decision to enter

To complete the equilibrium, we still have to endogenize the arbitrageurs' decision to enter. In the previous stage arbitrageurs always had the option to buy no shares. As a result, the expected profits of an arbitrageur who has chosen to enter are always non-negative. However, when the announcement of the takeover bid is made, some of the arbitrageurs are engaged in other operations and their financial resources—including their bounded debt capacity—are tied up. Shleifer and Vishny (1997) show that arbitrageurs do not have unlimited capital they can invest and this is crucial in determining their strategy. Jindra and Walkling (1999) argue that there is a cost to arbitrageurs in tying up the funds during the acquisition process and find empirical support to the hypothesis that tender offers with longer time horizon are more costly to arbitrageurs. In this paper we abstract from the temporal dimension, but try to depict an analogous situation, where arbitrageurs have to free some resources and this is costly for them. This cost may also be interpreted as a lost opportunity to invest in different deals, because the supply of funds is inelastic. As we will argue later, this is consistent with the finding of Mitchell and Pulvino (1999) that in the period between 1963 and 1987 arbitrageurs returns are higher when takeover activity is higher, which they justify arguing that the opportunity costs of investing in other deals is higher. Moreover, even before they start buying, arbitrageurs must collect some information, which is costly. We therefore assume that arbitrageurs decide whether to enter or not, where entry has a cost $c > 0$, which can be arbitrarily small.[18]

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and therefore take into account the effect that their demand has on the price. As a result, arbitrageurs may buy less than $\bar{\delta}$ shares. Notice, however, that the incentive to buy less shares is lower than in Kyle (1984, 1989), since here, as δ_i decreases, also the informational advantage of the risk arbitrageur decreases. Finally, it is also possible to consider a model as in Kyle (1985) and Giammarino, Heinkel and Hollifield (1994), where risk arbitrageurs demand an amount δ_i independent from the volume and the market makers set the price at the reservation value of the small shareholders based upon total trading volume. In this last case, for some level of volume arbitrageurs' profits are negative, so it is not necessary to introduce a cost of entry in the next section.

[18] Arbitrageurs never earned negative (interim) expected profits in the trading stage, since trading was modelled as a one-shot game. In a dynamic game, arbitrageurs may start buying shares before realizing that too few (or too many) arbitrageurs are likely to be around. As a result, they may have bought shares at a premium and they realize a loss. The cost of entry we impose may be seen also as an attempt to capture that effect.

We focus on the symmetric equilibria. It is clear that the case in which no arbitrageur enters is always an equilibrium. In fact, one arbitrageur alone, who deviates and enters, can at most buy $\bar{\delta}$ shares without revealing his presence, and that is not enough for the takeover to succeed. We want to show, however, that there exists another equilibrium where the takeover can succeed. First of all, there cannot exist an equilibrium where all arbitrageurs enter with probability one. In this case, the small shareholders would know that there are exactly N arbitrageurs buying with probability one. Therefore the arbitrageurs would have no information advantage at all and there would be no room for trade. The only other possibility for a symmetric equilibrium is a mixed strategy equilibrium where each arbitrageur enters with a probability p which makes him indifferent between entering and not entering (i.e. such that the expected profits from entering are equal to c).

Given p, the probability that exactly n out of the N potential arbitrageurs entered is

$$g(n) = \begin{pmatrix} N \\ n \end{pmatrix} p^n (1-p)^{N-n} \tag{7}$$

Each arbitrageur is able to forecast what is the equilibrium corresponding to each realization of ω and n. If the equilibrium implies they buy 0 shares, then entry implies a loss equal to c. The ex-ante expected profits can therefore be written as

$$\Pi(p, N, c) \equiv E_{n,\omega}\left[\pi(n,\omega)\right] \tag{8}$$

where $\pi(n,\omega)$ are the ex post profits for each realization of n and ω.

Proposition 3: *If the cost c is not too high and N not too low, there always exist two symmetric equilibria where each arbitrageur enters with probability p such that $0 < p < 1$ and the ex-ante expected profits in (8) are equal to 0. This is also an equilibrium of the general game.*

Proof: See Appendix II.

The condition that c is not too high makes it more likely that arbitrageurs can break-even after incurring the cost c and the condition on N guarantees that there are potentially enough risk arbitrageurs for the takeover to be successful.

In equilibrium, each of the N arbitrageurs randomizes between entering and not entering the contest with a probability p, where p is endogenously determined so that each risk arbitrageur is indifferent between entering and not entering. Out of the N potential arbitrageurs, n will enter the contest and invest in shares of the target company. Depending on the realization of n and of ω, arbitrageurs either buy no shares (and therefore bear a loss c) or buy $\bar{\delta}$ shares. In the first case the takeover will be for sure unsuccessful, while in the second case the takeover has a positive probability to be successful: it is the probability that $0.5 \leq \bar{\delta} n \leq \bar{y} - \omega$ (where $\bar{y}$ is the maximum level of volume for which profits are non-negative).

If S is the ex-ante probability of success of the takeover, in equilibrium the ex-ante expected profits of the small shareholders is $S\Delta P + P_0$. The reason is that each of them is indifferent between selling and holding and, if they hold, they get $\Delta P + P_0$ if the takeover is successful and P_0 otherwise.

6. Empirical Implications

In this section, we derive several empirical implications of the model. In the previous section we showed that there are two equilibria where arbitrageurs randomize between entering and not entering with two different probabilities p^*. Since the two equilibria had similar characteristics, we did not distinguish among them. In this section, however, to derive empirical implications, we focus only on one equilibrium, the one where risk arbitrageurs randomize with the highest probability p_2^*.

We focus on that equilibrium since the other is not stable, as it is not immune to deviations from coalitions. The intuition of this can be seen by looking at Figure 3. If all arbitrageurs are randomizing with probability p_1^*, one arbitrageur alone obtains zero ex-ante expected profits if he enters. However, if two arbitrageurs agree between themselves to enter with probability 1, they increase the probability that the takeover will succeed and obtain strictly positive ex-ante expected profits. Since arbitrageurs usually talk among small groups, this equilibrium is not stable to deviations by small groups of arbitrageurs. If instead all arbitrageurs are randomizing with probability p_2^*, two arbitrageurs cannot ensure themselves, by deviating, positive ex-ante expected profits. In fact, if they agree to enter with probability 1, ex-ante expected profits are negative. If instead they agree not to enter then their profits are equal to zero. Therefore a coalition of two risk arbitrageurs does not disrupt the equilibrium. For this reason, we focus on this second equilibrium.

6.1. Volume and Price. It can be easily computed that $\frac{d\tau_i^a}{dy} > 0$ and $\frac{d\tau}{dy} > 0$ for $0.5 \leq y \leq \bar{y}$. Therefore, if $\bar{y} = 1$ the probability of success of the takeover (both the one perceived by the arbitrageurs and the one perceived by other shareholders) increases with y and is equal to zero if $y < 50\%$.[19] If instead $\bar{y} < 1$, then the probability of success of takeover becomes 0 also for very high volumes ($y > \bar{y}$).

Moreover, if $0.5 \leq y \leq \bar{y}$, the higher is the volume the higher is the price of the shares in the market ($P_0 + \Delta P \tau$).

6.2. Number of Arbitrageurs and their Profits. In Appendix II we derived the function $\bar{\pi}(n) \equiv E_\omega [\pi(n, \omega) \mid n]$, which gives the expected profits per arbitrageur, given the number n of risk arbitrageurs who took positions in the deal. This function is represented in Figure 2. If there are few risk arbitrageurs (less than $\frac{0.5}{\delta}$) the takeover cannot succeed and expected profits are negative, since the arbitrageur bears the cost of entry plus he overpaid for the shares. When n reaches $\frac{0.5}{\delta}$, the profits become positive, because the takeover now has a positive chance to be successful. As n increases, however, expected profits decrease, because of two effects. First, as n increases the expected volume increases and the share price increases. Second, for high levels of y the profits $\pi(y)$ may become negative again (and for $y > 1$ they surely are negative): in those cases the arbitrageurs do not buy any shares and the takeover will not be successful. This again reduces expected profits.

[19]This is due to the fact that we assumed that initially there are no large shareholders. If we relax this assumption, the takeover may succeed also for lower trading volumes.

Therefore, if $n < \frac{0.5}{\delta}$ the takeover never succeeds. If instead $\frac{0.5}{\delta} \leq n \leq \frac{\bar{y}}{\delta}$ the probability of a takeover is equal to $\int_0^{\bar{y}-n\bar{\delta}} 1f(\omega)d\omega = \bar{y} - n\bar{\delta}$ and is decreasing with n. For higher values of n the probability is again zero. The intuition is quite simple: if not enough arbitrageurs are around the takeover has zero chances to be successful. Once the minimum number is reached, however, the probability of a takeover decreases with n since competition among them may drive up the share price so much that they decide not to buy shares at all.

Moreover, ex-ante arbitrageurs profits are higher the higher is the cost c. This derives immediately from the fact that in equilibrium ex-ante expected profits (net of the cost c) are equal to 0. This result is compatible with Mitchell and Pulvino (1999): they find that, for the period between 1963 and 1987, arbitrageurs' returns are higher when takeover activity (measured as the ratio between the sum, across all deals, of the target firms' equity values divided by the total value of all equities traded on the NYSE, AMEX and NASDAQ) is higher. When the takeover activity is higher more deals are available. If we interpret the cost of entry c as the opportunity cost of investing in one deal, then if more alternative deals are around c is higher, which means the returns of those who entered must be higher.

6.3. Shares Liquidity. We assumed that noise trade volume was distributed uniformly on $[0, 1]$. In reality, some companies' shares are traded more frequently than other ones. When shares are traded more often, risk arbitrageurs can hide their trade more easily.[20] We can then look at the implications of a different level of liquidity.

When shares are less liquid, the trading volume we would normally observe, in absence of arbitrageurs' activity, is lower. We therefore assume that the less liquid the shares are, the lower is the maximum trading volume which can be observed (when no arbitrageurs are around). Therefore, the noise trading volume has a maximum

[20]Mitchell and Pulvino (1999), when simulating a risk arbitrage strategy, impose a constraint which limits the investments in illiquid securities. They do so by restricting the amount invested in any single deal such that the price impact on both the target and acquirer's stock is less than 5%. This is consistent with the analysis we conduct in this section, where we claim that the main effect of a lower liquidity is a larger impact of risk arbitrageurs' activity on the price.

value which is now less than one and it is distributed uniformly in the interval $[0, \bar{\omega}]$, with $0.5 < \bar{\omega} \leq 1$. We then study how the equilibrium changes as $\bar{\omega}$ decreases. Using (3) and the fact that $f(y - s\delta) = \frac{1}{\bar{\omega}}$ if $y - s\delta \leq \bar{\omega}$ and 0 otherwise, we obtain

$$\tau = \frac{\int_{\frac{0.5}{\delta}}^{\frac{y}{\delta}} g(s)ds}{\int_{\frac{y-\bar{\omega}}{\delta}}^{\frac{y}{\delta}} g(t)dt}. \tag{9}$$

Moreover, if $y \geq \bar{\omega} + \delta$, τ^a is equal to τ while if $y < \bar{\omega} + \delta$

$$\tau^a = \frac{\int_{\frac{0.5}{\delta}}^{\frac{y}{\delta}} g(s)ds}{\int_{1}^{\frac{y}{\delta}} g(t)dt}. \tag{10}$$

The intuition is the following. If $y > \bar{\omega} + \delta$, everybody knows that there is at least one risk arbitrageur, therefore the arbitrageur's advantage disappears. If instead $y < \bar{\omega} + \delta$, the risk arbitrageur still has an advantage.[21] As the liquidity decreases, the risk arbitrageur can hide less well: in fact, the difference $\tau^a - \tau$ decreases as $\bar{\omega}$ decreases. As a result, when the liquidity of the target stock decreases, the (interim) expected profits of the risk arbitrageurs decrease. This can be summarized in the following remark:

Remark 1: The more liquid the shares are, the higher the expected profits of the arbitrageurs who bought shares.

However, we know that ex ante profits in equilibrium must be equal to 0, therefore the equilibrium p_2^* must change. In the Appendix we prove that p_2^* increases, which gives us the following remark:

Remark 2: The more liquid the shares are, the higher is the expected number of arbitrageurs who invest in the deal.

Proof: See Appendix III.

[21] Note that for $y > \bar{\omega}$ it is already clear that at least one risk arbitrageur is buying shares. However, the risk arbitrageur knows that he is in and this still constitutes an advantage.

6.4. The takeover premium. Until now we took the takeover premium P_T for given. We now look at how the takeover premium affects the number and profits of the arbitrageurs and study the choice of P_T by the bidder and show that the optimal P_T is always an interior solution.

Remark 1. The (interim) expected profits of the risk arbitrageurs, $\pi(y)$, increase with P_T.

Proof: This is easy to see, since $\frac{d\pi}{dP_T} = \tau^a \gamma^e \delta(y) \geq 0$ and the inequality is strict when $\delta(y) = \bar{\delta}$. Moreover, the interim expected profits are positive over a larger range of values of y, since by implicit function theorem $\frac{d\bar{y}}{dP_T} > 0$.

Remark 2. The higher is the takeover premium P_T, the higher is the expected number of risk arbitrageurs who invest in the deal.

Proof: We start by looking at how $\bar{\pi}(n)$ (derived in Appendix II and represented in Figure 2) changes as P_T increases. If $n < \frac{0.5}{\bar{\delta}}$, assume first that $\bar{y} < 1$. Then as P_T increases, $\bar{y}$ increases which implies that $E(\tau)$ increases (i.e. the share price that the risk arbitrageur expects to pay increases) and as a consequence $\bar{\pi}(n)$ decreases. If instead $\bar{y} = 1$ then $\bar{\pi}(n)$ remains constant. If $\frac{0.5}{\bar{\delta}} \leq n \leq \frac{\bar{y}}{\bar{\delta}}$, $\bar{\pi}(n)$ increases (since $\pi(y)$ increases).

Therefore, if $\bar{y} = 1$ the ex-ante expected profits have increased and the equilibrium p_2^* must increase so that they are again equal to 0. If instead $\bar{y} < 1$, then the sign is not obvious. However, this is exactly the same situation we analyzed in Appendix III for shares liquidity, so we can use the same logic and show that p_2^* increases. ¶Q.E.D.

Finally, let us look at the bidder's choice of P_T. The bidder chooses P_T in order to maximize:

$$\begin{aligned}
\mathrm{MAX}_{P_T} \quad & (P_0 + \Delta P - P_T) E\left[\bar{\delta} n \mid \delta(y) n > 0.5\right] \mathrm{Prob}\left\{\delta(y) n > 0.5\right\} = \qquad (11)\\
= \quad & (P_0 + \Delta P - P_T)\bar{\delta} \int_{\frac{0.5}{\bar{\delta}}}^{\frac{\bar{y}}{\bar{\delta}}} \int_0^{\bar{y} - \bar{\delta} n} n g(n \mid P_T) d\omega dn = \\
= \quad & (P_0 + \Delta P - P_T)\bar{\delta} \int_{\frac{0.5}{\bar{\delta}}}^{\frac{\bar{y}}{\bar{\delta}}} (\bar{y} - \bar{\delta} n) n g(n \mid P_T) dn
\end{aligned}$$

The first order condition is:

$$
\begin{aligned}
- &\quad \bar{\delta}\int_{\frac{0.5}{\delta}}^{\frac{\bar{y}}{\delta}} (\bar{y}-\bar{\delta}n)ng(n \mid P_T)dn + (P_0+\Delta P - P_T)\bar{\delta}\frac{d\bar{y}}{dP_T}\int_{\frac{0.5}{\delta}}^{\frac{\bar{y}}{\delta}} ng(n \mid P_T)dn + \quad (12)\\
+ &\quad (P_0+\Delta P-P_T)\bar{\delta}\int_{\frac{0.5}{\delta}}^{\frac{\bar{y}}{\delta}} (\bar{y}-\bar{\delta}n)n\frac{\partial g(n)}{\partial p}\frac{dp}{dP_T}dn
\end{aligned}
$$

the first term represents the direct cost of increasing the takeover premium (the bidder has to pay more for the shares) and is negative. The second term is positive and reflects the fact that when P_T increases, profits are positive on a larger range ($\bar{y}$ increases) and this increases the probability of success of a takeover. The last term is the effect that a change in P_T has on the distribution $g(n)$; in other words, this reflects the change in p^* discussed above. This last term can be positive or negative.

The bidder will never choose a $P_T < \underline{P}$ (where $\underline{P}$ was defined in Appendix I as the minimum takeover premium for $\pi(y)$ to be positive for at least some y) since in that case the probability of success of the takeover is zero. We now show that it is never optimal for the bidder to set P_T equal to either $\underline{P}$ or $P_0 + \Delta P$. In other words, the bidder always offer a strictly positive takeover premium, but still keeps a positive surplus from the takeover.

If P_T is close to $P_0+\Delta P$ (all the takeover premium is paid out) the last two terms disappear and the first order condition is strictly negative: the bidder has an incentive to reduce P_T and keep some of the surplus created by the takeover. If instead P_T is very close to $\underline{P}$ this implies that $\bar{y}$ is close to 0.5: as $\bar{y} \to 0.5$ all three terms go to zero. However, if we define V as the first order condition in (12), it can be easily computed that $\frac{\partial V}{\partial P_T}$, at $\bar{y} = 0.5$, is equal to 0 while $\frac{\partial V}{\partial \bar{y}} = (P_0 + \Delta P - P_T)\frac{\bar{y}}{\delta^2}\frac{d\bar{y}}{dP_T}g(\frac{\bar{y}}{\delta})$ which is positive when $\bar{y} = 0.5$. Therefore at $P_T = \underline{P}$ (i.e. $\bar{y} = 0.5$) the objective function in (11) increases when P_T increases (through the increase in $\bar{y}$) and it is never optimal to set $P_T = \underline{P}$. The bidder has an incentive to increase P_T since this increases the probability that the arbitrageurs actually buy shares and the takeover is successful.

7. Concluding remarks

We have provided an explanation of why arbitrageurs have an incentive to enter the market of corporate control and why in so doing they do not drive the price up until the returns are 0 (which would discourage them from bearing the cost of entry). We have also characterized the equilibrium in which arbitrageurs enter, buy shares and tender a fraction of them. Such characterization has allowed us to derive some relationships that link the trading volume and the number of arbitrageurs buying shares to the success rate of the takeover and the market price. Moreover, the role of the arbitrageurs in determining the success of the takeover influences the acquirer's choice of the takeover premium.

The model can be extended to take into account other characteristics. One possibility would be to allow noise traders and risk arbitrageurs to short sales. Risk arbitrageurs may have an incentive to short sell if their assessment of the probability that the takeover will be successful is lower than the assessment of the small shareholders. Therefore, in the cases where we found it was optimal for arbitrageurs not to buy shares, it may become optimal to short sell shares.

Another interesting extension would be to allow risk arbitrageurs to communicate between themselves. Usually risk arbitrageurs belong to small "clubs" and they talk only to arbitrageurs in the same club. In this case they will be informed not only of their own presence, but also of the presence of everybody in the same club. We could model this behavior by assuming that risk arbitrageurs in the same club commit, before randomizing, to inform each other if they choose to enter the contest. In general, it will be incentive compatible for a risk arbitrageur who entered to tell the truth. If the agreement is among two people only, their ex-ante expected profits increase, so that they will choose to enter more often. However, as the number of members of each club increases, the expected number of arbitrageurs entering increases, until it starts having an adverse effect. As a result, there is an optimal size of the club. An exhaustive treatment of this aspect is however beyond the scope of this paper.

Finally, this paper allows to study the role of the legal limit to the number of shares an arbitrageur can buy without disclosing his presence (such as the 5% limit according to Section 13D of the Security Exchange Act). This limit is usually interpreted as an obstacle to takeovers, while we show that it can actually favor takeovers. In fact, it reduces competition among risk arbitrageurs. It would therefore be interesting to study how different limits across countries influence the takeover activity.

APPENDIX

Appendix I. Proof of Proposition 2

Let us first look at the choice of δ_i by an arbitrageur, for given beliefs τ and τ_i^a and given that any risk arbitrageur who entered has bought δ shares. The arbitrageur chooses δ_i in order to maximize

$$MAX_{\delta_i} \delta_i \{[\Delta P - \gamma^e (P_0 + \Delta P - P_T)]\ \tau^a{}_i + P_0 - P_1\} \tag{A1}$$

where P_1 is the price paid, which is given by (1) if $n\bar{\delta} + \omega \leq 1$ and by (2) if $n\bar{\delta} + \omega > 1$ and γ^e is the fraction of shares the arbitrageur expects to tender:

$$\gamma^e = E\left[\gamma(n) \mid y - \delta_i\right].$$

Moreover, define

$$P_{\tau_i} \equiv [\Delta P - \gamma^e (P_0 + \Delta P - P_T)]\, \tau_i^a$$

If the total demand $n\bar{\delta} + \omega > 1$, P_1 is equal to the reservation price V^a and the risk arbitrageurs will not buy any shares. If the total demand $n\bar{\delta} + \omega \leq 1$, the objective function becomes

$$MAX_{\delta_i} \delta_i \left[P_{\tau_i} - \Delta P \tau\right] \tag{A2}$$

the first order conditions are

$$P_{\tau_i} - \Delta P \tau + \delta_i \frac{\partial \tau_i^a}{\partial \delta_i} [\Delta P - \gamma_i (P_0 + \Delta P - P_T)] - \delta_i (P_0 + \Delta P - P_T) \tau_i^a \frac{d\gamma^e}{d\delta_i} \tag{A3}$$

where $\frac{\partial \tau_i^a}{\partial \delta_i} > 0$ and by implicit function theorem $\frac{d\gamma^e}{d\delta_i} < 0$, since, for each possible realization of n, the larger is δ_i the lower is the fraction γ that the arbitrageur has to tender.

If $\delta_i = \delta \leq \bar{\delta}$ (the risk arbitrageur A_i buys as many shares as the others), then $\tau_i^a = \tau^a > \tau$. Moreover, $P_{\tau_i} - \Delta P \tau$ is increasing in δ_i. Therefore, given τ_i^a and τ, define $\hat{\delta}_i$ such that $P_{\tau_i} = \Delta P \tau$ (if such $\hat{\delta}_i > 0$ exists). Then for all $\delta_i < \hat{\delta}_i$ the objective function is negative and the optimum is $\delta_i = 0$. For all $\delta_i \geq \hat{\delta}_i$, the first order conditions are always strictly positive. Therefore, the solution is always a corner solution: the arbitrageur wants to buy either no shares at all or as many shares as possible. The intuition is quite simple: P_{τ_i} is the expected benefit from holding one share, while $\tau\Delta P$ is the actual cost (both in excess of P_0); since risk arbitrageurs are risk neutral, they will buy shares if and only if the expected benefit is higher than the cost.

Assume now the risk arbitrageur buys more than $\bar{\delta}$ shares. He should then declare his transaction and the price will become $P_0 + \Delta P \tau_i^a$, at which he has no interest in buying shares at all. Therefore the arbitrageur will buy either $\bar{\delta}$ or 0 shares.

Let us check if it is ever an equilibrium to buy $\bar{\delta}$ shares. For each n and ω, if the arbitrageurs buy $\bar{\delta}$ shares, the volume is $y = n\bar{\delta} + \omega$ and τ and τ^a depend on such a y. We have therefore to see if the profits from buying $\bar{\delta}$ shares are positive or negative. First of all, if $y < 0.5$, then $\tau_i^a = \tau = 0$. The share price is P_0 and the risk arbitrageurs are indifferent between buying and not buying shares. Given that they are not interested in holding long term positions, we assume that in this case they do not buy any shares. This gives us case (a) of Proposition 3.

Let us now consider the case with $0.5 \leq n\bar{\delta} + \omega \leq 1$. If the risk arbitrageur bought $\bar{\delta}$ shares, $\tau^a(y)$ is given by (5). The expected (interim) profits of arbitrageur A_i are

$$\pi(y) \equiv \bar{\delta}\,[P_{\tau_i} - \Delta P \tau] \tag{A4}$$

where

$$\frac{d\pi}{dy} = \frac{\partial \pi}{\partial y} + \frac{\partial \pi}{\partial \gamma^e}\frac{d\gamma^e}{dy}.$$

We know that $\frac{\partial \pi}{\partial \gamma^e} < 0$, moreover by implicit function theorem $\frac{d\gamma^e}{dy} < 0$, since the higher is y the larger is the expected n, therefore the second term is always positive. The intuition is quite clear: the larger is y the less shares the arbitrageur expects to tender and this increases his profits. The first term is given by

$$\frac{\partial \pi}{\partial y} = \bar{\delta}\left[\frac{\partial \tau^a}{\partial y}[\Delta P - \gamma^e(P_0 + \Delta P - P_T)] - \frac{\partial \tau}{\partial y}\Delta P\right] \tag{A5}$$

where $\frac{\partial \tau^a}{\partial y} > 0$ and $\frac{\partial \tau}{\partial y} > 0$. To find out whether $\frac{\partial \pi}{\partial y} > 0$, note that $\frac{\partial \tau^a}{\partial y} > \frac{\partial \tau}{\partial y}$ if and only if $\left[G(\frac{y}{\delta})\right]^2 < G(\frac{0.5}{\delta})\left[2G(\frac{y}{\delta}) - G(1)\right]$, which is always satisfied if y is close to 0.5. Therefore, if P_T is sufficiently high, profits are positive and increasing and it is an equilibrium to buy $\bar{\delta}$ shares at least for values of y sufficiently close to 0.5. If instead P_T is low, profits are negative and decreasing. Let us define as $\underline{P}$ the P_T at which (A5) is equal to zero at $y = 0.5$. If $P_T > \underline{P}$, interim expected profits are positive at least for y close to 0.5.

For higher level of volumes, however, the profits can be a non-monotonic function of y. The intuition is clear: as y increases it becomes more and more likely that there are arbitrageurs around: the price of the shares goes up more than the value of the shares to the arbitrageurs, since their informational advantage is becoming thinner, so that $\frac{\partial \tau}{\partial y} > \frac{\partial \tau^a}{\partial y}$ and profits start decreasing.

One may wonder whether the profits from buying $\bar{\delta}$ shares can ever become negative (in which case the only equilibrium for that level of y would be not to buy shares at all, that is $\delta_i = 0$). This is possible, because although $\tau^a > \tau$ always, the arbitrageur tenders some of its shares and the benefit from one share is therefore less than ΔP. Profits are equal to zero if $P_{\tau_i} = \Delta P \tau$. If we divide both sides by $\Delta P + P_0 - P_T$, define $\phi \equiv \frac{\Delta P}{\Delta P + P_0 - P_T}$ and express τ^a and τ in terms of the distribution $G(s)$, some simple manipulation of the zero profits equation yields $\gamma^e G(\frac{y}{\delta}) = \phi G(1)$. The left hand side is always less than 1 and as $P_T \to P_0 + \Delta P$, $\phi \to \infty$. Therefore, if $G(1)$ is not infinitesimal (which we can check when we endogenize $G(n)$), and P_T is sufficiently high, profits can never become zero, which means the profits are always positive for any $y \geq 0.5$.

To summarize, if P_T is not too low ($P_T \geq \underline{P}$) there exists values of n and ω (and therefore y) for which profits are positive and if P_T is sufficiently high profits are positive for any volume larger than 50%. In all this cases it is an equilibrium for the risk arbitrageurs who entered to buy $\bar{\delta}$ shares.

Finally, we have to show that when it is not an equilibrium to buy $\bar{\delta}$ shares, the only symmetric pure strategy equilibrium is the one in which risk arbitrageurs buy 0 shares. Let us assume that each risk arbitrageur buys a quantity $\delta_0 \leq \bar{\delta}$. Then, for given beliefs $\tau(\delta_0)$ a single risk arbitrageur always has an incentive to deviate and buy up to $\bar{\delta}$ shares. Therefore δ_0 cannot be an equilibrium. This gives us case (b) of Proposition 3.

To see case (c) notice that if $n\bar{\delta}+\omega > 1$ the price should increase up to $P_0+\tau^a \Delta P$. However, at that price the risk arbitrageurs will not buy shares.

Appendix II. Proof of Proposition 3

For each (n, ω), we can compute the ex-post profits $\pi(n, \omega)$. We consider the cases in which $P_T \geq \underline{P}$, otherwise the arbitrageurs never buy shares and the profits are never positive. If (n, ω) is such that $n\bar{\delta} + \omega < 0.5$ or $n\bar{\delta} + \omega > \bar{y}$ (where $\bar{y}$ is either the volume level at which profits become equal to zero or is equal to one) then the arbitrageurs who chose to enter would buy no shares at all and $\pi(n, \omega) = -c$. If instead (n, ω) are such that $0.5 \leq n\bar{\delta} + \omega \leq \bar{y}$ but $n\bar{\delta} < 0.5$ then the arbitrageurs buy $\bar{\delta}$ shares but the takeover fails and their profits are $\pi(n, \omega) = -c - \Delta P \tau \bar{\delta}$. Finally, if $0.5 \leq n\bar{\delta} + \omega \leq \bar{y}$ and $n\bar{\delta} \geq 0.5$ then the takeover succeeds and the arbitrageurs make positive profits $\pi(n, \omega) = \bar{\delta}\left[\Delta P(1 - \tau) - \gamma(n)(\Delta P + P_0 - P_T)\right] - c$.

Let us define $\bar{\pi}(n) \equiv E_\omega\left[\pi(n, \omega) \mid n\right]$ to be the expected profits for a given n of an arbitrageur who has entered. If $n < \frac{0.5}{\delta}$

$$\bar{\pi}(n) = -c - \bar{\delta}\Delta P \int_{0.5 - n\bar{\delta}}^{\bar{y} - n\bar{\delta}} \tau(n\bar{\delta} + \omega)d\omega = -c - \bar{\delta}\Delta P \int_{0.5}^{\bar{y}} \tau(\hat{\omega})d\hat{\omega}. \tag{A6}$$

Rewriting the profit as a function of $\hat{\omega}$ shows that the profits are independent of n and therefore constant in that range.

If $\frac{0.5}{\bar{\delta}} \leq n \leq \frac{\bar{y}}{\bar{\delta}}$

$$\bar{\pi}(n) = -c + \bar{\delta} \int_{0}^{\bar{y} - n\bar{\delta}} \pi(n\bar{\delta} + \omega)d\omega = -c + \bar{\delta} \int_{\bar{\delta}n}^{\bar{y}} \pi(\hat{\omega})d\hat{\omega}. \tag{A7}$$

If we take the derivative with respect to n, the lower bound of the integral increases, therefore the profits are a decreasing function of n. It can easily be seen that if $n = \frac{\bar{y}}{\bar{\delta}}$, $\bar{\pi}(n) < 0$, since for any $\omega > 0$ the arbitrageurs earn $-c$. We define $\hat{y}$ as the level of volume such that $\bar{\pi}(\frac{\hat{y}}{\bar{\delta}}) = 0$ (as it is shown in Figure 2).

Finally, if $n > \frac{\bar{y}}{\bar{\delta}}$, $\bar{\pi}(n) = -c$.[22] Figure 2 represents $\bar{\pi}(n)$ as a function of n.

[22] If $N < \frac{\bar{y}}{\bar{\delta}}$ this case never arises.

The ex-ante expected profits are equal to

$$\Pi(p, N, c) \equiv E_{n,\omega}\left[\pi(n,\omega)\right] = E_n\left[\bar{\pi}(n)\right] \tag{A8}$$

To find the symmetric equilibria we proceed in the following way. Let us define a distribution of n, $\hat{g}(n)$, such that $\hat{g}(n+1) = g(n)$, where $g(n)$ is the binomial defined in (7). Suppose $N-1$ arbitrageurs randomize their entry decision with probability p and let us consider the entry decision of the N-th arbitrageur. If he decides not to enter, the expected payoff is 0. Suppose he decides to enter with probability 1, then his expected payoff defined as $\pi(p)$ can be expressed in the following way

$$\pi(p) = \sum_{n=1}^{N}\left[\bar{\pi}(n)\hat{g}(n)\right] = AProb\{n < \frac{0.5}{\bar{\delta}}\} + BProb\{\frac{0.5}{\bar{\delta}} \leq n \leq \frac{\bar{y}}{\bar{\delta}}\} - cProb\{n > \frac{\bar{y}}{\bar{\delta}}\},$$

where, $\hat{g}(n)$ is the distribution of the number of arbitrageurs in the game taking into account the decision of the last arbitrageur and A and B are the expressions of in (A6) and in (A7), respectively. Notice that $A < 0$ always and $B > 0$.

If all other arbitrageurs randomize with probability $p = 0$, then $E_n\left[\bar{\pi}(n)\right] < 0$, since one arbitrageur alone is not enough for the takeover to succeed and the profits are given by (A6). As p increases the ex-ante expected profits increase. We want to prove that, under some conditions, there exists a p, such that $\pi(p) > 0$. If c is sufficiently low, A is negative and B is positive. Then, let us take $p = \frac{(0.5+\hat{y})/(2\bar{\delta})}{N}$. The mean of the binomial is $Np = (0.5+\hat{y})/(2\bar{\delta})$. Therefore if N is large enough, the realized n should be concentrated around Np, falling in the interval corresponding to the expression of (A7). Thus, in the expression of $\pi(p)$, the weight of B is high enough for $\pi(p)$ to be positive.

By continuity, there must exist a value of p—let us call it p_1^*—at which $\pi(p) = 0$. As n increases, however, more and more weight is given to the case in which $n > \frac{\hat{y}}{n}$. Therefore, ex-ante profits become negative again. In particular, take $p = \frac{\bar{y}/\bar{\delta}}{N}$ (or higher) : again, if N is large enough all the weight of the probability is on $\bar{\pi}(n) = -c$ and $\pi(p)$ is negative. By continuity, there must exist another p—let us call it p_2^*, at which ex ante profits are equal to 0.

These two probabilities, p_1^* and p_2^* are the equilibria. Intuitively, when the other arbitrageurs randomize with a probability p such that the expected profits from entering are zero, the last arbitrageur is indifferent between entering and not and then he might as well randomize between the two with probability p. Therefore these two values of p are the two equilibria of the game (in addition to the equilibrium in which nobody enters). Figure 3 represents the ex-ante profits of a risk arbitrageur who enters, given that all other risk arbitrageurs are randomizing with probability p. The level of p for which the ex-ante profits are equal to zero are the equilibria.

Appendix III. Proof of Remark 2 of Section 6.3

Let us first see how the expected profits $\bar{\pi}(n)$—derived in Appendix II and represented in Figure 2—change when $\bar{\omega}$ increases. As $\bar{\omega}$ increases, $\tau(y)$ and $\bar{y}$ increase. As a consequence $\bar{\pi}(n)$ can either decrease or increase for $n < \frac{0.5}{\delta}$, as can be seen in (A6). For $\frac{0.5}{\delta} \leq n \leq \frac{\bar{y}}{\delta}$, $\pi(y)$ increases and as a consequence $\bar{\pi}(n)$ increases, as can be seen in (A7).

If $\bar{\pi}(n)$ increases for $n < \frac{0.5}{\delta}$, the ex-ante expected profits increase for any level of p and therefore p_2^* must increase. If instead $\bar{\pi}(n)$ decreases for $n < \frac{0.5}{\delta}$, then the effect on the ex-ante profits is ambiguous. However, we can show that the ex-ante profits at p_2^* (which were equal to zero) become strictly positive and therefore the equilibrium p_2^* must increase. To see this we proceed in the following way. We know that at the equilibrium p_2^* an individual who unilaterally increases p must receive zero expected profits. We then look at the trade-off facing an individual increasing p: that trade-off guarantees that the ex-ante expected profits are equal to zero. We show that when $\bar{\omega}$ increases the trade-off becomes more favorable. That means that now an individual increasing p would earn strictly positive ex-ante profits. Therefore, p_2^* has to increase in equilibrium.

If one individual increases p from p_2^*, $G(\frac{.5}{\delta})$ (the probability of obtaining a negative payoff, since $n < \frac{0.5}{\delta}$) decreases, this increases this individual payoff. The disadvantage, however, is that $1 - G(\frac{\bar{y}}{\delta})$ (the probability of obtaining the negative payoff $-c$, since $n > \frac{\bar{y}}{\delta}$) increases. By definition, these effects must compensate so that ex-ante expected profits are equal to zero. However, when $\bar{\omega}$ increases, the payoff obtained when $n < \frac{0.5}{\delta}$ becomes even more negative and the gain from increasing p is higher. At the same time the cost if $n > \frac{\bar{y}}{\delta}$ is still $-c$, but $\bar{y}$ has increased, so it is less likely that the arbitrageur will bear that cost. The remaining mass of probability is in the central part of the support, where profits increased. The trade-off is surely more favorable now, so the equilibrium p_2^* must increase.

¶Q.E.D.

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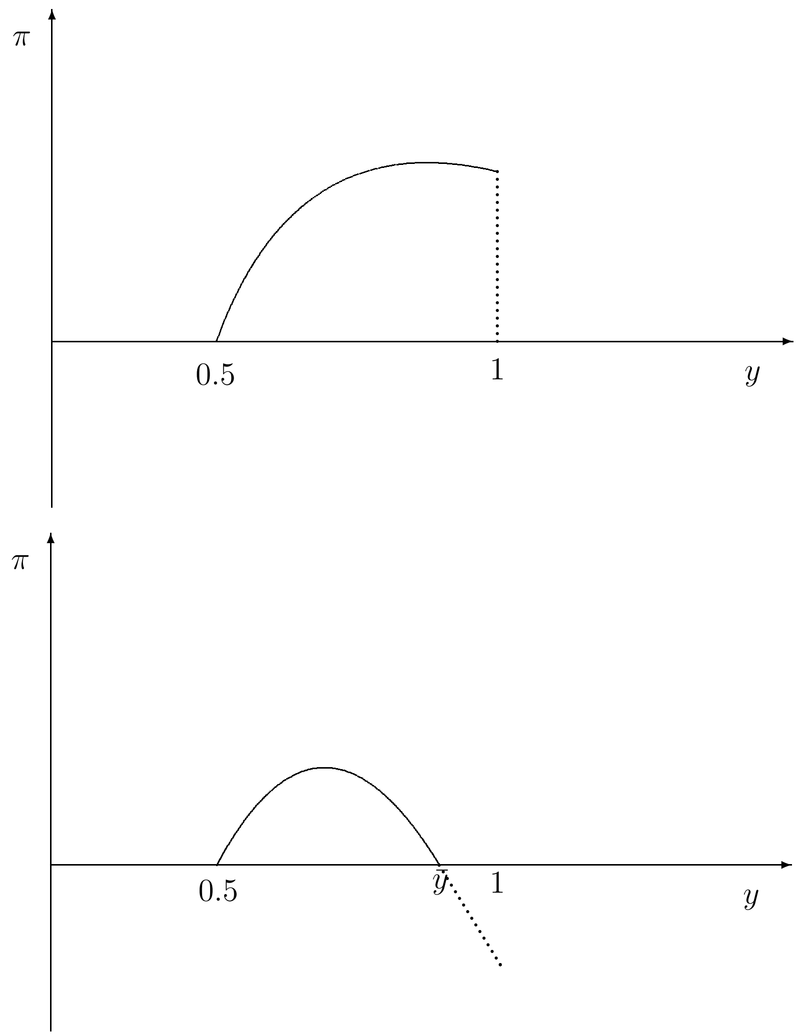

Figure 1: Expected Interim Profits

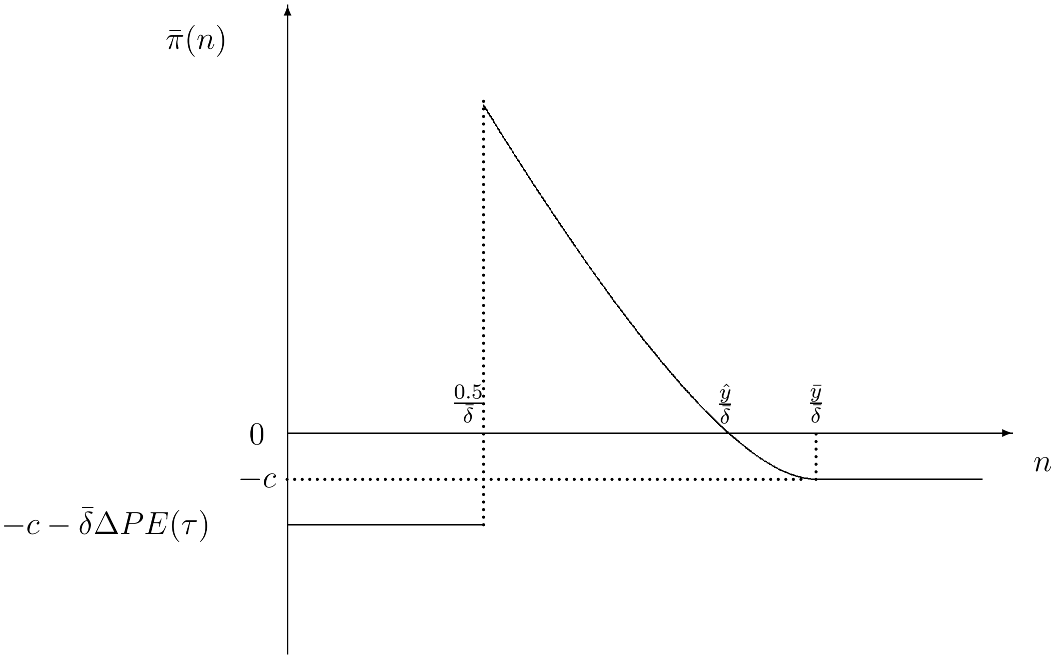

Figure 2: Arbitrageurs profits as a function of their number

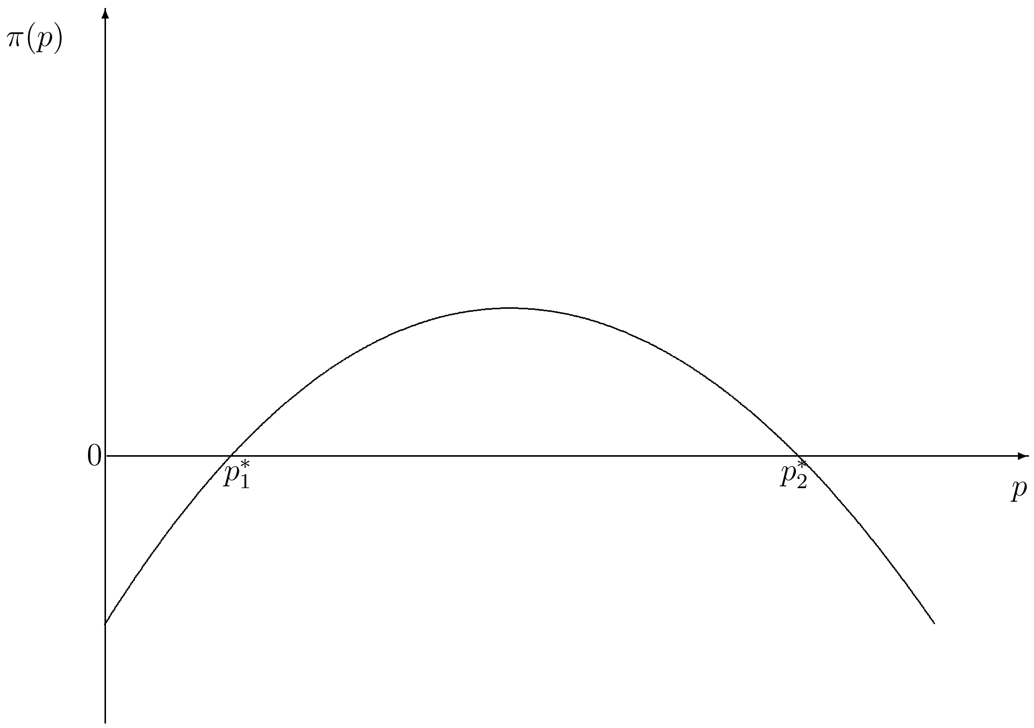

Figure 3: Ex-ante Expected Profits as a function of the randomization p