Exchange Rate Volatility and International Trade: A General Equilibrium Analysis

by

Piet Sercu† Raman Uppal‡

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† K.U.Leuven, Naamsestraat 69, 3000 Leuven, Belgium, piet.sercu@econ.kuleuven.ac.be.
‡ University of British Columbia and Massachusetts Institute of Technology, 50 Memorial Drive, E52-456, Cambridge, MA 02142, USA, uppal@mit.edu.
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Abstract
In this paper, we use insights from the literature on financial options to analyze the effect of exchange rate volatility on the volume of trade between countries. In contrast to existing work, this analysis is carried out in a general-equilibrium stochastic-endowment economy with imperfect international commodity markets in which both trade and exchange rate volatility are endogenous quantities. Our main objective is to examine the popular conjecture that an increase in exchange rate volatility is associated with a decrease in the volume of international trade. We show that, even in a simple model, the relation between trade volume and exchange rate volatility can be either negative or positive depending on the underlying source for the change in exchange rate volatility. Specifically, when the source of the increase in exchange rate volatility is an increase in the volatility of the endowment processes, our model predicts an increase in the expected volume of trade. On the other hand, when there is an increase in the segmentation of commodity markets, exchange rate volatility increases but the volume of trade decreases. In both cases there is a drop in welfare, but in the first case this is associated with an increase in trade and in the second case with a decrease in trade.

JEL Classification: F31, F32, F11

Key words: Exchange risk, trade volume, option pricing, non-traded goods
Our objective in this paper is to evaluate the conjecture that an increase in exchange rate volatility leads to a decrease in the volume of international trade. Perée and Steinherr (1989) raise two weaknesses in the existing literature on exchange rate volatility and international trade: first, the existing theoretical models—for example, De Grauwe (1992), Franke (1991), Sercu (1992), and Viaene and de Vries (1992)—are partial equilibrium in nature, and second, in the empirical work a linear relationship between trade and exchange rate risk is postulated while the true relation might be non-linear.\(^1\) The model we develop is of a general-equilibrium economy with stochastic endowments, and in our model both trade and exchange rate volatility are endogenous quantities. Moreover, our work provides the exact nature of the non-linear relation between exchange rate volatility and the volume of international trade.

The general equilibrium model we construct to illustrate our arguments is that of a two-country, one-good, complete-markets Lucas (1982) model that is extended to allow for imperfect international commodity markets. In contrast to existing partial-equilibrium work on the relation between international trade and exchange rate volatility, in our model the exchange rate and the prices of financial securities are determined endogenously. Our major result is that in this general equilibrium setting an increase in exchange rate volatility may be associated with either an increase or a decrease in the volume of international trade, depending on the source of the change in volatility. An attractive feature of our analysis is that, even though the model we work with is of a general equilibrium economy, all our results can be obtained in closed-form by taking advantage of the insights from the literature on financial options.

We now discuss the existing literature on the relation between exchange rate volatility and international trade, starting first with an overview of the theoretical models and then a survey

\(^1\)Perée and Steinherr (1989) also mention that it is not clear how one should measure exchange rate risk and that the aggregate trade equations ignore the competitive structure of product markets. While the appropriate definition of exchange rate volatility in our theoretical model will be clear, we do not address the issue of industrial structure.
of empirical work. Following this discussion, we describe how our analysis extends the existing
models.

In the early theoretical literature, a number of models were constructed to support the
view that an increase in exchange rate volatility leads to a reduction in the level of international
trade. These models (for example, Clark, 1973; Baron, 1976a; Hooper and Kohlhagen, 1978;
Broll, 1994; and Wolf, 1995) consider firms exposed to exchange risk. A typical argument in
this literature is that higher exchange risk lowers the risk-adjusted expected revenue from
exports, and therefore reduces the incentives to trade. However, these results are derived from
partial equilibrium models. For example, most of this literature assumes that exchange rate
uncertainty is the sole source of risk in the decision-maker’s portfolio, and either ignores the
availability of hedges (forward contracts, or non-linear hedges like options and portfolios of
options) or takes the prices of the hedge instruments (or at least some of the determinants of
these prices) as given.

Taking into account the firm’s option to (linearly) hedge its contractual exposure, some
other partial-equilibrium models question whether risk-averse entrepreneurs would always view
a higher exchange risk as a deterrent to trade. For example, Ethier (1973) and Baron (1976b)
show that exchange rate volatility may not have any impact on trade volume if firms can hedge
using forward contracts. Viaene and de Vries (1992) extend this analysis to allow for the
endogenous determination of the forward rate; then, exchange rate volatility has opposing effects
on importers and exporters (who are on opposite sides of the forward contract). In this case,
Viaene and de Vries find that the net effect of exchange rate volatility on trade is ambiguous.

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2 See IMF (1984), Perée and Steinherr (1989), Edison and Melvin (1990), and Côté (1994) for a more detailed
review of the theoretical models and empirical work examining the relation between trade volume and exchange rate
volatility.

3 Cushman (1983) argues that the relevant source of uncertainty for the firm is about real rather than the nominal
exchange rate; he finds similar results for the case where profits depends on the real exchange rate.
Also De Grauwe (1988) shows that risk aversion is not sufficient to obtain a negative link between exchange risk and expected trade; the direction of the association depends critically on the degree of risk aversion. This is because, in general, an increase in risk has both an income effect and a substitution effect that work in opposite directions (Goldman and Kahn, 1985). Thus, even though firms are worse off with an increase in exchange rate risk, their response may be to export more rather than less. Dellas and Zilberfarb (1993) make a similar point using a portfolio-choice model.

While these models allow the firm to hedge or at least diversify its exchange risk, they ignore the firm’s option to adjust its production in response to the exchange rate. Models that focus on the firm’s flexibility tend to conclude that a higher exchange risk actually stimulates trade. The reason is that, when firms are allowed to optimally respond to exchange rate changes, the revenue per unit of an exportable good (De Grauwe, 1992, and Sercu, 1992) or the entire cashflow from exporting (Franke, 1991, and Sercu and van Hulle, 1992) become convex functions of the exchange rate. From this it follows that expected unit revenue or the expected cashflow increases when the volatility of the exchange rate increases, which then acts as a stimulant to trade rather than a deterrent.4 These models, however, still take the demand functions or the cashflow function as given, and therefore ignore the issue of how the demand or cashflow function is affected by the changes in the economy that cause an increase in exchange risk. Moreover, all the existing models assume that the exchange rate is exogenous, and is therefore not affected by the actions of the firms. Lastly, the existing models typically analyze a single firm, while the data that are used in the empirical tests described below are of the aggregate economy (Bini-Smaghi, 1991; Goldstein and Khan, 1985).

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4This literature is similar to the trade literature on hysteresis; see, for example, Baldwin (1988). For a comprehensive review of this modeling approach, see Dixit and Pindyck (1994).
We now discuss some of the empirical work studying the relation between trade volume and exchange rate volatility.\(^5\) Koray and Lastrapes (1989) and Lastrapes and Koray (1990) use VAR models to examine whether exchange rate volatility affects the volume of trade. They find that exchange rate volatility explains only a small part of imports and exports. Gotur (1985) also finds that there is little support for a relation between exchange rate volatility and trade. Gagnon (1993) finds similar results based on simulations of a dynamic optimizing model with adjustment costs. In cross-section work, Brada and Mendez (1988), using a gravity model of bilateral trade, find that even though exchange rate volatility reduces trade, its effect is smaller than that of restrictive commercial policies. Frankel and Wei (1993), using an instrumental variables approach, also conclude that the effect of exchange rate volatility on trade is small. On the other hand, Asseery and Peel (1991) using an error-correction framework, and Kroner and Lastrapes (1993) using a multivariate GARCH-in-mean model, find that an increase in volatility may be associated with an increase in international trade. McKenzie and Brooks (1997) even find that US-German imports and exports are positively and significantly associated with ARCH-varying exchange rate volatility. Thus, the overall conclusion is that the negative effects of exchange rate volatility, if present, are small.

To examine the relation between international trade and exchange rate volatility in a framework that does not have the limitations of the theoretical models discussed above, we need a general-equilibrium model of the aggregate economy as in, for instance, the (neo-)classical trade literature. However, the standard free-trade models assume that all commodity markets are perfect; that is, the drawback of the neoclassical approach is that Commodity Price Parity is

\(^5\)In empirical tests, typically, it is the real rather than the nominal exchange rate that is studied [see, for instance, Asseery and Peel, 1991; De Grauwe, 1988; Gagnon, 1993; Gotur, 1985; Koray and Lastrapes, 1989; Kroner and Lastrapes, 1993; and Peréé and Steinherr, 1989]. Exchange rate risk is measured using one of the following: the standard deviation of the level of the exchange rate or the standard deviation of the percentage change in the exchange rate, the difference between the actual and predicted forward rate (so as to measure the unanticipated change), or a time-series model for exchange risk such as GARCH (Asseery and Peel, 1991; Pozo, 1992; and, McKenzie and Brooks, 1997).
postulated to hold at all times and for all goods, implying that there is no real exchange rate risk. Another drawback of the standard free-trade models is that, by requiring period-by-period equilibrium on the trade balance, they ignore the existence of capital markets.

Accordingly, our objective is to develop a model of the macroeconomy that has the internal consistency of the general-equilibrium models of international trade, but where capital markets are allowed to play their normal economic roles, and where commodity markets are sufficiently segmented to allow for deviations from Commodity Price Parity and changes in the real exchange rate. In our model, the financial markets are assumed to be complete and perfectly integrated, reflecting the fact that, at least for developed economies, international capital markets are far less subject to restrictions than commodity markets. Thus, in our model consumers can make cross-border financial investments to finance or hedge future imports; likewise, firms can make optimal hedging decisions; and the prices of all contracts are determined in a general-equilibrium framework. Commodity markets, on the other hand, are assumed to be segmented internationally. We model this segmentation by introducing a cost for transferring goods across countries, as in Dumas (1992) and Sercu, Uppal and Van Hulle (1995). This transfer cost may be considered a proxy for transportation costs, contracting costs, or any other hindrance to international trade.

The rest of the paper is organized as follows. In Section I, we describe the economy that we use in our analysis. In Section II, we show that in our one-good setting the relation between exchange rate volatility and the volume of international trade is positive when output risk increases, and negative when the shipping cost increases. In Section III, we discuss the implications of our modeling assumptions, and relate the results of our theoretical model to the

—in a perfect-markets setting, PPP-deviations can still arise because of international differences in commodity preferences. However, Engel (1993) and Rogers and Jenkins (1995) find that, as a source of PPP deviations, violations of Commodity Price Parity are far more important than differences in commodity preferences. Engel and Rogers (1995) also show that within-country deviations from the Law of One Price are much smaller than cross-country deviations, which is consistent with the friction in our model for trading goods across countries.
empirical evidence on the relation between exchange rate volatility and trade. We conclude in Section IV. The major results of each section are collected in propositions, while intermediate results are presented in lemmas. Proofs for these propositions and lemmas are given in the appendix.

I. The Economy

In this section, we present a model of two countries \((k = 1, 2)\) that have perfectly integrated financial markets but segmented commodity markets. That is, capital markets are assumed to be complete and frictionless (implying that asset prices are equal across countries, after conversion into the same reference currency), but it is costly to trade goods internationally. In what follows, we describe the endowment process and the preferences of consumers.

In every period, each country has a stochastic endowment of a single non-storable good that is homogenous across countries and can be traded internationally only at a cost. The endowment in country \(k\) at time \(t\) of this good is denoted by \(q_k(t)\). These stochastic endowments are given exogenously, as in Lucas’ (1982) exchange economy. Although the results in this section on the equilibrium levels of trade and the real exchange rate are distribution-free, in the next section we do specify a distribution for the endowment processes to identify the implications of various shocks for both expected trade and the variance of the real exchange rate; specifically, we shall let the endowment processes for the goods be given by log random walks, with constant mean, \(\mu_k - 0.5 \sigma_k^2\), and variance, \(\sigma_k\), \(k\)={1,2}:

\[
\ln q_k(T) \sim N\left(\ln q_k(t) + [\mu_k - \frac{1}{2} \sigma_k^2] (T - t), \sigma_k \sqrt{T - t}\right), \quad k\in\{1,2\}.
\]  

The correlation between the outputs of the good at home and abroad, \(\rho\), is assumed to be constant and less than unity.
The home and foreign country \((k = 1\) and \(2\), respectively) are assumed to be populated by a large and equal number of infinitely-lived consumers with identical preferences over the single good, and identical, constant relative risk aversion:

\[
U_k[c_k(t)] = \frac{1}{1-\eta} [c_k(t)]^{1-\eta}, \quad 0 \leq \eta \leq \infty, \eta \neq 1, \quad (2)
\]

where \(c_k(t)\) denotes the consumption of the good in country \(k\). Thus, in this model, differences in utility functions are not a source of trade. To ensure symmetry, we also assume that the initial wealth of the home country, which depends on its lifetime endowment stream, was the same as that of the foreign country. The factors that do distinguish one country from another are the stochastic output in each country and the presence of costly shipping. As a result of the output risk, there almost surely is a divergence between the two countries’ outputs. It is true that trade can (and will) reduce the resulting imbalance of the consumptions, but the shipping cost means that there never can be perfect pooling of the risks.

In our model, this physical cost of shipping represents all the imperfections that segment the commodity market in one country from that in another. This exogenously determined shipping cost is modeled, following Dumas (1992), as a waste of resources: if one unit is shipped, only \(1/(1+\tau)\) units actually arrive \((\infty > \tau > 0)\). This transfer cost reflects not just a transportation cost (since it is independent of the distance shipped) but serves also as a modeling device to capture the various factors that inhibit international trade. Support for this specification is provided by the work of Engel and Rogers (1996) and by the Threshold AutoRegressive statistical model developed by O’Connell and Wei (1997) and Obstfeld and Taylor (1997), and the Smooth Transition AutoRegressive model studied in Michael, Nobay and Peel (1997). The

7 The special case where \(\eta = 1\) is represented by the log utility function. This utility specification yields the same first-order conditions as the ones obtained by setting \(\eta\) equal to unity in the case for the utility function in (1), and the same expressions for trade and the real exchange rate. Thus, the implications for the log utility function are similar to the ones we derive for the case \(\eta \neq 1\).
shipping cost implies that, within a certain region, it will be optimal not to trade even when the price of the tradable good at home is different from that abroad (see Figure 1). Thus, the different outputs generally imply international deviations from Commodity Price Parity.

Given that within each country individuals have identical, homothetic utility functions, the model can be expressed in terms of two representative consumers, one for each country. Rather than considering decentralized decision-making, we look at the problem from a central planner's perspective. With our assumption of complete and frictionless financial markets and competitive goods markets, the decentralized solution is identical to that of the central planner, but analyzing the central planner's problem allows us to identify the optimal policies for consumption and trade in a relatively straightforward way.

Let \( x_k(t) \) denote the amount of the good exported from country \( k \) at time \( t \) (measured before transactions costs). The central planner's objective is to choose the decision rule for exports so as to maximize the equally-weighted aggregate utility of the two countries:\(^8\)

\[
\text{Max}_{\{x_k(t)\}} \quad E \left\{ \sum_{t=0}^{\infty} \beta^{-t} U_1[c_1(t)] \right\} + E \left\{ \sum_{t=0}^{\infty} \beta^{-t} U_2[c_2(t)] \right\},
\]

subject to:

\[ c_1(t) = q_1(t) - x_1(t) + \frac{x_2(t)}{1+\tau}, \]  
\[ c_2(t) = q_2(t) - x_2(t) + \frac{x_1(t)}{1+\tau}, \]  
\[ x_k(t) \geq 0 \text{ and } k=1, 2, \]

\(^8\)The equal weights reflect the assumptions that (i) at the time the economy started both countries had identical endowments and (ii) the endowment processes have identical distributions.
where $0 < \tau < \infty$, $\beta$ is the subjective discount factor, and $U_k[c_k(t)], k = \{1, 2\}$ is as defined in equation (2).

The central planner's decision rules for consumption and trade are summarized in Propositions 1.1 and 1.2, and the equilibrium real exchange rate is derived in Proposition 1.3. Before presenting these results, we describe the intuition underlying the optimal consumption and trade policies.

Consider the solution we would have obtained if the two goods were costlessly tradable across countries. In this case, it would be optimal for the central planner to equate the marginal utility of consumption for the goods across the two countries. However, in the presence of the shipping cost it is not optimal to equate marginal utility. Thus, with a strictly positive shipping cost, the first-order conditions imply that there will be a no-trade zone within which international imbalances in the weighted marginal utility of consuming the good will be left uncorrected—notably when the cost of shipping outweighs the utility gained by reducing the international imbalance in the consumption of this good. In the corresponding decentralized economy, these no-trade states correspond to situations where the deviation from Commodity Price Parity is too small, relative to the shipment cost, to justify trade. Similarly, even when this good is actually transferred across countries, shipments will be restricted to the level where the cost of shipping the last unit has become equal to the incremental gain in aggregate weighted utility; that is, such shipments will still fall short of equating the weighted marginal utilities. In the corresponding decentralized economy, the matching feature is that commodity trade, if any, can only reduce the percentage deviation from Commodity Price Parity to the level of the transaction cost.

The consumption behavior described above has the following implications for international trade: trade occurs only when the ratio of the two outputs falls outside a particular region. Thus, it is possible to divide the state space into three critical regions or meta-states, indexed by $i$, where $i = \{0, 1, 2\}$: in region 0 of the state space, there is no trade; in region 1,
country 1 exports the good; and, in region 2, country 2 exports the good. These three regions are shown in Figure 1. It will be convenient to express our results in terms of these three regions. Many of our results also depend on the ratio of optimal consumption rather than levels; we will use \( \kappa_i(t) \equiv [c_2(t)/c_1(t)] \) to denote this ratio at time \( t \) and in region \( i \).

**Proposition 1.1:** In each of the states \( i = \{1, 2, 3\} \), the optimal consumption ratio across countries for the good is:

\[
\kappa_i(t) \equiv \frac{c_2(t)}{c_1(t)} = \begin{cases} 
\left( \frac{1}{1+\tau} \right)^{1/\eta} & \text{if } \frac{q_2(t)}{q_1(t)} < \left( \frac{1}{1+\tau} \right)^{1/\eta} \quad [i = 1: \text{country 1 is exporting}] \\
\left( 1+\tau \right)^{1/\eta} & \text{if } \frac{q_2(t)}{q_1(t)} > \left( 1+\tau \right)^{1/\eta} \quad [i = 2: \text{country 2 is exporting}] \\
\frac{q_2(t)}{q_1(t)} & \text{otherwise} \quad [i = 0: \text{no trade}] 
\end{cases}
\] (4)

The optimal export policies, presented in Proposition 1.2, follow from the consumption behavior described above. Equation (5) gives the optimal amount of the good that should be exported from country 1, and (6) gives the optimal exports of the good from country 2.

**Proposition 1.2:** The optimal levels of trade for the goods are given by:

\[
x_1(t) = \frac{1}{1/[\kappa_1 (1+\tau)] + 1} \max \left( q_1(t) - \frac{1}{\kappa_1} q_2(t), 0 \right), \quad (5)
\]

\[
x_2(t) = \frac{1}{\kappa_2/(1+\tau) + 1} \max \left( q_2(t) - \kappa_2 q_1(t), 0 \right). \quad (6)
\]
Lastly we derive the real exchange rate, which in a one-good economy is given by the ratio of the marginal utility of consumption abroad to that at home.

Proposition 1.3: The real exchange rate can be expressed as:

\[
S(t) = \begin{cases} 
\kappa_1 - \eta = 1 + \tau & \text{if } i = 1 \\
\left(\frac{q_2(t)}{q_1(t)}\right)^{\eta} & \text{if } i = 0 \\
\kappa_2 - \eta = \frac{1}{1 + \tau} & \text{if } i = 2 
\end{cases}
\]  

(7)

Proposition 1.3 implies that, in the absence of trade, the real exchange rate is bounded by \(1 + \tau\) and \(1/(1 + \tau)\), and equals the upper bound \(1 + \tau\) [the lower bound \(1/(1 + \tau)\)] when country 1 [2] is exporting. This reflects the no-arbitrage conditions on deviations from commodity price parity that hold in a decentralized economy.

From Propositions 1.1, 1.2 and 1.3, we see that it is possible to express explicitly the real exchange rate and the volume of international trade as functions of \(\tau\), \(\eta\), and the state variables (the outputs of the two goods). In the next section we will examine how a change in either \(\tau\) or the volatility of the relative endowment process affects the expected volume of trade and the volatility of the exchange rate.

II. The Relation between Trade and Exchange Rate Risk

In this section, we examine the change in exchange rate volatility and expected trade for two experiments: one, where there is a change in the volatility of the relative endowment processes; two, where there is a change in the degree of commodity market segmentation, \(\tau\).
II.A. The Effect of Output Volatility on Exchange Risk and Expected Trade

We start by considering the effect of an increase in volatility in the endowment processes on the expected level of trade. To do this, we wish to obtain analytical expressions for the expected volume of domestic and foreign exports. We do this by noting that the expression for the realized volume of domestic [foreign] exports in (5) [(6)] is similar to the payoff of an option to exchange two risky assets at the rate $\kappa_2 [1/\kappa_1]$. The properties of such options have been studied in the finance literature by Margrabe (1978). Thus, we can use the insights from the theory of option pricing to determine the expected volume of trade.

Given the assumption that the distribution of the two endowment processes is jointly lognormal, we obtain an expression for the expected foreign exports that is similar to the value of an option to exchange two risky assets whose prices are lognormally distributed. From option-pricing theory, we also know that the value of an option is increasing in the volatility of the underlying stochastic process. In our context, we show in Lemma 2.1 below, that the expected volume of foreign exports is a positive function of the volatility of the relative output process, $q_2(T)/q_1(T)$.

Lemma 2.1: The conditional expectation at time $t$, of foreign exports at a later date $T$, is given by the expression in (8), which is a non-linear positive function of the variance of $q_2(T)/q_1(T)$:

\[ E_t(x_2(T)) = \frac{E_t[q_2(T)] N(d_1) - \kappa_2 E_t[q_1(T)] N(d_2)}{\kappa_2/(1+\tau) + 1}, \tag{8} \]

where $E_t(q_k(t)) = q_k(t) \exp\{\mu_k (T-t)\}$, $k = \{1, 2\}$

$\phi^2 \equiv \sigma_1^2 - 2 \rho \sigma_1 \sigma_2 + \sigma_2^2$, the p.a. variance of $\ln \frac{q_2(T)}{q_1(T)}$

\[ d_1 = \frac{\ln \frac{E_d q_2(T)}{E_d q_1(T)} + \frac{1}{2} \phi^2 (T-t) - \ln \kappa_2}{\phi \sqrt{T-t}}, \]

\[ d_2 = \frac{\ln \frac{E_d q_2(T)}{E_d q_1(T)} - \frac{1}{2} \phi^2 (T-t) - \ln \kappa_2}{\phi \sqrt{T-t}}, \]
\[ N(d) \equiv \text{the probability that } z \leq d, \text{ } z \text{ a unit normal random variable.} \]

Lemma 2.1 shows that the expected volume of foreign exports is increasing in the volatility of the relative endowment process. Similarly, we can show the same is true for domestic exports. Given that total trade is the sum of domestic and foreign exports, we conclude that the expected volume of trade increases with an increase in the volatility of the relative endowment process.

We now evaluate the effect of an increase in output risk on the variance of the real exchange rate. Like others before us, we choose to study the log of the exchange rate because of its symmetry. From (7):

\[
\ln S(t) = \begin{cases} 
-\eta \ln \kappa_1 = \ln(1+\tau) & \text{if } \frac{q_2(t)}{q_1(t)} \leq \kappa_1; \\
-\eta \ln \frac{q_2(t)}{q_1(t)} & \text{if } \kappa_1 \leq \frac{q_2(t)}{q_1(t)} \leq \kappa_2; \\
-\eta \ln \kappa_2 = -\ln(1+\tau) & \text{if } \frac{q_2(t)}{q_1(t)} \geq \kappa_2. 
\end{cases} \tag{9}
\]

Thus, the log real exchange rate is proportional to a truncated variate, \( \ln\left[\frac{q_2(t)}{q_1(t)}\right] \), where the values of the log output ratio that fall outside the no-trade region \([\ln \kappa_1, \ln \kappa_2] \) are replaced by the (constant) bounds \( \ln \kappa_1 \) and \( \ln \kappa_2 \). Such truncation problems are studied in the insurance literature, and Sercu (1997) shows that an increase in the volatility of a normally distributed underlying variable leads to an increase in the volatility of the truncated variable.

**Lemma 2.2:** Given the expression for the exchange rate in (9) and the assumption that \( \ln\left[\frac{q_2(T)}{q_1(T)}\right] \) is normally distributed, \( \text{var}(\ln S(T)) \) is a positive function of the volatility of relative output.

To illustrate the lemma, consider, for simplicity, a situation where the bounds on the log exchange rate, \( \pm \ln(1+\tau) \), are symmetric around the expected log exchange rate. An increase in the variance of the relative output does not affect these bounds. Given the symmetry of the
bounds around the mean, the effect of an increase in volatility of \( q_2(T)/q_1(T) \) is to increase the probability that \( S(T) \) is at one of its bounds. That is, when the volatility of either \( q_1(T) \) or \( q_2(T) \) increases, more and more of the probability mass of \( \ln S(T) \) is shifted away from the middle of the distribution towards the bounds. This leads to an increase in the variance of the log exchange rate. Sercu (1997) shows that this conclusion holds also in situations where the bounds on the log exchange rate are not symmetric around its expected value.

Thus, from the first experiment on the relation between trade and exchange rate volatility, we conclude the following.

**Proposition 2.1:** When there is an increase in the volatility of the relative endowment process, there is an increase in both exchange rate volatility and the expected volume of trade. Thus, when the source of the shock in the exchange risk is a change in the risk of the outputs there is a positive association between expected trade and exchange rate volatility.

**II.B. The Effect of Segmentation on Exchange-Rate Volatility and Trade**

In the preceding section, the changes in the moments of trade and the exchange rate were assumed to be driven by a shift in the riskiness of the relative output, and we found that such a shift induces a positive relationship between trade and exchange risk. Many modifications of the model could however lead to a different conclusion. In this section, our aim is to provide just one such counterexample. Specifically, we now analyze how exchange rate volatility and the expected volume of trade change as we vary \( \tau \), the parameter that determines the degree of segmentation between international commodity markets.

Let us consider the effects of a drop in the shipment cost. Figure 1 implies that a decrease of \( \tau \) boosts expected trade, for two (related) reasons. First, the zone of no trade shrinks; that is, the probability of trade becomes larger. Second, for any given output point outside the no-trade zone, a more narrow no-trade zone also means that a larger amount of trade is needed to reduce
the difference in the international consumption levels to the level justifiable by transaction costs. The shrinking no-trade zone also means that the bounds on the exchange rate become tighter; therefore, the variance of the exchange rate decreases.

**Proposition 2.2:** With a lognormal output ratio, a drop in the shipment cost implies (i) a decrease in exchange rate volatility and (ii) an increase in the expected volume of trade. Thus, when there is a change in the shipment cost, there is a negative association between expected trade and exchange rate volatility.

While a proof of the proposition is provided in the appendix, we illustrate Proposition 2.2 by comparing an economy with international commodity markets that are partially segmented (0 < \( \tau \) < \( \infty \)) to one where they are perfectly integrated (\( \tau = 0 \)). Let us first examine how expected trade volumes would react to a complete elimination of the shipment cost. Recall, from Propositions 1.2 and 1.3, that there is no trade in the region where

\[ \kappa_1 \equiv \left( \frac{1}{1+\tau} \right)^{1/\eta} < \left( \frac{q_2(T)}{q_1(T)} \right)^{1/\eta} < \left( 1+\tau \right)^{1/\eta} \equiv \kappa_2. \]

With zero shipping costs, this region of no-trade shrinks to a single line—the 45-degree line—because \( \kappa_1 \) and \( \kappa_2 \) collapse to unity. Thus, the probability of trade increases; in addition, for a given output combination for which trade is non-zero, also the amount of exports is higher than it would have been under a positive \( \tau \). Thus, compared to an economy with segmented commodity markets (\( \tau > 0 \)), the expected volume of trade will be larger in an economy where \( \tau \) equals zero.\(^9\)

We next compare the volatility of the exchange rate in an economy with partially segmented markets to one where \( \tau \) equals zero. When \( \tau = 0 \), the real exchange rate always equals a constant (unity), implying that the variance of the exchange rate vanishes entirely regardless of

\(^9\)In terms of options, when one shrinks \( \tau \) to zero, the trade function becomes like the payoff from a straddle rather than that from a strangle; and the former is more sensitive to changes in volatility than the latter.
the riskiness of relative output. To sum up the example, when \( \tau \) is reduced to zero, expected trade rises and exchange rate volatility drops to zero.

Compared to Proposition 2.1, where trade and exchange rate volatility were positively related, the predictions about the relation between the expected volume of trade and exchange rate volatility are reversed in Proposition 2.2: here, with increased segmentation of the commodity markets, the expected volume of trade decreases while exchange rate volatility increases. Thus, from these two experiments we conclude that an increase in exchange rate volatility may be associated with either an increase or a decrease in the volume of trade.

III. Discussion of the model

In the previous section, we showed that one needs to be cautious in concluding that an increase in exchange rate volatility is always associated with a decline in trade. This is because, in a general equilibrium setting, both the volume of trade and the volatility of exchange rates are endogenous quantities; thus, the relation between the volume of trade and the volatility of exchange rates can be either positive or negative depending on the underlying source for the change in exchange rate volatility. In this section, we discuss (a) the sensitivity of these results to our modeling assumptions, and (b) the implications of the results of our theoretical model for empirical work.

Our modeling choices have been motivated by the desire to obtain all results analytically, without having to resort to numerical methods. However, it is possible to extend the model in several directions, as described below. We first consider the sensitivity of our results to our assumption of lognormal distributions for the output shocks. From equations (5) and (6), we see that the expressions for trade are convex in \( q_1(t) \) and \( q_2(t) \). Thus, the result in Lemma 2.1, that an increase in the riskiness of \( q_1(t) \) and \( q_2(t) \) leads to an increase in the expected volume of trade, does not depend on a particular distribution for the endowment process. Similarly, our
conclusion that a drop in the shipment cost stimulates expected trade is distribution-free. Also the results on the variance of the exchange rate may hold for non-normal distributions; for instance, Sercu (1997) shows that, for any underlying distribution, the variance of the truncated variable increases when the riskiness of the underlying is increased by adding binomial noise.

While the distributional assumptions do not seem to be crucial for our results, the approach we adopted to make the existence of two countries economically meaningful may have a bigger impact on the conclusions. Recall that, to segment the countries, we considered a one-good international market with a cost for shipping goods internationally. An alternative common device to make the two countries distinct is to assume that, besides a perfectly tradable good, there is a second, perfectly non-tradable good. With a non-traded good added to the model, our conclusions would remain unchanged as long as the output process for this good is non-stochastic or the two goods are separable in utility. Indeed, under these assumptions $\kappa_1$ and $\kappa_2$ would still be non-stochastic and all our earlier inferences would, therefore, continue to hold. However, in a more general model with non-traded goods, the $\kappa$'s would become functions of a second stochastic variable, relative output of the non-tradable good. In the "option" interpretation of the trade functions, these $\kappa$'s determine both the strike price and the size of the option contract (as can be seen from equation of (5) and (6)), and the variability of the $\kappa$'s is directly proportional to the variability of the relative output of the non-tradable good. Not surprisingly, then, a higher output risk in the non-tradable goods sector has an ambiguous effect on trade. The same holds if there are multiple, imperfectly tradable goods: the $\kappa$'s for each good then depend on the outputs of other good, making it difficult to make general inferences about the effect that increased output risk in one sector has on trade in another good. Similar conclusions hold with respect to exchange rate volatility.

10 Examples of this approach include Backus and Smith (1993), Stockman and Dellas (1989), Stulz (1987), and Tesar (1993).
We now make an observation about the implications of our theoretical results for empirical tests of the relation between the volume of international trade and the volatility of exchange rates. As noted in the introduction, existing empirical evidence on this relation is mixed: evidence of a negative relation between trade and exchange rate is weak, at best. A potential reason why empirical studies may have failed to detect a strong relation between trade and exchange rate volatility is that the relation implied by our general equilibrium model is non-linear, implying that the linear regression model frequently used by empirical studies to estimate the relation between trade and exchange rate volatility is misspecified.

IV. Conclusions

In this paper, we examine the conjecture that exchange rate volatility leads to a decline in trade. We do this by developing a model of a stochastic general-equilibrium economy with international commodity markets that are partially segmented. In contrast to existing work on the effects of exchange rate volatility on trade, in our model the exchange rate is determined endogenously. We argue that both trade and exchange rate volatility are endogenous quantities, and thus, it is misleading to relate one to the other as if one of them were exogenous. We show that it is possible to have either a negative or a positive relation between trade and exchange rate volatility, depending on the source of the increase in exchange rate volatility. In particular, we consider two experiments. In the first experiment, the source of increased volatility in the exchange rate is an increase in the volatility of the endowment processes; in this case, trade increases along with exchange rate volatility. In the second experiment, the degree of segmentation of commodity markets increases. In this case, too, exchange rate volatility increases, but it is now associated with a decrease in trade. Since both kinds of change can occur in the real world, our model provide a potential explanation for the results of empirical studies that typically fail to find a strong negative relation between exchange rate volatility and the volume of international trade.
Our analysis also offers new insights as to the relationship between exchange risk, trade, and welfare. The model implies that the volatility of the real exchange rate may be associated with (a) a drop in the volatility of fundamentals and (b) a reduction in the physical impediments to trade. In both cases, the decrease in real exchange risk is beneficial in terms of welfare: to risk-averse agents, a lower consumption risk increases expected utility regardless of whether consumption risk is reduced by lower costs in international trade or by lower output risk. While a reduction in trade barriers is associated with an increase in the volume of trade, a drop in the volatility of fundamentals may be associated with a fall in trade. That is, even though welfare increases in both cases, the effect on the endogenous variable, trade, may differ.\textsuperscript{11} Thus, in pursuing a policy of reducing exchange rate variability, policymakers should not consider, as a separate factor, the effect that their policy has on trade.

Understanding the relation between exchange rate volatility and commodity trade is fundamental for choosing between different exchange rate regimes and for the setting of commercial policy. Our model is a first step in analyzing these issues. However, to make inferences about tariff and monetary policy in the context of exchange rate volatility, our model would have to be extended. To study tariff policy, one would have to interpret the shipping cost in our model as a non-dissipative tariff that is determined endogenously. One would also have to model how the proceeds of the tariff are to be used in the economy. Similarly, to study the role of monetary policy, one needs to introduce money in the model in a way that it affects real allocations and also specify how the government’s seignorage income is used in the economy.

\textsuperscript{11}Manuelli and Peck (1990) show, in the context of an overlapping generations model, that the volatility of the \textit{nominal} exchange rate may not necessarily have negative implications for welfare. This is because different nominal exchanges may be associated with the same real allocation. The fact that the nominal exchange rate may be indeterminate in such models was first discussed in Kareken and Wallace (1981). King, Wallace and Weber (1992) also illustrate that the nominal exchange rate may not be related to fundamentals, though the real allocations differ across equilibria only if financial markets are incomplete. In contrast to these models, our focus is on the volatility of the real exchange rate, and in our model financial markets are complete.
Appendix

Proof of Proposition 1.1

Given that the utility function in (2) is time-separable, and the constraints in (3) apply period-by-period, we can rewrite the intertemporal problem of the central planner as a static optimization program. Thus, the central planner’s problem at time $t$ is:

$$\text{Max}_{x_k(t)} \frac{1}{1-\eta} [c_1(t)]^{1-\eta} + \frac{1}{1-\eta} [c_2(t)]^{1-\eta},$$

subject to the constraints in (3). Letting $\Lambda(t)$ denote the Lagrangian function and $\lambda_k(t)$ the Lagrangian multipliers on constraints (3b) and (3c), we get the following first-order conditions:

$$0 = \frac{\partial \Lambda(t)}{\partial c_k(t)} = (1 - \eta) \left[ \frac{1}{c_k(t)} \right]^{1-\eta} - \lambda_k, \quad k = 1, 2;$$

$$0 = x_k(t) \frac{\partial \Lambda(t)}{\partial x_k(t)}, \quad k = 1, 2;$$

$$0 \geq \frac{\partial \Lambda(t)}{\partial x_1(t)} = -\lambda_1(t) + \frac{\lambda_2(t)}{1+\tau} \Rightarrow \frac{\lambda_1(t)}{\lambda_2(t)} \geq \frac{1}{1+\tau};$$

$$0 \geq \frac{\partial \Lambda(t)}{\partial x_2(t)} = -\lambda_2(t) + \frac{\lambda_1(t)}{1+\tau} \Rightarrow \frac{\lambda_2(t)}{\lambda_1(t)} \geq \frac{1}{1+\tau}.$$

The first-order conditions yield the following bounds on relative marginal utility:

$$\frac{1}{1+\tau} \leq \frac{\partial U_2(t)/\partial c_2(t)}{\partial U_1(t)/\partial c_1(t)} = \left( \frac{c_2(t)}{c_1(t)} \right)^{-\eta} \leq 1+\tau.$$
with the lower [upper] bound holding with equality when country 1 is importing [exporting]. Thus, it is optimal to trade only when these bounds are violated by the autarky solution. This gives the following bounds on relative consumption:

\[ \kappa_1 \leq \frac{c_2(t)}{c_1(t)} \leq \kappa_2 , \]

where \( \kappa_1 \) and \( \kappa_2 \) are as defined in (4).

**Proof of Proposition 1.2**

To obtain equations (5) and (6), note that the relevant state space can be divided into three distinct regions: a) where there is no trade (region \( i = 0 \)); b) where the good is exported by country 1 (region \( i = 1 \)); and c) where the good is exported by country 2 (region \( i = 2 \)).

**a. No trade.** In the absence of trade, we have \( \frac{c_2(t)}{c_1(t)} = \frac{q_2(t)}{q_1(t)} \), implying that

\[ x_1(t) = x_2(t) = 0. \]

**b. Exports from country 1.** From (4), in region \( i = 1 \), country 1 must be exporting an amount \( x_1(t) \) such that \( \frac{c_2(t)}{c_1(t)} = (1+\tau)^{-1/\eta} \). The amount of good being exported from country 1 can be identified from the sharing rule \( c_2(t) = \kappa_1 c_1(t) \) (with \( \kappa_1 \) defined in (4)) and the market-clearing condition \( c_1(t) = q_1(t) - x_1(t) \) and \( c_2(t) = q_2(t) + x_1(t)/(1 + \tau) \). The solution is

\[ x_1(t) = \frac{q_1(t) - q_2(t)/\kappa_1}{1/[\kappa_1(1+\tau)] + 1} , \]

which is positive since we are considering states where \( q_1(t) \kappa_1 > q_2(t) \). This condition also implies that, in these states, \( x_2(t) = 0 \).

**c. Exports from country 2.** In this state, country 2 must be exporting an amount \( x_2(t) \) such that \( \frac{c_2(t)}{c_1(t)} = (1+\tau)^{1/\eta} \). Imposing the market-clearing condition, the volume of foreign exports is:
\[ x_2(t) = \frac{q_2(t) - \kappa_2 q_1(t)}{\kappa_2/(1+\tau) + 1}, \]

which is positive since we are considering states where \( q_2(t) > \kappa_2 q_1(t) \). This condition also implies that, in these states, \( x_1(t) = 0 \).

**Proof of Proposition 1.3**

In an economy with complete financial markets, the real exchange rate is the ratio of the marginal utilities of real consumption (as shown in, for example, Backus, Foresi and Telmer, 1996). In our one-good model, from (2), this simplifies to

\[ S(t) = \left( \frac{c_2(t)}{c_1(t)} \right)^\eta. \]

To obtain Proposition 1.3, it suffices to substitute into this expression the consumption ratios derived in Proposition 1.1.

**Proof of Lemma 2.1**

We consider foreign exports \( x_2(T) \) as given in (6), and rewrite \( \text{Max}[q_2(T) - \kappa_2 q_1(T), 0] \) as \( q_2(T) - \kappa_2 q_1(T) \) times an indicator function, \( I(.) \):

\[ x_2(T) = \frac{[q_2(T) - \kappa_2 q_1(T)] I\left(\frac{q_2(T)}{q_1(T)}\right)}{\kappa_2/(1+\tau) + 1}, \]

where \( I\left(\frac{q_2(T)}{q_1(T)}\right) = \begin{cases} 1 & \text{if } \frac{q_2(T)}{q_1(T)} > \kappa_2(t) \\ 0 & \text{otherwise} \end{cases} \).

Thus, the expectation to be evaluated can be written as
\[
E_t(x_2(T)) = \frac{E\left[ q_2(T) I\left(\frac{q_2(T)}{q_1(T)}\right)\right] - \kappa_2 E\left[ q_1(T) I\left(\frac{q_2(T)}{q_1(T)}\right)\right]}{\kappa_2/(1+\tau) + 1}.
\] (A1)

To solve the expectations in the above expression, we use the following result:

**Lemma A.1:** Let \( X \) and \( Y \) (where \( Y \) may be a vector) be joint lognormal with means of the log-transforms denoted by \( m_X \) and \( m_Y \), variances of the log-transforms denoted by \( \nu_X \) and \( \nu_Y \), and covariance between the log-transforms denoted by \( c_{XY} \). Let \( f(Y) \) be a function of \( Y \). Then, provided the expectation exists,

\[
E(X f(Y) ; m_X, \nu_X, m_Y, \nu_Y, c_{XY}) = E(X ; m_X, \nu_X) E(f(Y) ; m_Y + c_{XY}, \nu_Y).
\]

That is, in \( E(f(Y) ; m_Y + c_{XY}, \nu_Y) \) the mean(s) of \( \ln Y \) has (have) been shifted by adding the covariance(s) of \( \ln Y \) with \( \ln X \).

**Proof:** Beckers and Sercu (1985).

We apply Lemma A.1 to each term in equation (A1), choosing the corresponding tradable good output as the \( X \)-variable, and the indicator \( I(q_2(T)/q_1(T)) \) as the function \( f(Y) \). Noting that the expectation of this indicator function is a probability, the expected volume of foreign exports can be written as the difference of the two expected values, each of them multiplied by a cumulative normal probability, \( E_t[I(q_2(T)/q_1(T))] = N(d) \), evaluated on the basis of an appropriately shifted distribution function:

\[
E_t\left[ q_2(T) I\left(\frac{q_2(T)}{q_1(T)}\right)\right] - \kappa_2 E_t\left[ q_1(T) I\left(\frac{q_2(T)}{q_1(T)}\right)\right] = E_t[q_2(T)] N(d_1) - \kappa_2 E_t[q_1(T)] N(d_2),
\]

where \( N(d) \) is the cumulative standard normal probability (prob \((z \leq d)\)). To obtain the argument for the (shifted) normal probability function, we rewrite the shifted mean in the first expectation on the left-hand side of the above expression as follows:
\[
E \left[ \ln \frac{q_2(T)}{q_1(T)} \right] + \text{cov} \left( \ln q_2(T), \ln \frac{q_2(T)}{q_1(T)} \right) \\
= [\ln q_2(t) + (\mu_2 - \frac{1}{2} \sigma_2^2)(T-t)] - [\ln q_1(t) + (\mu_1 - \frac{1}{2} \sigma_1^2)(T-t)] + \left[ \sigma_2^2 - \rho_2 \sigma_1 \sigma_2 \right] (T-t) \\
= [\ln q_2(t) + \mu_2 (T-t)] - [\ln q_1(t) + \mu_1 (T-t)] + \frac{1}{2} \left[ \sigma_1^2 - 2 \rho_2 \sigma_1 \sigma_2 + \sigma_2^2 \right] (T-t) \\
= \ln \frac{E_t[q_2(t)]}{E_t[q_1(t)]} + \frac{1}{2} \phi^2 (T-t),
\]

where \( \phi^2 \equiv \sigma_1^2 - 2 \rho \sigma_1 \sigma_2 + \sigma_2^2 \) is the variance of the log output ratio. Thus, the probability associated with \( E_t(q_2(t)) \) is

\[
E \left[ \ln \frac{q_2(T)}{q_1(T)} \right]: \ln \frac{E_t[q_2(t)]}{E_t[q_1(t)]} + \frac{1}{2} \phi^2 (T-t), \phi^2 (T-t) \right] = \ln \frac{E_t[q_2(t)]}{E_t[q_1(t)]} + \frac{1}{2} \phi^2 (T-t) - \ln \kappa_2; \quad \phi^2 (T-t) = N(d_1), \quad (A2)
\]

Similarly, the shifted mean in the second expectation in equation (A1) can be rewritten as

\[
E \left( \ln \frac{q_2(T)}{q_1(T)} \right) + \text{cov} \left( \ln q_1(T), \ln \frac{q_2(T)}{q_1(T)} \right) = \ln \frac{E_t[q_2(t)]}{E_t[q_1(t)]} - \frac{1}{2} \phi^2 (T-t),
\]

implying that the associated probability is

\[
E \left[ \ln \frac{q_2(T)}{q_1(T)} \right]: \ln \frac{E_t[q_2(t)]}{E_t[q_1(t)]} - \frac{1}{2} \phi^2 (T-t), \phi^2 (T-t) \right] = \ln \frac{E_t[q_2(t)]}{E_t[q_1(t)]} - \frac{1}{2} \phi^2 (T-t) - \ln \kappa_2; \quad \phi^2 (T-t) = N(d_2), \quad (A3)
\]

where

\[
d_2 = \frac{\ln \frac{E_t[q_2(t)]}{E_t[q_1(t)]} - \frac{1}{2} \phi^2 (T-t) - \ln \kappa_2}{\phi \sqrt{T-t}}.
\]
Using Lemma A.1, and equations (A2) and (A3), we obtain

\[
E_t \left[ q_2(T) - \kappa_2 q_1(T) \right] \frac{d}{dq_2(T)} \frac{q_2(T)}{q_1(T)} = \frac{E_t[q_2(T)] N(d_1) - \kappa_2 E_t[q_1(T)] N(d_2)}{\kappa_2/(1+\tau) + 1} ,
\]

(A4)

which is equation (8). We can rewrite the numerator of the right hand side of (A4) as

\[
E_t[q_2(T)] N(d_1) - \kappa_2 E_t[q_1(T)] N(d_2) = E_t[q_1(T)] \left[ \frac{E_t[q_2(T)]}{E_t[q_1(T)]} N(d_1) - \kappa_2 N(d_2) \right].
\]

The part in the curly brackets is identical to the valuation formula of Black and Scholes (1973) and Merton (1973) for a call option on an asset with current price \(E_t[q_2(T)]/E_t[q_1(T)]\), strike price \(\kappa_2\), a zero interest rate and variance \(\phi^2\) p.a. Because option prices increase, \textit{ceteris paribus}, when the variance increases, the expression in square brackets is a positive function of \(\phi^2\). Lastly, from \(\phi^2 \equiv \sigma_1^2 - 2\rho \sigma_1 \sigma_2 + \sigma_2^2\), the total effect of equal increases in the standard deviations is positive:

\[
\frac{\partial \phi^2}{\partial \sigma_1} + \frac{\partial \phi^2}{\partial \sigma_2} = 2 (\sigma_1 + \sigma_2) (1-\rho) > 0.
\]

**Proof of Lemma 2.2**

For notational convenience, we set

\[
\beta_1 = \ln(1+\tau)
\]

\[
\beta_2 = -\ln(1+\tau)
\]

\[
y = -\eta \ln \frac{q_2(T)}{q_1(T)}
\]

With this notation, the random component in the real exchange rate is given as
\[
\ln Z(T) = \ln S(T) = \begin{cases} 
\beta_2 & \text{if } y \leq \beta_2 \\
y & \text{if } \beta_2 \leq y \leq \beta_1 \\
\beta_1 & \text{if } y \geq \beta_1 
\end{cases}, \quad (A5)
\]

with as the conditional density of \( y \).

\[
\pi(y) = \frac{1}{\delta \sqrt{2\pi}} \exp\left\{-\frac{1}{2} \left( \frac{y - \gamma}{\delta} \right)^2 \right\}
\]

\( \gamma = \mathbb{E}_t(y) \), the conditional expected value of \( y \)

\( \delta^2 = \text{var}_t(y) \), the conditional variance of \( y \)

That is, the exchange rate is a normally distributed variable truncated above and below. From Sercu (1997), the variance of such a truncated variable increases with \( \delta \). Still from Sercu (1997), when the distribution is not Gaussian, a sufficient condition for the same result is that the riskiness of the underlying is increased by adding a binomial perturbation to \( y \).

**Proof of Proposition 2.1**

This follows from Lemmas 2.1 and 2.2.

**Proof of Proposition 2.2**

The most transparent way to prove the first part of the proposition is to rely on the geometry of Figure 1. This figure immediately implies that, as \( \tau \) decreases—that is, with a narrower no-trade zone—the volume of trade will be higher for any endowment vector outside the no-trade zone.

To identify the effect of a small change of \( \tau \) on the variance of the exchange rate, one can differentiate the variance with respect to \( \ln(1+\tau) \). It is easily verified that this derivative equals
the derivative with respect to the variance of the underlying, $\delta$ (as in the proof of Lemma 2.2), times $2\delta$. This multiplication by $2\delta$ does not affect any conclusion regarding the sign of the derivative. Thus, the conclusion is that a rise (fall) in $\tau$ increases (decreases) the volatility of the log exchange rate.
When the weights assigned by the central planner to the two countries are the same, then the critical loci that separate the no-trade domain from the region with trade are symmetric around the 45-degree line and are as follows: domestic exports if \( q_2(t)/q_1(t) < \kappa_1 \); foreign exports if \( q_2(t)/q_1(t) > \kappa_2 \); no-trade otherwise. The figure also shows the amounts of exports from country 1, \( x_1 \), and (net) imports into country 2, \( x_1/(1+\tau) \), that arise if the output point is given by \((q_1^*, q_2^*)\) in the zone of domestic exports. For any given output vector \((q_1^*, q_2^*)\) outside the no-trade zone, a smaller \( \tau \) (that is, a narrower no-trade zone) requires a larger amount of trade to bring consumption to the nearest bound.
References


