Behavioral Biases of Dealers in U.S. Treasury Auctions∗

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Abstract

This paper provides evidence of behavioral biases and bounded rationality by large dealers in U.S. Treasury auctions. I argue that these dealers use a heuristic of yield-space bidding which leads to biases manifested in three ways: the submission of dominated bids - those that could be improved without raising the bidding price; bidding in a manner that disregards the unevenly spaced grid; and rounding of bids in yield space. Consistent with bounded rationality, I show that bidders are less susceptible to this bias when the cost of suboptimal bidding is high. While the literature provides substantial evidence of behavioral biases among individual investors, it is less well documented for large sophisticated institutions that are likely to be important for setting of asset prices. These primary bond dealers who regularly bid for billions of dollars in Treasury bill auctions are precisely such economic agents.

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1 Introduction

Behavioral biases among individual investors have been documented by a growing literature. In fact, casual empiricism or simple introspection confirms that economic agents do not always behave fully rationally. However, this type of behavior has been less well documented for investors who are likely to be important in the setting of asset prices, i.e., large sophisticated institutions that participate repeatedly in the marketplace.

Primary bond dealers who participate regularly in U.S. Treasury bill auctions are precisely the type of economic agents that should be expected to act according to models of rational behavior. They regularly bid for billions of dollars worth of securities in competitive auctions, which are held a number of times each week.

In this paper, I document that in auctions for very-short-term Treasury bills, these dealers often submit suboptimal bids, i.e., they could increase the probability of winning the auction without changing the price they would pay for the securities. The observed suboptimal bids could be explained by bounded rationality (as in Gabaix and Laibson (2000)). Under bounded rationality, economic agents use heuristics to approximate an optimal action. Since Treasury auctions are held in yield space and fixed-income securities are usually quoted in yield space, I argue that the heuristic used by dealers in Treasury auctions involves choosing a bid in yield space rather than in price space. As explained below, certain bids in yield space can be shown to be suboptimal because of the details of the yield-price conversion in U.S. Treasury auctions.

Treasury auctions are conducted in yield space, and bidders that submit the lowest yields are awarded the securities. The yield is then converted into a price to determine the amount to be paid for the securities. For very-short-term Treasury bills, when converting between yield and price, the rounding rules are such that two different yields can correspond to the same price. This occurs because the grid in yield space can be finer than the grid in price space. Since the auction rules specify that the lowest submitted yields win the auction, there
can be two (or more) bids that are the same in price space, of which one will win the auction and one will lose. I refer to the bid with the higher yield as “dominated” since it is at the same price as the undominated bid, but with a lower probability of winning the auction.

This could simply be a curiosity in the auction rules, except that the allocation of many billions of dollars of securities are at stake in each auction; a large proportion of possible bids on the bidding grid are dominated, (including more than half of the allowable bids for the weekly four-week Treasury bill auction); and most importantly, auction participants submit dominated bids regularly.

I examine the phenomenon of dominated bids under both the traditional “multiple-price” auction format and the current “single-price” auction format. Under the multiple-price format, dominated bids are never optimal, and any dominated bids should be viewed as evidence of a behavioral bias. Under the single-price format, in certain cases dominated bids could be consistent with rationality, as they could be chosen with the intention of reducing the probability of being rationed at a lower price rather than to win at the bid price; but such bids should be observed only very infrequently. Empirically, under both auction formats I find a large proportion of dominated bids. Of observed bids in auctions for securities that have the potential for dominated bids, 28% of bids in multiple-price auctions and 52% of bids in single-price auctions are dominated. These results show that when bidding for Treasury bills, dealers do not fully optimize.

Since boundedly rational agents trade off the cost of optimization and its benefits, when a bidder has a high valuation and the benefit to optimizing his bid is large, he should be less likely to use the yield-space heuristic. Therefore, I distinguish between marginal bids (i.e., those at the market clearing price) and inframarginal bids (i.e., winning bids with a bid price above the market clearing price). I find that among inframarginal bids under the multiple-price format dominated bids relatively infrequent. The bias is reduced - but not eliminated - when there is both a high probability of winning the auction and the price to
be paid depends on the bid.

In addition to the frequency of dominated bids, I provide further evidence that bidders use the heuristic of yield space. In the bidding for three-month Treasury bills there is no possibility of dominated bids, but for related reasons, although the set of possible bids is evenly spaced in yield space, it is unevenly spaced in price space - the bid increment alternates between larger and smaller steps. Nevertheless, I find that in a sample of 760 auctions, both the marginal bids and the inframarginal bids are almost exactly evenly split between bids before the smaller increment and before the larger increment. While this is not definitive on its own, as the optimal bidding strategy is model dependent, it does add to the evidence that bidders think in yield space.

As a third piece of evidence of behavioral bias, I examine the tendency of bidders to bid in round numbers (e.g., 5.20% rather than 5.19% or 5.21%). This in itself is unlikely to be consistent with optimization. But the fact that the tendency to bid round numbers is manifested in yield space rather than price space further suggests that bidders use a yield space heuristic. Moreover, the nature of the rounding differs between marginal and inframarginal bidders.

The difference in the frequency of dominated bids between marginal and inframarginal bidders also has implications for the nature of Treasury auctions which are often modeled as common-value auctions. It suggests that inframarginal bids are not just those submitted by dealers who observe a high signal of a common value. Rather, the fact that inframarginal bidders submit dominated bids at a different frequency than marginal bidders indicates that inframarginal bidders know that they have a high probability of winning the auction, while marginal bidders know that they have a lower probability of winning. This suggests either a private-value component to the auction or a downward sloping demand curve for Treasury bills.

Because this paper considers large dealers, it complements the growing literature on
individual investors which documents behavior inconsistent with rational behavior.\textsuperscript{1} The behavioral finance literature is supported by a very large experimental literature showing how individual decision making is influenced by psychological effects. In addition, a series of papers going back to Simon (1955) argues that economic agents are only boundedly rational, i.e., they only approximately optimize because of the costs of perfect optimization.\textsuperscript{2}

One response in support of rational models is that individual investors are unlikely to be marginal in the pricing of securities. Individual investors are often small infrequent traders and unimportant in the pricing of securities. More important, the argument goes, is the behavior of large institutions that participate on a large scale on a regular basis. So the main point of this paper is to demonstrate that even large sophisticated dealers, repeatedly bidding for billions of dollars of securities, are subject to bounded rationality.

The rest of the paper proceeds as follows. Section 2 describes the market and the institutional details that allow for dominated bids. Section 3 presents the empirical evidence of behavioral biases and bounded rationality. In Section 4, I show formally the conditions under which dominated bids are compatible or incompatible with rationality. Section 5 concludes.

2 The auction and the rounding rules

In this section, I describe the Treasury auction, and explain how it is possible for two bids of the same price to have different priorities in Treasury auctions, i.e., for a bid to be dominated.

The focus of this paper is very-short-term Treasury bills, including “cash management

\textsuperscript{1}A very small sample of that literature includes the results that investors do not diversify their portfolios sufficiently (French and Poterba (1991), Huberman (2001), and Grinblatt and Keloharju (2001)); investors trade too frequently (Barber and Odean (2000)); and investors trade based on irrelevant past purchase prices (Odean (1998)). Recently, Elton, Gruber and Busse (2004) show that investors buy S&P 500 index mutual funds that are dominated by others with lower expenses. Rashes (2001) shows that even mistakes over ticker symbols can lead to substantial trading activity. See Barberis and Thaler (2003) for a recent review of theoretical and empirical behavioral finance.

\textsuperscript{2}See Camerer (1995) for a survey of the experimental literature on psychology and decision making. See Conlisk (1996) for a review of the literature on bounded rationality.
bills” (CMBs) of varying maturities auctioned irregularly to manage the short-term cash needs of the government, and four-week Treasury bills auctioned on a weekly basis since mid 2001. The maturity of CMBs can be from a single day to a year, but in recent years the maturity has typically been just a few weeks. The Treasury also auctions 13-week and 26-week bills on a weekly basis, as well as longer-term coupon-bearing notes, but for reasons that will soon be clear these securities are not subject to the phenomenon of dominated bids.

The issue size of CMBs varies substantially, but in recent years has averaged over $20 billion per auction. Since first issued, the weekly issue size of 4-week bills has averaged over $16 billion. Some 20 to 30 primary dealers submit the vast majority of bids in the auctions for these securities. Table 1 presents summary statistics of Treasury bill auction characteristics. The auctions are oversubscribed, with the quantity of bids exceeding supply by a factor of 2.3 to 4.3 (depending on the subset considered). For the purposes of this paper, the summary statistics are reported broken up by auction format and by tick size.

The auction proceeds as follows: A number of days prior to the auction the quantity and maturity of the securities to be issued are announced. Immediately before the auction deadline, each bidder submits (possibly multiple) yield-quantity pairs. The yields submitted by bidders are constrained to fall on a discrete grid. Currently, the bidding increment for all Treasury bills is a half basis point (0.005%), e.g., 3.240%, 3.245% etc. In the past, bids were submitted as multiples of a whole basis point.

The auctioneer determines the stop-out yield as the lowest yield at which the quantity demanded equals or exceeds the supply of securities. Bids at lower yields are awarded their demand in full. Bids exactly at the stop-out yield are awarded a fraction of their demand on a pro-rata basis to clear the market.

Under the traditional “multiple-price” format, used until November 1998, each winning bidder pays a price corresponding to his submitted yield. Under the newer “single-price” auction format, all winning bidders pay a price corresponding to the market-clearing stop-out
yield. This paper considers auctions under both formats.

Given the yield in the auction, the price per $100 of face value is

\[ P = 100(1 - yield \times ndays/360) \text{ rounded to the nearest } 0.001, \]

where \( ndays \) is the maturity of the bill measured in days. The yield used in this calculation is known as the “banker’s discount rate”.

The rounding of \( P \) to the nearest $0.001 is crucial to the present puzzle, since it is a second grid: first, the bids are constrained to a grid of one basis point (or, more recently, a half basis point) in yield space (the “bidding grid”), second, the corresponding dollar price is rounded to fall on a grid with a tick size of $0.001 (the “pricing grid”).

Consider a hypothetical Treasury bill with 36 days until maturity and a tick size of one basis point. For such a security, each basis point increment in the yield corresponds to a reduction of $0.001 in price. Hence, there is a one-to-one correspondence between the bidding grid and the pricing grid.

However, for bills of less than 36-days maturity, each basis point change in yield corresponds to a change of less than $0.001 in price space, leading to the possibility that two different bids in yield space could be rounded to the same price. Nevertheless, a pair of such bids would have different priorities in the determination of who wins the auction. The bid with the higher yield is dominated by the bid with the lower yield, since by lowering the yield a bidder could increase his probability of winning the auction without changing the price.

With a one basis point bidding grid, there is potential for a bid to be dominated for securities that have a maturity of less than 36 days. Under the half basis point tick size, auctions of securities with maturities of less than 72 days may have dominated bids. For

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3 In the auction literature, these formats are usually referred to as "uniform-price" and "discriminatory" mechanisms. In this paper, I use the Treasury’s terminology.

4 See Federal Register (1999) for the official rules of the auction including the yield-price conversion.
four-week Treasury bills (which have a half basis point tick size), for example, the pricing grid is \(72/28 = 2.57\) times as wide as the bidding grid. As a result, some 61\% (\(= 1.57/2.57\)) of all possible bids are dominated. See Table 2 for a numerical example of dominated bids.

For longer term bills there is no possibility of dominated bids. Nevertheless, the rounding rule results in a bidding grid that is not uniformly spaced in price space. For example, in a 13-week Treasury bill auction with a one basis point bidding increment in yield space, the increments in price space would alternate between $0.002 and $0.003.

The interpretation of dominated bids depends on the auction format. Under the multiple-price format, a rational bidder trying to win the auction would never submit a dominated bid. In such case, by submitting a bid at the next lower yield increment (corresponding to the same price), he could increase the likelihood of being awarded the securities in those states when the auction outcome suggests a high value. Under the single-price format dominated bids are not necessarily suboptimal, but should be an infrequent occurrence. It can be rational only if the bidder wants to ensure that his allocation will not be reduced by rationing at the next lower price, but values the bills at less than the price of his bid. In Section 4, I formalize these arguments.

3 Empirical results

In this section, I present the empirical results documenting behavioral biases in the bidding for Treasury bills. I show that the observed patterns are consistent with boundedly rational bidders using a heuristic of choosing bids in yield space. Such bidders are more likely to abandon the heuristic and optimize when the cost of deviating from optimality is high. The main evidence is the frequency of dominated bids. As corroborating evidence, I discuss the bidding for 13-week bills which have a bidding grid that is unevenly spaced in price space. I also discuss the tendency to bid in round numbers in yield space as further evidence of bounded rationality.
The data used in this paper are drawn from the summary statistics revealed by the Treasury after each auction. The statistics released by the Treasury depend on the auction format. For multiple-price auctions, the Treasury reveals the market-clearing yield and the lowest winning yield. For single-price auctions, the statistics include the market-clearing yield, the median winning yield, and the 95th-percentile winning yield (i.e., the lowest winning yield excluding the 5% tail of winning bids). Individual bids are never revealed. So although we do not observe each individual bid, the summary statistics allow one to observe the yield of the marginal bidder, as well as certain inframarginal bids.

3.1 Dominated bids

As explained above, since there is both a bidding grid in yield space and a second grid in price space, it is possible for a bid to be dominated when the maturity of a security is very short. In this section, I examine the frequency of dominated bids under both the multiple-price auction format and the single-price format. I distinguish between marginal bids and inframarginal bids to determine how a bidder’s valuation affects his tendency to submit a dominated bid.

First consider multiple-price auctions. If bidders are perfectly rational, we should not observe any dominated bids. At the other extreme, if bidders only think in terms of yield, we should observe dominated bids in proportion to the number of potential bids on the bidding grid that would be dominated. The middle case is that bidders are boundedly rational and, using a heuristic of choosing bids in yield space, are likely to submit dominated bids when the cost of doing so is low, but less likely to do so when the cost is high.

In the sample, there are 65 CMB auctions conducted under the multiple-price format. The frequency of dominated bids is reported in Panel A of Table 3. Among market-clearing bids, 43% are dominated. Among observed inframarginal bids, 14% are dominated. The existence of dominated bids, especially the high percentage among market-clearing bids is
strong evidence that bidders do not optimize, but are subject to a behavioral bias.\footnote{One could argue that bidders want to lose the auction because they view it as negative NPV, and are only bidding to satisfy the requirement that primary dealers bid meaningfully. However, this would contradict substantial evidence that bidders in Treasury auctions earn positive rents. (See Cammack (1991), Goldreich (2004), Nyborg and Sundaresan (1996).) Furthermore, the high levels of oversubscription suggest that bidders are actively trying to win the auctions.}

Of all possible yields on the bidding grid for these bills, an average of 58% would be dominated, so if bidders completely disregard the rounding rule, approximately 58% of all observed bids should be dominated. In fact, fewer bids are dominated, indicating that at least some bidders are aware of this possibility and, at least sometimes, avoid dominated bids. This difference between the potential frequency of dominated bids and the observed frequency is highly statistically significant.

Moreover, the difference in the frequency of dominated bids between marginal and inframarginal bids is statistically significant at the 1% level. This is suggestive of bounded rationality. Inframarginal bidders are those that choose to bid in a manner that has a high probability of winning, while at the same time paying the higher price of such a bid (because of the multiple-price format). Therefore, an inframarginal investor has incentive to choose a bid that simultaneously increases the probability of winning without raising the price excessively. Dominated bids are precisely those that an inframarginal bidder should avoid, as they lower the probability of winning without lowering the price. In comparison, marginal (market-clearing) bidders have a lower ex-ante probability of winning the auction, so their incentive to choose an undominated bid decreases accordingly. Thus, under the multiple-price format, bounded rationality implies that marginal bidders should be more likely to use the yield-space heuristic.

Panel B of Table 3 presents the results for 172 single-price auctions in which each winning bidder pays a price corresponding to the market-clearing yield. Note that under this format, the observed inframarginal bids in the data include the median and 95th-percentile winning yields, rather than the lowest winning yield.
The results are superficially similar to those of the multiple-price auction. A large percentage of both marginal and inframarginal bids are dominated. The percentage of dominated bids among market-clearing bids and median winning bids is statistically significantly less than the 64 percent of all possible yields on the bidding grid that would be dominated. (In the case of the 95th percentile winning bids, this difference is only significant at the 10% level.)

The striking difference in contrast to the multiple-price auction is that under the single-price auction, inframarginal bids are more likely to be dominated than market-clearing bids. This difference is perfectly consistent with bounded rationality. Inframarginal bidders may be those with a high private valuation who knowingly submit a bid with a very high probability of winning, but since under the single-price format all winning bidders pay the same price, they have little incentive to carefully choose a bid from among those that are likely to win. However, marginal bidders are more likely to be those that only want to win if the price is sufficiently low, and are more likely to exert the effort to choose a bid carefully. Thus, marginal bidders are less likely to use a yield-space heuristic and submit dominated bids.

To summarize, the observation that bidders often submit dominated bids is evidence of a behavioral bias on behalf of the large primary dealers who participate in Treasury auctions. However, it is the pattern in the frequency of dominated bids – more frequent for marginal bids under the multiple-price format and more frequent for inframarginal bids under the single-price format – that provides evidence of bounded rationality. Bidders use the yield-space heuristic and submit dominated bids more frequently when the benefits of optimization are low.

This distinction between the bidding strategies of inframarginal bidders across auction formats, that they are more cautious under the multiple-price format, is consistent with patterns in the observed spread of winning bids. The difference between the marginal bid and the most inframarginal observed bid is typically much wider under the single-price
format than under the multiple-price format. Although this could be interpreted as more price uncertainty surrounding single-price auctions, in light of the evidence above it seems more likely that inframarginal bidders under the single-price format choose to bid more aggressively since their bids are unlikely to directly affect the price.

The differences between marginal and inframarginal bidders have implications for the nature of Treasury securities as common-value or private-value goods. Bidders in Treasury auctions are often modeled as homogenous investors, each with a signal of a common value. However, the results in this section suggest that bidders are not homogenous, and that inframarginal bidders are not simply those that happen to observe a high signal. Instead, they actively choose to submit a bid that has a higher probability of winning the auction. This would occur if, besides the obvious common-value component, Treasury securities also have a private value component. Bidders that have a high private value submit higher bids to ensure winning the auction and are more likely to be inframarginal.

While the evidence of behavioral bias inherent in dominated bids is clear, it is also important to discuss economic significance. The pricing grid is very small as a percentage of face value—only $0.001 per $100 of face value, but it is large when multiplied by more than $40 billion of Treasury bills issued each week. However, it is not the dollar value of the tick size that is important here, since the phenomenon of dominated bids relates to allocation rather than price.

The relevant economic measure is the quantity of bills awarded to bids at the margin. Bidders who submit a dominated bid are reducing their probability of being awarded these securities. Although this quantity is not known, it can be roughly estimated based on the summary statistics revealed after each auction. On average, the difference between the market-clearing bid and the median winning bid is 1.5 basis points, or three ticks, for four-week Treasury bills. Conservatively assuming that the distribution of bids is uniform over
this range, this corresponds to $2.7 billion at the margin for a typical $16 billion auction.\footnote{Since the average difference between the median winning bid and the 95th percentile winning bid (3.5 basis points) is much wider than the average difference between the market-clearing bid and the median winning bid, the assumption of a uniform distribution is conservative and it is likely that the average quantity bid at the market-clearing bid exceeds $2.7 billion.}

After accounting for rationing among bidders at the margin, there are some $1.3 billion of securities that are allocated to bidders who submit the market-clearing bid. A bidder who submits a dominated bid reduces his probability of being allocated these securities.

### 3.2 Unevenly spaced bidding grid

The actions of bidders in the presence of the rounding rule leads to additional evidence that bidders use a heuristic of determining their bids in yield space.

The rounding rule in the yield-price conversion described above leads to an unevenly spaced bidding grid in price space. Since prices are rounded to the nearest $0.001, the grid step in price space (i.e., the price difference between two adjacent bids) is sometimes larger and sometimes smaller. For example, changing a bid by one basis in yield may correspond to a price change of $0.001, but the next basis point change in yield may correspond to an additional $0.002, or vice versa. (See Table 4 for an example of uneven grid steps for 13-week bills.) Dominated bids are simply an extreme form of this in which the size of a step is zero.

The average step size in price space corresponding to an incremental change in the bid yield is $0.001 \times (\delta \times \text{ndays}/36)$, where $\delta$ is the bidding increment in basis points and $\text{ndays}$ is the maturity of the bill in days. Only when the maturity of a bill is such that $\delta \times \text{ndays}/36$ is equal to an integer are grid steps of constant size in price space. Otherwise there will be large steps and small steps. For thirteen-week Treasury bills (under a one basis point bidding grid), the step size averages approximately $0.0025$, so the step size almost always alternates between $0.002$ and $0.003$.

The equilibrium bidding strategy in the presence of an uneven grid is model dependent.
For the purposes of this paper I do not impose any one model. Nevertheless, a bidder facing an uneven grid should consider the step size when evaluating potential bids. A bidder may optimally raise his yield by one tick if it corresponds to a substantially lower price, but not if it lowers the price by only a small amount. Of course, in equilibrium he will have to consider how other bidders respond to the uneven grid.

However, if a bidder uses a yield-space heuristic, the distribution of bids in yield space will be unaffected by the uneven steps in the pricing grid.

Since the pricing grid alternates between $0.002 and $0.003 for the 13-week bill (for a one basis point tick size, as was the case from 1983 to 1997), it provides an opportunity to observe bidders’ response to an uneven grid. A bid can either be before a “large” step or before a “small” step (i.e., so that an increase in yield of one basis point corresponds to a reduction of $0.003 or $0.002 in price space, respectively). Table 5 reports the distribution of bids between those before a large step and those before a small step for 760 13-week auctions conducted under the multiple-price format. I exclude the small number of bids that are in between two large steps or in between two small steps. If bidders rationally optimize, we are likely to observe different probabilities of bids before a large step and bids before a small step. However, if bidders think in yield space, they do not distinguish between large and small steps, and we should observe each with equal probability.

I find that almost exactly 50% of bids are before large steps and 50% before small steps. This holds for both marginal and inframarginal bids. This is consistent with bidders using a yield-space heuristic and not paying attention to the uneven grid in price space. Of course, without a well-defined alternative hypothesis, which is avoided here to keep the argument model independent, one cannot reject rationality. However, the tight confidence intervals around 50% limit the possible rational equilibria and are suggestive of bidders thinking in yield-space.
3.3 Bidding in round numbers

The third piece of evidence of behavioral biases comes from the tendency of bidders to choose “round” numbers in yield space when submitting bids. For example, in the auctions since the bidding grid was reduced from one basis point to a half basis point, more than 75% of observed bids were still submitted as multiples of whole basis points. This tendency to submit bids in round numbers is in itself difficult to reconcile with optimization, but the fact that the bids are rounded in yield space adds extra support to the hypothesis that investors think in yield space. Moreover, there is evidence that the rounding of bids is different between marginal bids and inframarginal bids in a manner suggestive of the behavioral biases proposed in this paper.

I study the rounding tendency by considering 1,826 multiple-price auctions of Treasury bills of all maturities in which there is a tick size of one basis point. This includes every Treasury bill auction until the end of 2003 conducted in yield space that had a one basis point tick size, except for the small number of single-price auctions with a one basis point tick size. Table 6 reports the distribution of the final digit in observed bids. The first result is that the distribution of final digits is not uniformly distributed (and rejected at the 1% level). Consistent with a rounding tendency, the most common final digit is zero for both market-clearing bids and for the most inframarginal bids. The frequency of zero as a final digit is statistically significantly different from 1/10 at the 1% level.

The extent of the bias towards a final digit of zero should be interpreted in light of the fact that the entire spread of winning bids in these auctions averages only 3.2 basis points. So any rounding to zero would only occur when the bid would have been very close to zero anyway, for example when the optimal bid would have had a final digit of one or nine.

In fact, the distribution of bids allows us to see the direction of the bias. Among market-clearing bids, the least frequent final digit is nine, suggesting that the rounding tendency is manifested by bidders rounding up in yield space from nine to zero, (i.e., rounding down in
price space). In contrast, the frequency of a final digit of one is not statistically different from 1/10, so the rounding tendency is only in one direction.

The opposite result is obtained for inframarginal bids. A final digit of one is observed least frequently, while a final digit of nine occurs with a frequency not statistically different from 1/10. Bidders round down (in yield space).

The difference in the direction of rounding between market-clearing and inframarginal bids suggests that rounding is not only a phenomenon of bidders being unable to refine their bids sufficiently, but rather it relates to bidders’ valuations. As such, it is consistent with the argument that there is a private-value component to bidders’ valuations. Market-clearing bidders are those that are only willing to purchase the bills if the price is sufficiently low. To the extent that they have a tendency to round, it is manifested by a tendency to bid more conservatively. In contrast, inframarginal bidders are those that deliberately submit bids with a high probability of winning, and rounding is manifested in an even more aggressive bid.

The tendency to choose round numbers in Treasury auctions is not at all the same as the tendency for stocks, or other assets, to trade on round prices (as in Harris (1991)). Harris attributes the tendency to trade in round numbers to the resulting reduction in negotiation costs between the two counterparties. By contrast, in the case of Treasury auctions, there is no negotiation between counterparties. Each bidder unilaterally chooses a yield, and the auctioneer just calculates the market clearing price. Rather than reducing transaction costs, favoring round numbers can only reduce the thinking cost to the bidder. This is exactly the same concept as boundedly rational investors using a heuristic; in this case the heuristic is to evaluate potential bids in yield space and to favor round yields.

In summary, the evidence of rounding suggests a bias in its own right, but additionally it is consistent with boundedly rational dealers using a yield space heuristic, especially when considered in conjunction with the observed frequency of dominated bids (Section 3.1) and
bidders’ tendency to ignore unevenness in grid steps in price space (Section 3.2).

4 Are dominated bids ever rational?

One of the central points of this paper is that the observed frequency of dominated bids is evidence of behavioral bias. In this section, in a fairly general setting, I show that dominated bids are not consistent with optimal bidding under the multiple-price auction format, and I show the conditions under which dominated bids are incompatible with optimization under the single-price format.

Consider an expected profit maximizing bidder who must choose a price for his bid. Normalize his bid quantity to one and, for simplicity, assume that this bid quantity is sufficiently small that it does not affect the market-clearing stop-out price.

Denote the bidding grid (in price space) of allowable prices as \( P_i, i = 1, 2, 3..., \) where \( P_i \) is (weakly) increasing in \( i \). The probability, from the bidder’s perspective, of \( P_i \) being the market-clearing price is denoted \( \pi_i(> 0) \).

Because Treasury bills presumably have a common value component, the bidder’s valuation of the security depends on the outcome of the auction. Denote \( V_i \) as the value of the security to the bidder if the market-clearing price is \( P_i \). (The function \( V_i(P_i) \) is likely to depend on the auction format.)

Denote the bid price submitted by the bidder as \( P_b \). If this bid is above the market-clearing price (or more precisely, if \( b > m \), where \( P_m \) is the market-clearing price), the bidder wins the auction and is awarded the security. Because in practice there is always rationing at the market-clearing price, assume that if the bid is equal to the market clearing price, (i.e., if \( b = m \)) the bidder is awarded half a security.

The price paid by a winning bidder depends on the auction format.
4.1 Multiple-price format

Under the multiple-price format, each winning bidder pays the price in his bid regardless of the market-clearing price. Thus, for a bid price $P_b$, the expected profit to a bidder is

$$Profit_b = \sum_{j=1}^{b-1} \pi_j (V_j - P_b) + \frac{1}{2} \pi_b (V_b - P_b)$$  \hspace{1cm} (1)

For a bid $P_b$ to be optimal, it is necessary that

$$Profit_b > 0$$  \hspace{1cm} (2)

$$Profit_b \geq Profit_{b+1}$$  \hspace{1cm} (3)

$$Profit_b \geq Profit_{b-1}$$  \hspace{1cm} (4)

Let us now consider the possibility of a bidder submitting a dominated bid $P_b$, where $P_b = P_{b+1}$.

Consider the expected profit (1). By assumption, $V_j$ is increasing in $j$. So $(V_b - P_b)$ is similarly increasing in $j$. Since $Profit_b$ is positive (from (2)) and the weights $\pi_j$ are non-negative, the state with the highest per-unit profit must have positive profit, i.e.,

$$V_b - P_b > 0.$$  \hspace{1cm} (5)

For the dominated bid to be optimal, $Profit_{b+1} - Profit_b$ must be negative. From (1), and using the definition of a dominated bid, $P_b = P_{b+1}$,

$$Profit_{b+1} - Profit_b = \frac{1}{2} \pi_b (V_b - P_b) + \frac{1}{2} \pi_{b+1} (V_{b+1} - P_b)$$  \hspace{1cm} (6)

From (5), the first term is positive, and since $V_i$ is an increasing function, the second term is also positive.
Thus $\text{Profit}_{b+1} - \text{Profit}_b > 0$, contradicting the optimality of the dominated bid.

Intuitively, the dominated bid $P_b$ is suboptimal for the following reason. First, a bid of $P_{b+1}$ does not change the profit when the market-clearing bid is below $P_b$. However, it awards more securities when the market-clearing bid is $P_b$ or $P_{b+1}$. But since these are the states in which $V_i$ is highest, and since $P_{b+1} = P_b$, then if it is worthwhile to bid $P_b$ (i.e., if expected profits are positive), changing the bid to $P_{b+1}$ can only increase expected profits.

Thus, observed dominated bids under the multiple-price format are inconsistent with dealers acting optimally.

### 4.2 Single-price format

Under the single-price format, all winning bidders pay the market clearing price. For a bid price $P_b$, the expected profit to a bidder is

$$\text{Profit}_b = \sum_{j=1}^{b-1} \pi_j (V_j - P_j) + \frac{1}{2} \pi_b (V_b - P_b) \quad (7)$$

Unlike the case of the multiple-price format I show below that under certain conditions it is possible for a rational bidder to submit a dominated bid under the single-price format. This can occur because increasing a bid from $P_{b-1}$ to $P_b$ (which is dominated and equal to $P_{b+1}$) has two effects on the bidder’s allocation. It results in a (rationed) allocation when the market clearing bid is $P_b$, and it also results in a full allocation (rather than rationing) when the market-clearing bid is $P_{b-1}$. Thus, a bidder may choose to submit a dominated bid $P_b$ if the profit conditional on a market clearing price of $P_b$ is negative, but the profit conditional on a market clearing price of $P_{b-1}$ is positive and more than offsets the negative profit. Such a bidder will not increase his bid to the undominated $P_{t+1}$ since that only increases his allocation in those states for which his profit is negative.

I now formalize the argument to show the conditions under which dominated bids are
consistent with rationality and the conditions under which they are not.

As before, for a bid $P_b$ to be optimal, it is necessary that

$$\text{Profit}_b > 0$$  \hfill (8)

$$\text{Profit}_b \geq \text{Profit}_{b+1}$$  \hfill (9)

$$\text{Profit}_b \geq \text{Profit}_{b-1}$$  \hfill (10)

Substitute (7) into (9) and (10) to obtain

$$\frac{1}{2} \pi_{b+1} (V_{b+1} - P_{b+1}) + \frac{1}{2} \pi_b (V_b - P_b) \leq 0$$  \hfill (11)

$$\frac{1}{2} \pi_b (V_b - P_b) + \frac{1}{2} \pi_{b-1} (V_{b-1} - P_{b-1}) \geq 0$$  \hfill (12)

Noting that $(V_i - P_i)$ decreases in $i$, a bid $P_b$ is optimal if both $\pi_{b+1} (V_{b+1} - P_{b+1})$ is sufficiently negative and not fully offset by (a possibly positive) $\pi_b (V_b - P_b)$, and also $\pi_{b-1} (V_{b-1} - P_{b-1})$ is sufficiently positive and not fully offset by (a possibly negative) $\pi_b (V_b - P_b)$.

For the purposes of clarity and to create a measure of the possibility that a bid will be optimal, let us add some structure and make some simplifying assumptions. Assume the following: $\pi_{b-1} = \pi_b = \pi_{b+1}$; the tick size in price spice (other than for dominated bids) is a constant $\Delta = P_i - P_{i-1}$; and the increase in the bidder’s valuation for an increase of $\delta$ in price (other than for dominated bids) is a constant $\alpha = V_i - V_{i-1}$, where $0 \leq \alpha < \Delta$.

With these assumptions, substituting into (11) and (12) results in

$$(\alpha - \Delta) + 2 (V_b - P_b) \leq 0$$  \hfill (13)

$$(\Delta - \alpha) + 2 (V_b - P_b) \geq 0$$  \hfill (14)

or,

$$-\frac{(\Delta - \alpha)}{2} \leq (V_b - P_b) \leq \frac{(\Delta - \alpha)}{2}$$  \hfill (15)
Under this condition, i.e., if the value of the security to the bidder is close to the bid price, $P_b$ is optimal. The range of possible values for $V_b - P_b$ is of width $\Delta - \alpha$. (Note that if we allow $\alpha$ to be larger than $\Delta$ then there would be no finite optimal bid, as bidders would have upward sloping demand curves.)

Now consider the possibility of a dominated bid $P_b$. In such case $P_b - P_{b-1} = \Delta$, and $P_{b+1} - P_b = 0$. We also have to address the value function $V_i$. Below the dominated bid, $V_b - V_{b-1} = \alpha$ as before, but $V_{b+1} - V_b$ is ambiguous. An increase in the market-clearing bid from $P_b$ to $P_{b+1}$ signifies increased demand, so the value of the security surely increases. However, since this doesn’t correspond to an increase in the actual price of the market-clearing bid, the increase in value may be less than $\alpha$. Denote the value increase as $\beta \alpha = V_{b+1} - V_b$, where $0 < \beta < 1$. In other words, $\beta$ represents the incremental increase in the value of the security as a proportion of the “normal” incremental value increase.

It follows that a dominated bid, $P_b$, is optimal if

$$\frac{-(\Delta - \alpha)}{2} \leq (V_b - P_b) \leq \frac{0 - \beta \alpha}{2}$$

(16)

The width of the range of possible values of $V_b - P_b$ which allow dominated bids to be optimal is

$$\frac{\Delta - (1 + \beta) \alpha}{2}$$

which is much narrower than the width $\Delta - \alpha$ that allows a bid to be optimal in the absence of dominated bids.

In particular, if $\Delta < (1 + \beta) \alpha$, then dominated bids are never optimal. In words, for an increase in the market-clearing bid from $P_{b-1}$ to $P_{b+1}$, if the increase in the bill’s value to the bidder exceeds a single price tick $\Delta$, then dominated bids are incompatible with rationality. If the inequality is reversed, then dominated bids would be observed, albeit with less frequency than undominated bids.
So while the existence of dominated bids under the single-price format is not *per se* proof of behavioral bias, the frequency of the dominated bids, the differences between marginal and inframarginal bids, and the existence of dominated bids under the multiple-price format, is evidence of behavioral bias and is consistent with bounded rationality.

5 Conclusion

Complementing the literature on behavioral biases in individual investors, I have shown that even large sophisticated institutions are subject to behavioral biases. In auctions for U.S. Treasury bills, primary bond dealers regularly submit dominated bids, they disregard uneven steps in the pricing grid, and they round their bids in yield space. Most importantly, when comparing single-price and multiple-price auctions, and when comparing marginal bids and inframarginal bids, I find that dealers are less likely to be subject to this bias when the cost of suboptimal bidding is higher. These results are consistent with boundedly rational dealers using a yield-space heuristic when choosing their bids.
References


Table 1: Short-Term Treasury Bill Auction Summary Statistics

This table summarizes the auction characteristics for Treasury bills, including cash management bills (CMBs), four-week bills, and longer term (13-week, 26-week and 52-week) bills from April 1983 to December 2003. (Four-week bills were first auctioned in July 2001. Fifty-two week bills were discontinued in February 2000.) The statistics for CMBs are also reported excluding those with a maturity long enough to preclude the possibility of dominated bids. Under the multiple-price format, each winning bidder pays a price corresponding to his submitted bid. Under the single-price format, all winning bidders pay a price corresponding to the yield of the market-clearing bid. Tick size refers to the grid in yield space. Maturity is the number of days in the life of the security. Bid-to-cover is the ratio of the quantity of tenders to supply. Bid spread is the difference between the market clearing yield and the lowest winning yield (or, under the single-price format, the difference between the market clearing yield and the 95th percentile winning yield).

<table>
<thead>
<tr>
<th>Date range</th>
<th># of auctions</th>
<th>Auction format</th>
<th>Tick size (basis points)</th>
<th>Avg auction size ($ billion)</th>
<th>Avg maturity (days)</th>
<th>Average bid-to-cover</th>
<th>Bid spread (basis points)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5/1983 - 8/1998</td>
<td>116</td>
<td>Multiple Price</td>
<td>1.0</td>
<td>11.6</td>
<td>44.3</td>
<td>4.3</td>
<td>4.6</td>
</tr>
<tr>
<td>exclud. &gt; 35 days</td>
<td>65</td>
<td></td>
<td></td>
<td>12.4</td>
<td>15.2</td>
<td>4.3</td>
<td>6.0</td>
</tr>
<tr>
<td>CMBs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11/1998 - 3/2002</td>
<td>33</td>
<td>Single Price</td>
<td>1.0</td>
<td>24.5</td>
<td>26.2</td>
<td>2.4</td>
<td>5.9</td>
</tr>
<tr>
<td>exclud. &gt; 35 days</td>
<td>24</td>
<td></td>
<td></td>
<td>24.1</td>
<td>14.7</td>
<td>2.5</td>
<td>10.1</td>
</tr>
<tr>
<td>4/2002 - 12/2003</td>
<td>22</td>
<td>Single Price</td>
<td>0.5</td>
<td>16.6</td>
<td>8.7</td>
<td>3.2</td>
<td>4.5</td>
</tr>
<tr>
<td>exclud. &gt; 71 days</td>
<td>22</td>
<td></td>
<td></td>
<td>16.6</td>
<td>8.7</td>
<td>3.2</td>
<td>4.5</td>
</tr>
<tr>
<td>4-Week Bills</td>
<td>7/2001-12/2003</td>
<td>126</td>
<td>Single Price</td>
<td>0.5</td>
<td>16.3</td>
<td>28.0</td>
<td>2.3</td>
</tr>
<tr>
<td>Longer-Term Bills</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4/1983-11/1997</td>
<td>1710</td>
<td>Multiple Price</td>
<td>1.0</td>
<td>9.5</td>
<td>161.8</td>
<td>3.4</td>
<td>3.1</td>
</tr>
<tr>
<td>11/1997-10/1998</td>
<td>114</td>
<td>Multiple Price</td>
<td>0.5</td>
<td>7.4</td>
<td>160.4</td>
<td>3.3</td>
<td>2.2</td>
</tr>
<tr>
<td>11/1998-12/2003</td>
<td>592</td>
<td>Single Price</td>
<td>0.5</td>
<td>12.0</td>
<td>145.0</td>
<td>2.0</td>
<td>6.0</td>
</tr>
</tbody>
</table>
Table 2: Example of Possible Dominated Bids

This table displays a portion of the bidding grid for a 20-day cash management bill to illustrate the possibility of dominated bids. In this example, bids are submitted in one basis point increments. Prices are rounded to the nearest $.001 per $100 face value. Bids marked with "D" are "dominated".

<table>
<thead>
<tr>
<th>Rate</th>
<th>Price (unrounded)</th>
<th>Price (rounded)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.02%</td>
<td>99.83222</td>
<td>99.832</td>
</tr>
<tr>
<td>3.03%</td>
<td>99.83167</td>
<td>99.832 D</td>
</tr>
<tr>
<td>3.04%</td>
<td>99.83111</td>
<td>99.831</td>
</tr>
<tr>
<td>3.05%</td>
<td>99.83056</td>
<td>99.831 D</td>
</tr>
<tr>
<td>3.06%</td>
<td>99.83000</td>
<td>99.830</td>
</tr>
<tr>
<td>3.07%</td>
<td>99.82944</td>
<td>99.829</td>
</tr>
<tr>
<td>3.08%</td>
<td>99.82889</td>
<td>99.829 D</td>
</tr>
<tr>
<td>3.09%</td>
<td>99.82833</td>
<td>99.828</td>
</tr>
</tbody>
</table>
Table 3: Frequency of Dominated Bids

This table displays the frequency of dominated bids among observed bids for short-term Treasury bill auctions under both auction formats. Cash management bill (CMB) auctions are included only when the maturity is short enough for the possibility of dominated bids. Under the multiple-price format, the market-clearing yield and the lowest winning yield are observed. Under the single-price format, the observed bids include the market-clearing yield (i.e., highest winning yield), the median winning yield, and the 95th-percentile winning yield (i.e., the lowest winning yield excluding the 5% tail of winning bids).

Panel A: Multiple-price auctions (cash management bills)

<table>
<thead>
<tr>
<th></th>
<th>Number of dominated bids</th>
<th>Percentage of observed bids that are dominated</th>
<th>Percentage of bids on the bidding grid that would be dominated (assuming random bidding)</th>
<th>P-value of difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market clearing yield</td>
<td>28</td>
<td>43%</td>
<td>58%</td>
<td>1.08%</td>
</tr>
<tr>
<td>Lowest winning yield</td>
<td>9</td>
<td>14%</td>
<td>58%</td>
<td>0.00%</td>
</tr>
<tr>
<td># of auctions = 65</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Single-price auctions (cash management bills and four-week bills)

<table>
<thead>
<tr>
<th></th>
<th>Number of dominated bids</th>
<th>Percentage of observed bids that are dominated</th>
<th>Percentage of bids on the bidding grid that would be dominated (assuming random bidding)</th>
<th>P-value of difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market clearing yield</td>
<td>75</td>
<td>44%</td>
<td>64%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Median winning yield</td>
<td>92</td>
<td>53%</td>
<td>64%</td>
<td>0.47%</td>
</tr>
<tr>
<td>95th percentile winning yield</td>
<td>100</td>
<td>58%</td>
<td>64%</td>
<td>9.51%</td>
</tr>
<tr>
<td># of auctions = 172</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4: Example of Uneven Grid Steps (13-Week Treasury Bill)

This table displays a portion of the bidding grid for 91-day Treasury bills to illustrate the uneven grid steps in price space. In this illustration, the bidding grid is one basis point in yield space. Prices are rounded to the nearest $.001 per $100 face value. *Step size* is the price difference between two adjacent bids.

<table>
<thead>
<tr>
<th>Rate</th>
<th>Price</th>
<th>Step size</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.00%</td>
<td>98.989</td>
<td>0.003</td>
</tr>
<tr>
<td>4.01%</td>
<td>98.986</td>
<td>0.002</td>
</tr>
<tr>
<td>4.02%</td>
<td>98.984</td>
<td>0.003</td>
</tr>
<tr>
<td>4.03%</td>
<td>98.981</td>
<td>0.002</td>
</tr>
<tr>
<td>4.04%</td>
<td>98.979</td>
<td>0.003</td>
</tr>
<tr>
<td>4.05%</td>
<td>98.976</td>
<td>0.002</td>
</tr>
<tr>
<td>4.06%</td>
<td>98.974</td>
<td></td>
</tr>
</tbody>
</table>
**Table 5: Bidding and Uneven Grid Steps**

Under a one basis point bidding grid (as was the case from 1983 to 1997), the pricing grid for 91-day Treasury bills alternates between $0.003 and $0.002 with few exceptions. A *bid before a "large" step* is an observed bid such that an increase in one basis point in yield would correspond to $0.003 decrease in price, i.e., \( P(r) - P(r+\delta) = .003 \), where \( P(r) \) is the price corresponding to a yield bid of \( r \), and \( \delta \) is the one basis point bidding increment. A *bid before a "small" step* is an observed bid such that an increase in one basis point in yield would correspond to $0.002 decrease in price, i.e., \( P(r) - P(r+\delta) = .002 \).

Occasionally, bids are in between two large steps or in between two small steps and are thus excluded from the data. The auctions were conducted under the multiple-price format, and the observed bids are the market clearing yield and the lowest winning yield.

<table>
<thead>
<tr>
<th></th>
<th>Bids before a &quot;large&quot; step</th>
<th>Bids before a &quot;small&quot; step</th>
<th>( \chi^2_i )</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market clearing yield</td>
<td>353 (49.2%)</td>
<td>365 (50.8%)</td>
<td>0.201</td>
<td>65.4%</td>
</tr>
<tr>
<td>Lowest winning yield</td>
<td>358 (50.7%)</td>
<td>348 (49.3%)</td>
<td>0.142</td>
<td>70.7%</td>
</tr>
</tbody>
</table>

# of auctions = 760
Table 6: Rounding in Yield Space

This table reports the distribution of the last digit of observed bids in yield space. The data includes all Treasury bill auctions conducted under the multiple-price format with a bid increment of one basis point. The data includes 13-week, 26-week and 52-week Treasury bill auctions from 1983 until 1997 and cash management bills from 1983 until 1998. The observed bids are the market-clearing yield and the lowest winning yield.

<table>
<thead>
<tr>
<th>Last digit in bid yield</th>
<th>Frequency</th>
<th>H₀: p = 10%</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Least frequent</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.</td>
<td>9</td>
<td>140</td>
<td>0.1%</td>
</tr>
<tr>
<td>.</td>
<td>6</td>
<td>154</td>
<td>2.6%</td>
</tr>
<tr>
<td>.</td>
<td>1</td>
<td>165</td>
<td>17.0%</td>
</tr>
<tr>
<td>.</td>
<td>3</td>
<td>175</td>
<td>55.3%</td>
</tr>
<tr>
<td>.</td>
<td>7</td>
<td>180</td>
<td>83.9%</td>
</tr>
<tr>
<td>.</td>
<td>2</td>
<td>188</td>
<td>67.4%</td>
</tr>
<tr>
<td>.</td>
<td>8</td>
<td>188</td>
<td>67.4%</td>
</tr>
<tr>
<td>.</td>
<td>5</td>
<td>207</td>
<td>5.7%</td>
</tr>
<tr>
<td>.</td>
<td>4</td>
<td>208</td>
<td>4.8%</td>
</tr>
<tr>
<td>Most frequent</td>
<td>0</td>
<td>221</td>
<td>0.3%</td>
</tr>
</tbody>
</table>

Joint test of discrete uniform distribution

\[ \chi_9^2 = 31.7 \]

P-value = 0.0%

<table>
<thead>
<tr>
<th>Last digit in bid yield</th>
<th>Frequency</th>
<th>H₀: p = 10%</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lowest winning yield</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.</td>
<td>1</td>
<td>151</td>
<td>1.4%</td>
</tr>
<tr>
<td>.</td>
<td>4</td>
<td>161</td>
<td>9.2%</td>
</tr>
<tr>
<td>.</td>
<td>6</td>
<td>168</td>
<td>25.5%</td>
</tr>
<tr>
<td>.</td>
<td>5</td>
<td>173</td>
<td>45.4%</td>
</tr>
<tr>
<td>.</td>
<td>3</td>
<td>174</td>
<td>50.2%</td>
</tr>
<tr>
<td>.</td>
<td>9</td>
<td>176</td>
<td>60.7%</td>
</tr>
<tr>
<td>.</td>
<td>8</td>
<td>195</td>
<td>33.3%</td>
</tr>
<tr>
<td>.</td>
<td>7</td>
<td>203</td>
<td>11.2%</td>
</tr>
<tr>
<td>.</td>
<td>2</td>
<td>209</td>
<td>3.9%</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>216</td>
<td>0.9%</td>
</tr>
</tbody>
</table>

Joint test of discrete uniform distribution

\[ \chi_9^2 = 23.4 \]

P-value = 0.5%

# of auctions = 1826