Rational Exuberance

or

Overinvestment as an Optimal Adaptive Control

Timothy C. Johnson*

November, 2002

Abstract

It is becoming conventional wisdom that major technological innovations inevitably spark speculative fever, resulting in wasteful overinvestment. This paper presents a general equilibrium model of investment in a new industry, whose production function is not known in advance. In the context of the model, overinvestment is actually socially optimal as the most efficient way to learn about the new technology. The model thus provides an explanation for the perceived linkage between innovation and apparent irrationality, even though no irrationality is at work. Further, the initial overinvestment is accompanied in the model by apparently inflated prices and apparently negative expected excess returns in the shares of the new industry, all of which are also fully rational. This suggests a new explanation for observed patterns of returns in new industries.

Keywords: bubbles, learning, adaptive control.

JEL CLASSIFICATIONS: G12, D8

*London Business School: tjohnson@london.edu. I am indebted to Viral Acharya, Suleyman Basak, Francisco Gomes, Denis Gromb, Anthony Neuberger, Anna Pavlova and seminar participants at LBS for many thoughtful comments.
Every previous technological revolution has created a speculative bubble, and there is no reason why IT should be different.
– The Economist 23 September, 2000

1 Introduction

It is now widely believed that technological breakthroughs inevitably entail economic excess. Parallels between the recent internet boom/bust and earlier waves of innovation going back to the eighteenth century have convinced many observers that something like a natural law takes over during these episodes, preventing agents from accurately assessing growth rate prospects. Moreover the mechanism of causality in this process is widely held to be obvious: the bubble is inevitably driven by the financial sector. The real economy merely responds logically, even helplessly, to the dictates of the stock market as it skews the price of capital first sharply towards, and then belatedly away from fledgling firms.

Anecdotal evidence is hard to assess. After all, revolutions mean high growth rates; growth rates are hard to forecast; and small errors can lead to huge mistakes in valuation. So perhaps inferences about inevitable folly are merely being drawn from a few highly visible errors, while instances of accurate (or under) estimation of growth rates go unremarked.

However there is academic support for the idea of systematic bias in the empirical finance literature. Although the number of true revolutions in available samples is small, clear patterns emerge from both the time-series and the cross-section of returns of high-growth or innovative firms. Shares of initial public offerings (Ritter (1991)) and other

---

1 “For a rising chorus of economists and business historians, the railroad’s parallels with the Internet are too striking to ignore...[T]he same pattern holds for three other tech-driven economic movements as well: the Industrial Revolution of the late 1700s in England, the age of cheap steel and electricity in the late 1800s in the U.S. and Germany, and the automobile and mass-production era starting about 1910. After a gestation period of a decade or more, the new technology usually sparks a boom followed by a sudden bust.” (Business Week May 13, 2002).

2 This view is stated succinctly in the IMF’s October 2001 WORLD ECONOMIC OUTLOOK (International Monetary Fund October 1, 2001). “As in past technological revolutions, the initial phase of the IT revolution appears to have been characterized by excessive optimism about the potential earnings of innovating firms. This over-optimism led for several years to soaring stock prices of IT firms, which made equity finance cheaper and more readily available, which in turn boosted investment by IT firms.”
“glamour” stocks (Lakonishok, Schleifer, and Vishny (1994)) invariably underperform, and this underperformance has not been convincingly attributed to any extraordinary (absence of) risk exposures. Similarly, evidence of time-series predictability in aggregate market returns is consistent with the view that high growth expectations tend to be disappointed (Campbell and Shiller (1998)). Meanwhile, survey evidence does seem to point to systematic overoptimism by analysts and predictable revision of growth rate forecast (Chan, Karceski, and Lakonishok (2001)).

Given these patterns, the large errors that seem to accompany large waves of new investment do not seem to be outliers, but just more dramatic instances of a recurring failure by people and institutions to learn the lessons of economic history and curb their overreactions. As to the source of this persistent fallibility, behavioralists cite bounded rationality while the popular press cites sinister manipulation and blind greed. From a normative perspective, neither offers much guidance about how correctly to form expectations when faced with a truly new technology.

This paper proposes a different view of the connection between innovation and overindulgence. In contrast to the conventional view, it does not involve any cognitive failing on the part of economic agents. Nor is it based on the transmission of financial signals to real investment. Instead, in the context of a general equilibrium, rational expectations model of a production economy, I show that overinvestment can be an optimizing policy when a new production technology is introduced. The key feature of the model is that the shape of the production function is not known \textit{a priori}. Indeed this is what distinguishes a truly new technology: there is no historical experience of how it will “scale up”. I contend that this is a particularly important type of uncertainty in financing nascent industries. No one knows how the delicate interplay of competition, demand, costs, and regulation will work out at vastly greater levels of production than have ever been seen before.

In essence, the agent in the model has an incentive to push investment beyond the level that would seem (myopically) optimal in order to learn the shape of the production function as efficiently as possible. As experience grows, this incentive vanishes and

\footnote{Other rational stories for overinvestment might be based on informational mirages (as in Bikhchandani, Hirshleifer, and Welch (1992) or Caplin and Leahy (1994)) or agency conflicts (Scharfstein and Stein (1990), Allen and Gale (2000)). As far as I know, these have not yet been applied in the context of an equilibrium macroeconomic model.}
investment declines. Market prices of installed capital mirror the gains to be had from learning. Tobin’s q for the new industry starts high and then subsides predictably over time.

Besides presenting an alternative mechanism behind technology-driven cycles, the model also clearly has different implications about their desirability. Perhaps instead of wastefully mis-allocating resources these episodes actually represent the quickest adjustment path to the long-run optimum. Of course, the story here does not preclude irrationality. The point is not to be doctrinaire, but rather to suggest that the conventional understanding of new technology bubbles may be incomplete.

In modelling learning as an active process to be optimally managed, this paper follows a line of literature begun by Prescott (1972) who first considered the problem of stochastic control when the control affects the information set. Since no “separation principle” applies in these settings, analytical solutions are usually impossible and numerical solutions are extremely computationally challenging. Nevertheless some aspects of the problem can be characterized by examining one or two period problems of an agent with linear reward structure (such as a monopolist maximizing profit in the face of an unknown demand curve). Indeed, the basic insight that the learning motivation can lead to increased output or investment has long been known. This paper’s contribution is to extend the simple stylized models to a more realistic context, incorporating standard utility functions, non-trivial constraints, and multiple periods. This allows me to deduce economically meaningful implications about important quantities.

Similar in spirit, the learning-by-doing literature primarily studies what might be called a reduced form of the production problem, whereby the production function itself is modified as experience accumulates. (See Jovanovic and Nyarko (1996) and Mazzola and McCardle (1996).) How this occurs is often not fully specified in these models, but the beneficial effects of knowledge are understood in advance by agents who thereby have an incentive to overinvest.

4 These are known as adaptive control problems in the systems literature. (See Åström (1987) or Goodwin and Sin (1984).)


6 Another branch of the learning literature focuses on long-run properties of active learning problems, such as whether or not learning is complete, and policies converge. See Aghion, Bolton, Harris, and
Finally, it may also be useful to see this paper in the context of learning models in the asset pricing literature, for example Detemple (1986) and Wang (1993). In these works, agents choose a portfolio under uncertainty while learning passively about the changing but unobservable drift rate of exogenous dividends or returns. Relative to these models, the problem here involves only a static parameter. But the endogeneity of the learning rate, and thus the absence of a separation principle, fundamentally alters the information dynamics. The present paper is the first to pose and solve an active learning problem in a macroeconomic context.

The paper is organized as follows. The next section gives the details of the economic setting. I describe the underlying information structure and the optimization problem, relate these to other similar formulations, and discuss the weaknesses and driving assumptions of the model. Section 3 presents solutions which establish the occurrence of the overinvestment effect and demonstrate that asset prices as well as investment are inflated. Section 4 examines patterns of expected returns that can arise under the model, and relates these to the empirical finance literature. I argue that the effects produced in the (uncalibrated) model can be large enough to account for the anomalous returns observed in new stocks, or when price levels seem to be inflated. On this basis, the model appears consistent with available measurement of bubbles. The final section summarizes the paper’s contributions and suggests directions for future research.
2 The Model

2.1 Description

Consider a single-agent production economy with one good. Time is discrete and is indexed by $t$. For simplicity, take the time intervals to be of unit size (i.e. one year). At the start of each period, the agent chooses to either consume his supply, $W$, of the good or invest it in one of two production technologies. The first is an elastically supplied linear storage technology, with certain return $R$, which is known to the agent. The second is a risky technology, new at $t = 0$, whose returns to scale are not precisely known. Denote the quantity invested in the riskless and risky technologies as $B$ and $K$ respectively. Let $C$ be consumption, and $I = W - C = B + K$ be total investment.

The agent’s problem is to choose policies $(C, B, K)$ to maximize lifetime expected utility, taking into account the effects of current actions, not just on future wealth, but also on the probability measure determining future expectations.

An important specification is that of the risky production function. I assume that, given time $t$ commitment of $K_t$, the new technology returns $Y_{t+1} + (1 - \delta)K_t$, where $\delta$ is a (known, constant) depreciation rate, and

$$Y_{t+1} = A f(K_t) \exp\{\epsilon_t\}$$

where $A$ is a known constant, $\epsilon_t$ is a mean-zero normal random variable with known variance $\sigma^2$ (assumed independent of past and future $\epsilon$), and

$$f(K) = \exp \left\{ \alpha \left( \frac{(K/K)^b - 1}{b} \right) \right\} \cdot 1_{\{K \geq K\}}.$$  

(2.1.1)

In this function, the parameter $b$ is another known structural constant, and $b < 0$ (discussed further below), but $\alpha$ is unknown and must be estimated. Thus the agent knows the production function up to a parameter which determines its curvature, and hence returns to scale.

The term involving $b$ permits me to generalize the constant elasticity model $K^\alpha$ which corresponds to $b = 0$, while working with functions that are bounded above. Whether or not that is considered realistic, it has important implications for tractability.
As we will see, the agent’s beliefs will not rule out $\alpha > 1$, which is problematic in the constant elasticity model, since there will always be some chance of an explosive production function.\(^7\) In the numerical work I will take $b = -0.25$. Figure 1 plots the production functions for various values of $\alpha$, with $b = -0.25$ on the left and $b = 0$ on the right. As the graph makes clear, for small values of $K$ the two models look quite similar. That is the point: the non-standard aspect of my production functions are not important locally. It is only as $K$ gets large that the difference matters. Figure 2 shows the first and second derivatives of the $b = -0.25$ curves for the same $\alpha$ values, which illustrates the agent’s uncertainty on two transformed scales.

Figure 1: Production Functions

The figure shows the form of the production functions used in the paper. The left panel shows curves corresponding to $\alpha = \{-1, -.5, 0, .70, 1.0, 1.25, 2\}$ for the case $b = -1/4$. The right panel is the constant elasticity case $b = 0$, and uses $\alpha = \{-.5, -.25, 0, .5, .70, 1, 1.5, \}$. A last point to clarify about the specification (2.1.2) is the indicator function on

\(^7\)Needless to say, allowing such explosions would lead to even more overinvestment as uncertainty increases. So taking $b < 0$ works against my main point.
The figure shows the first and second derivatives of the production functions plotted in the left panel of Figure 1.

The right. I include a minimum investment threshold $K$ in the new technology just to maintain the consistency of the interpretation that higher values of $\alpha$ are better in that they correspond to higher $Y$ curves for all $K$. This plays no important role in the results, and hereafter I normalize $K$ to unity.

The information structure of the problem is as follows. At time zero agents have a normal prior over $\alpha$ with mean $m_0$ and variance $v_0$ or precision $\pi_0 \equiv 1/v_0$. After making their allocation decisions, they observe $Y_1$ or equivalently $y_1 = \log Y_1$, where

$$y_{t+1} = a + \alpha \left( \frac{K^b_t - 1}{b} \right) + \epsilon_t$$

and $a = \log A$. Although they know the values of $a$ and $b$ they cannot separate the noise term from the term due to investment. But, conditional on the observation, they can and do update their beliefs according to Bayes law. Because of the normality of $\epsilon$ and
of the prior distribution, the posterior distribution stays normal at all times, with mean and variance updated according to the recursion

\[ \pi_{t+1} = \pi_t + \left(\frac{x_t}{\sigma}\right)^2 \] (2.1.3)

\[ \pi_{t+1} m_{t+1} = \pi_t m_t + \left(\frac{y_{t+1} - a}{x_t}\right) \left(\frac{x_t}{\sigma}\right)^2 \] (2.1.4)

where I have defined \( x_t = x(K_t) = \left(\frac{K_t-1}{b}\right) \). Hence, as the first equation shows, investing more in the risky technology directly buys more knowledge about it.

A second crucial consequence of the normal conjugate set up is that conditional beliefs about the the process \( y_t \) stay normally distributed at all times. Specifically, after integrating out uncertainty about \( \alpha \), \( y_{t+1} \) at time \( t \) is subjectively distributed as

\[ \mathcal{N} \left( a + m_t x_t, \sigma^2 + v_t x_t^2 \right) . \] (2.1.5)

This expression immediately demonstrates one of the key motivations for acquiring information. Although current investment increases next period’s return uncertainty (\( x \) is increasing in \( K \)) and so has an ambiguous impact on the attractiveness of next period’s payoff, it also always lowers \( v_{t+1} \) according to (2.1.3). This unambiguously decreases the variance of all future period’s returns. Hence \( K \) buys an improvement in the future investment opportunity set, through higher Sharpe ratios.

To formally define the agent’s problem, I assume a standard time-separable utility function, \( u(C) \), with subjective discount factor \( \beta \). The agent is to choose policies \( (C, B, K) \) to maximize

\[ E^{\mathcal{F}_0} \left[ \sum_{t=0}^{T} \beta^t u(C_t) \mid W_0, m_0, \pi_0 \right] . \]

The current state is characterized by current wealth and the two sufficient statistics describing agent’s beliefs. The somewhat redundant notation is meant to emphasize that those parameters also describe the current information set \( \mathcal{F}_0 \) and hence the probability measure with respect to which the expectation is taken.\(^8\)

A finite time horizon is used for computationally reasons.\(^9\) However, in a slight

---

\(^8\)When it is not necessary to refer to the information set specifically, I will just write, e.g. \( E_t \).

\(^9\)There is another motivation too. For the problem to be well-posed on an infinite horizon, a transver-
variation of the problem, I assume that at time $T$ only the risky investment opportunity expires, but the agent does not.\footnote{The modified lifetime objective can be described by replacing $u(C_T)$ in the summation by $J_T^\infty(W_T)$, the latter denoting the value function for the remaining infinite horizon problem with just the non-stochastic technology. This is a simple analytic function for the utility specifications used.} This has the benefit of disentangling investment effects that are due to the agent’s life cycle.

To complete the specification of the problem, I constrain the set of feasible policies by imposing $C_t \geq 0$, $B_t \geq 0$, $K_t \geq 0$, for all $t$, on the grounds that physical investment, like consumption, cannot be negative.\footnote{These restrictions do not affect any of the paper’s conclusions. In particular, solutions allowing net borrowing ($B < 0$) simply have unbounded overinvestment as $m$ rises.} Together with the budget constraint, $W = C + B + K$, the policy space may be described by the two variables $\iota \equiv I/W$ and $\omega \equiv K/I$.

### 2.2 Discussion

The problem outlined above is meant to capture the situation facing investors when confronted, at $t = 0$, with a totally new opportunity, whose ideal scale is hard to predict. While formally a real opportunity, the new technology could also represent an innovative financial opportunity, such as hedge funds or emerging markets, whose long-run capacity is hard to assess.

In reality, investors know even less about new opportunities than the curvature parameter $\alpha$. The simplified set up here is meant to focus on just one aspect of the full problem. In thinking about the forecasting task, $\alpha$ is fundamentally different from the other parameters of the production function $a$ (or $A$) and $\sigma$, in that the latter two can be learned locally. That is, even small scale experience with the technology will reveal them rather quickly. So treating these as known seems consistent with the objective of capturing the uncertainty that could not possibly have been learned for a fundamentally new business.

There are two different ways the model can be viewed as describing how bubbles arise. One case would be that agents initially underappreciate the new opportunity and $m_o < \alpha$ is biased downwards. Then, on average positive surprises will follow because, for any choice of $K$, the subjective mean of $y$ is below the true mean. With each positive

---

\textsuperscript{10}The modified lifetime objective can be described by replacing $u(C_T)$ in the summation by $J_T^\infty(W_T)$, the latter denoting the value function for the remaining infinite horizon problem with just the non-stochastic technology. This is a simple analytic function for the utility specifications used.

\textsuperscript{11}These restrictions do not affect any of the paper’s conclusions. In particular, solutions allowing net borrowing ($B < 0$) simply have unbounded overinvestment as $m$ rises.
surprise, $K$ will rise, until the bias disappears. Alternatively, one can simply view time zero expectations as unbiased, and favorable enough for a significant allocation $K$. This, in effect, depicts the inflating of the bubble in a single step, from $t= -1$ (when $K=0$, since the technology didn’t exist) to $t=0$. Both, of course, are inadequate depictions of the emergence of a new industry. A fuller model would allow the technology itself to develop (e.g. through a stochastic $A$ or $\alpha$). This would be a useful generalization, but one which is not necessary to make the paper’s main point. I will show below that, however the initial build-up happened, levels of investment and prices are increasing in uncertainty about $\alpha$, and that this effect has nothing to do with remaining expectational errors or biases in $m$.

Besides the information structure, there are a couple of other driving assumptions which should be acknowledged. One is that there are no irreversibilities or frictions in adjusting the level of $K$. This assumption might be defended by imagining the model embedded in an overall growing economy in which actual disinvestment (net of depreciation) would be rare. But clearly adjustment costs hurt the main argument by providing a disincentive to over commit.\(^{12}\)

Likewise a second disincentive would arise if individual agents could free-ride on the information externality created by the risky experimentation of others. So the assumption of a single agent is not as innocuous as it usually seems in business cycle models. Indeed, learning models with competing agents with private information have been used to explain the exact reverse of the phenomenon studied here: inefficiently cautious underinvestment.\(^{13}\) On the other hand, as in any model where information production is valuable, if ownership rights to the new technology are well protected, distortions in investment (relative to the single-agent economy) may be minimal.\(^{14}\) It is perhaps not a coincidence that many of the historical examples of new technology bubbles cited by commentators were of inventions with natural monopoly characteristics, such as electricity generation, canals, and railroads.

\(^{12}\)Still, extremely interesting effects can arise in active learning problems with adjustment costs. Balvers and Cosimano (1990) provide an example which also has a bubble-like feel in which the incentive to learn essentially cumulates over time when costs deter experimenting. Active learning then occurs in periodic bursts when the need exceeds a threshold.

\(^{13}\)See Caplin and Leahy (1993) for an example with both irreversibility and information externality.

\(^{14}\)The natural disaggregated version of my model then would consist of agents with proprietary opportunities in distinct technologies. This generalization is the subject of ongoing work.
It is worth emphasizing at this point that, throughout, the paper views the new industry as fairly small with respect to aggregate wealth. (In the numerical solutions, for example, the long-run optimal level of investment is usually in the vicinity of ten percent of wealth.) So, although the structure is that of a real business cycle model, the focus is not really on general equilibrium effects.\(^{15}\) In particular, the reader should not think of the risky sector as the entire equity stock of the economy. More consistent with the paper’s interpretation would be to view the riskless technology as “the market” i.e. just the aggregate of all other productive activity, so that \(R\) is the certainty-equivalent opportunity cost of funds. Indeed, the main conclusions are fully robust to making \(R\) stochastic, as long as its correlation with \(Y\) is low.

In using production-based models to account for patterns of return predictability, the paper contributes to an important branch of asset pricing research. Balvers, Cosimano, and McDonald (1990) showed how time-series predictability arises as a consequence of non-linear technology. There, declining returns to scale and delayed output (“time-to-build”) combine to drive a wedge between the first order conditions of producers and investors. Recently, Gomes, Kogan, and Zhang (2002) demonstrated that cross-sectional variation in productivity and scale can induce size and book-to-market effects in the cross-section of measured returns. In my model, both type of return pattern will be generated by the endogenous learning of agents about the non-linearity they face.

3 Solutions

3.1 Some Analysis

The model described in Section 2 must be solved numerically. But before proceeding to examine computational results, it is useful to consider analytically how learning alters the investment problem.

Without any uncertainty about the production function, standard manipulation of

\(^{15}\)This emphasis makes the paper quite distinct from the line of research that has used production based models to tackle e.g. the equity premium puzzle. (For example, see Rouwenhorst (1995) or Jermann (1998).)
first order conditions implies that the optimal time \( t \) investment \( K^*_t \) must satisfy

\[
E_t \left[ \beta \frac{u'(C^*_t+1)}{u'(C^*_t)} Y_{t+1}(K^*_t) \right] = \frac{(R + \delta)}{(1 + R)} \tag{3.1.6}
\]

where \( Y' \) denotes \( A f'(\epsilon_{t+1}) \exp(\epsilon_{t+1}) \) (c.f. equation (2.1.1)). In other words, the expected marginal product – risk adjusted – must equal the opportunity cost of the investment.

Now consider the problem when \( K \) also affects future information. Intuitively, the incentive to overinvest comes from the impact \( K \) has on future decisions. The agent knows that the closer he can come at \( t + 1 \) to achieving the full-information optimum, the better off he will be. To put it another way, he knows that all future allocations made under imperfect information will be suboptimal, and hence exact a cost in foregone utility.

Mathematically, it is always better to receive extra information before solving a maximization problem in the sense that

\[
E^F \max_a E^G X(a) \geq \max_a E^F E^G X(a) = \max_a E^F X(a)
\]

when \( E^F \subset E^G \) for any random function \( X \) of a parameter \( a \). Getting the new information does not always make you happier, but knowing you will get it does.

To map this intuition onto the investment problem here, consider an agent who cares only about the information production role of \( K \). For example, an agent whose time \( t+1 \) wealth is fixed in advance at \( \overline{W} \) but who will inherit the technology then, will choose \( K_t \) to maximize

\[
E^{F_t} \max E^{F_{t+1}} \left[ u(C_{t+1}) + \beta J_{t+2} | \overline{W} \right].
\]

Here the only effect of his choice is on the inner information set, \( F_{t+1} = F_t \otimes \sigma(Y_{t+1}) \). To him, increasing \( K \) increases the informativeness of the signal \( Y_{t+1} \). And, as the inequalities above indicate, the more informative \( Y_{t+1} \) is, the better.\(^{16}\)

Now returning to the actual maximization problem faced by the agent in the model,

\(^{16}\)Formally, the implication is that any signal \( Y_{t+1} \) produces an objective greater than

\[
\max E^{F_t} \left[ u(C_{t+1}) + \beta J_{t+2} | \overline{W} \right] = E^{F_t} \max E^{F_{t+1}} \left[ u(C_{t+1}) + \beta J_{t+2} | \overline{W} \right] \]

which is the objective at \( t \) without the signal.
the full first-order condition for \( K_t \) can be written

\[
E_t \left[ \frac{\partial J_{t+1}}{\partial W_{t+1}} \, dW_{t+1} + \frac{\partial J_{t+1}}{\partial m_{t+1}} \, dm_{t+1} + \frac{\partial J_{t+1}}{\partial \pi_{t+1}} \, d\pi_{t+1} \right] = 0
\]

The first term is the only term present for a non-learning investor. The second and third terms, meanwhile, represent exactly the quantities set to zero by the agent imagined above who was insulated from the wealth consequences of the \( K \) choice. The statement that more information is better (for that agent) can be rephrased now as saying that these two partial derivative terms should be positive in expectation.

Call these expectations \( \lambda \). Then again rearranging the first-order condition in the usual manner, the result now is

\[
E_t \left[ \beta u'(C'_{t+1}) \, v'_{t+1}(K_t) \right] = \frac{(R + \delta - \hat{\lambda})}{(1 + R)}
\]

(3.1.7)

where \( \hat{\lambda} = u'(C'_{t}) \beta (1 + R) \lambda \) is positive when \( \lambda \) is. This equation shows why the learning mechanism leads to overinvestment. The benefits of future information act – through \( \hat{\lambda} \) – to effectively lower the hurdle rate, or the opportunity cost of risky investment. This tends to push \( K^* \) beyond the level that would be myopically optimal.

The discussion here has been heuristic, and has overlooked some subtleties. The extra terms in the first-order condition can, in fact, switch sign.\(^\text{17}\) And one cannot categorically assert that \( K^* \) will always exceed the myopic level. I will demonstrate in the next section that, for reasonable parameter choices, the incentive to overinvest predominates for a broad range of states. The fact that the situation can be reversed, and the same underlying problem can produce underinvestment, is also intriguing, suggesting as it does an even richer variety of adjustment paths to the long-run optimum. However this is not to distract from the focus of the paper, which is to suggest that the learning facet of the investment problem offers a potential explanation for the seemingly repeated pattern of overinvestment bubbles in new industries.

\(^{17}\)This possibility was noted by Bertocchi and Spagat (1998).
### 3.2 Results

I turn now to numerical results. To start, this subsection examines the optimal level of investment in the risky asset. Section 3.3 investigates prices, and Section 4 explores expected returns.

To be clear on terminology, in this section “underinvestment” and “overinvestment” refer to the deviation of the optimal level $K^*$ (or $\omega^*$) from what it would be at the same point in the state space if the returns to scale parameter were known with certainty to be equal to its current estimate $m_t$, or equivalently, if $v_t$ were zero. In other words, the benchmark here is not one particular economy with a particular $\alpha$. In fact, since the true value $\alpha$ nowhere enters into the problem, my characterization of over- or underinvestment makes no assumption about the bias, $m_t - \alpha$, of the agent. The full information level of investment is the one that an observer of the economy with an infinite history – an idealized econometrician – would deem rational *ex post*. Hence this is the appropriate comparison in seeking explanations about what appear to be anomalies in the data.

Because solving the model is computationally demanding, I fix the parameters shown in Table 1, and explore the relative effect of the state and the utility specification. Recall also from Section 2 the background assumptions that the time interval is one year, and that the technology terminates at time $T$, but the agent does not.

**Table 1: Parameters for Numerical Solutions.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>current wealth</td>
<td>$W$</td>
<td>100</td>
</tr>
<tr>
<td>return on riskless technology</td>
<td>$R$</td>
<td>.05</td>
</tr>
<tr>
<td>subjective discount factor</td>
<td>$\beta$</td>
<td>1/1.05</td>
</tr>
<tr>
<td>depreciation rate</td>
<td>$\delta$</td>
<td>.10</td>
</tr>
<tr>
<td>production function (log) intercept</td>
<td>$a$</td>
<td>log(.20)</td>
</tr>
<tr>
<td>production function exponent</td>
<td>$b$</td>
<td>-1/4</td>
</tr>
<tr>
<td>output volatility</td>
<td>$\sigma$</td>
<td>.20</td>
</tr>
<tr>
<td>minimum investment</td>
<td>$K$</td>
<td>1</td>
</tr>
</tbody>
</table>

The assumptions about the production parameters are supposed to describe a hypo-

---

18Details of the dynamic programming algorithm are available upon request.
hlitical emerging industry, but otherwise are meant to be innocuous. They imply that a unit investment yields expected payoff 0.104, after depreciation, with a one standard deviation shock in either direction giving a range of 0.064 to 0.144. A risk-neutral investor with full-information setting $E_t [Y'(K)] = R + \delta$ would invest about 2.7 for $\alpha = 1$, rising to 6.4 and then 16.8 for $\alpha = 1.2$ and 1.4. The agent will be assumed to have a prior with $m_0$ near one, so these allocation numbers will be typical. Since wealth = 100, this shows the sense in which the industry is “small”.

To further illustrate the situation, imagine that the agent’s initial uncertainty (prior standard deviation) for $\alpha$ is $\sqrt{v_0} = 0.20$ and $m_0 = 1$. This normally distributed prior can be mapped into a prior on the “correct” investment level, the solution to equation (3.1.6). Figure 3 shows this distribution, computed from transforming prior draws for the case of a risk-neutral agent. As the figure shows, this agent thinks there is about a 7% chance that no investment is optimal, and less than 1% chance that investment should exceed 25. The prior looks economically sensible, and is mathematically well-behaved in the sense that the right tail is thin. The posterior mean and standard deviation on this scale are 4.1 and 4.6 respectively. However the median and 75th percentile are 2.7 and 4.7.

Figures 4 and 5 plot the optimal level of risky investment for a variety of utility functions. The first figure fixes $m_t = 1.2$ and the second uses $m_t = 1.4$. In each, risky investment is shown both as a raw number (bottom graphs) and as a share of total investment (top). The right-hand graphs employ CARA (exponential) utility with risk-aversion exponent 0.1, 0.01, and 0.001. The left graphs are for CRRA (power) utility, with powers 0.5, 1.0, and 2.0. The computation sets $T = 5$, that is, there are five periods of risky investment remaining.\textsuperscript{19}

The horizontal axis for all the graphs is the posterior standard deviation $\sqrt{v_t}$ for $\alpha$ which, reading to the right, goes from zero – corresponding to perfect information – to 0.20, which I suggested above might be a reasonable level of uncertainty for a new industry. So, reading backwards from the left, one can interpret these curves as the slopes down which investment would descend to its long run equilibrium level as information accumulates.

And the slope is downward. Overinvestment is indeed initially optimal for all the

\textsuperscript{19} Investment policies turn out to vary little for different choices of $T$.\textsuperscript{16}
The figure shows the prior distribution for the perfect-information level of investment, which corresponds to a normal prior over the returns-to-scale parameter $\alpha$ having mean 1.0 and standard deviation 0.20. The transformed prior was computed by drawing 10000 times from the normal prior, and, for each, solving equation (3.1.6) in the text.

cases shown. This is the heart of the paper’s results. In percentage terms the $m_t = 1.2$ plots show a steeper decline, about 30%, but the absolute overinvestment rises as the overall level does. On both scales the overinvestment shows no sign of being bounded above, suggesting that much larger levels would be observed as prior uncertainty increased further.

There is little difference between the utility specifications when $m_t = 1.2$ (and none at all for lower values of $m$) because there is little overall risk. Risk aversion starts to make a difference as the risky investment increases. With $m_t = 1.4$ the pattern becomes clear: risk aversion reduces the overinvestment. Intuitively, when the gamble starts to become large, the investor is less likely to risk a little more for the sake of learning, which, since
The figure shows the optimal level of investment, $K$, as a function of the posterior standard deviation for the returns to scale parameter. The top two graphs scale $K$ by total investment $I$. The bottom two use the raw $K$. The two left-hand graphs use CARA utility with parameter $\gamma = \{.001, .01, .1\}$. The right-hand graphs uses CRRA utility with parameter $\gamma = \{.5, 1.0, 2.0\}$. In every case, the lower graphs correspond to higher risk aversion. The posterior mean is set to $m = 1.2$. Other parameter settings are as in Table 1.

$K$ is big, will be substantial anyway. This might seem to suggest that for even bigger optimal investment the agent will behave in a certainty-equivalent fashion. As the next figure shows, this is in fact the case.

Figure 6 fixes one utility case (CRRA, $\gamma = 1$) and shows how optimal investment varies across $m$. The lines plotted are for successively smaller levels of the posterior standard deviation, $\sqrt{\nu_t} = 0.20, 0.18, 0.16, \ldots$ and now the investment allocation is plotted relative to the full-information $\sqrt{\nu_t} = 0$ case. This shows that, in percentage term, the biggest effect is near $m = 1$. The effect falls to zero as $m$ declines (for low $m$, $K$ should be zero regardless), as it does when $m$ rises above 1.0. For $m > 1.8$ the overinvestment effect
The figure shows the optimal level of investment, $K$, as a function of the posterior standard deviation for the returns to scale parameter. The top two graphs scale $K$ by total investment $I$. The bottom two use the raw $K$. The two left-hand graphs uses CARA utility with parameter $\gamma = \{.001, .01, .1\}$. The right-hand graphs uses CRRA utility with parameter $\gamma = \{.5, 1.0, 2.0\}$. In every case, the lower graphs correspond to higher risk aversion. The posterior mean is set to $m = 1.4$. Other parameter settings are as in Table 1.

vanished. For those levels of $m$, $K$ is a large fraction (e.g. over 50%) of wealth, and the agent treats parameter uncertainty as systematic risk, which matters much more than incrementally increasing learning. These cases, in effect, model a much larger shift in the structure of the economy than occurs in the new industry scenarios that are the main focus here.

From the graphical results, the dynamics of a new industry bubble in this model become clear. At $t = 0$ agents learn of the new opportunity and investment leaps from nothing to e.g. 7.5 if $m_0 = 1.2$. Suppose this is right and $\alpha$ is also 1.2. Then a simulation of the next few years would see $K$ decline, stochastically but systematically, to about
The figure shows lines of constant posterior standard deviation for optimal investment as a function of the posterior mean. The $K$ values are standardized by the corresponding perfect-information level $K_0$. The case shown is log utility with 5 years remaining. Other parameter settings are as in Table 1. Other parameter settings are as in Table 1.

5.5. Referring back to Figure 4, the rate of movement on to the right can be readily calculated. For $K \approx 7.5$ posterior precision increases by $\Delta \pi = [x(K)/\sigma]^2 \approx 62$. This suggests a deflation from $\pi = 1/0.20^2$ to $1/0.04^2$, for example, in about ten years. Smaller expected returns to scale, e.g. with $m_0 = 1.0$, experience a larger percentage decline but over a longer period of time.

Obviously the overinvestment in this model does not occur in response to signals from overheated asset markets: allocations are determined directly from the economic primitives. Nevertheless, the next section shows that the model can still account for the appearance of a link between investment and the cost of capital, as claims to the new technology can appear “overpriced” at the same time the overinvestment is occurring.
3.3 Asset Prices

No financial markets are involved in solving the model economy. To introduce financial assets, I follow the standard practice of computing shadow prices of claims to different cash-flows. In particular, the stock price of the new industry is the value placed on its flow of net dividends $D_t = Y_t - K_t + (1 - \delta)D_{t-1}$ until $T$. This exercise imagines a hypothetical portfolio problem faced by the representative agent – or a collection of households with identical preferences – at time $t$ but just after the time $t$ allocations have been made. Since the equilibrium allocation policies are optimal for the same agent, it makes sense to regard these as exogenous for purposes of the auxiliary portfolio problem.\(^{20}\) Hence the pricing calculation proceeds as if the same agent were living in an endowment economy.

To understand the resulting prices, it is important to realize that the same representative agent applies very different first order conditions to the portfolio problem from those that apply to the production decision. This statement is not true in many production-based asset pricing models, and so deserves comment.

In the portfolio problem, with exogenous quantities, the situation is standard, and the investor’s first order condition can be written

$$q_t \equiv \frac{P_t}{K_t} = E_t \left[ \beta \frac{u'(C_{t+1}^*)}{u'(C_t^*)} \left( \frac{P_{t+1} K_{t+1}}{K_{t+1}} + \frac{Y_{t+1}}{K_t} - \frac{K_{t+1}}{K_t} + (1 - \delta) \right) \right] \quad (3.3.8)$$

where $P_t$ is the dividend claim’s price. In a standard production problem, this could be combined with equation (3.1.6) and a simple backward induction argument to prove that $q_t = 1$ for all $t$. Now that argument would fail for several reasons.

First, the argument uses $Y' = Y/K$. Most production models employ output functions that satisfy this, but the present model does not. So even under full information, the price-capital ratio will vary with the ratio $KY'/Y = mK^b$. Second, the learning aspect of the problem implies that the $K$ first order condition is not given by (3.1.6), as was discussed in Section 3.1. Finally, the shadow price computation is made without imposing portfolio constraints, whereas the production decision is made under non-negativity quantity requirements.

For these reasons, there can be multiple “wedges” between investment and production

\(^{20}\)This might not be true if, as sometimes assumed, production decisions were made by firms with different objective functions (e.g. risk-neutral profit maximization) from households.
marginal rates of substitution in this economy, and the $q$ ratio cannot be deduced ana-
ytically. Instead, I perform the integration in (3.3.8) numerically over the state space, using the optimal quantities computed in the last section.

Before going to the results, one other price computation should be pointed out. In the usual endowment economy fashion, one can imagine a net-zero-supply riskless one-period claim, and immediately deduce its price

$$E_t \left[ \frac{\beta u'(C_{t+1})}{u'(C^*_t)} \right] = \frac{1}{1 + r_t}$$

which also defines the risk-free rate $r_t$. It is conceivable, however, that this might not be equal to $R$, the one-period return on the riskless technology. Claims on that technology are in fixed positive supply. And hence if that production decision is ever constrained, such a claim’s price can exceed $1/(1 + R)$. When thinking about excess returns or discounting claims, $r_t$ is the relevant riskless rate, and it is stochastic.

Turning now to the results, Figure 7 shows the price-capital ratio as a function of $m$ for the log utility case with $T = 10$ years. Separate lines are plotted for values of the posterior standard deviation from 0 to 0.2. For all levels of uncertainty, it is clear that the model can produce $qs$ much greater than unity, particularly as $m$ rises. Figure 8 clarifies the role of uncertainty by standardizing all the $q$ values in the previous picture by the perfect information value $q_0$. In this plot the outer-most lines correspond to higher standard deviation. For low values of $m$, uncertainty does not actually lead to higher asset prices, and can even deliver slightly lower ones. For higher $m$, the reverse is true, and uncertainty substantially inflates prices over and above any other effects in the model. For $m$ in the range 1.2 to 1.5 the effect is over 10%. This extra inflation grows rapidly with the level of uncertainty, and for values of $\sqrt{v}$ over 0.25, for example, the effect is over 20%.

The effect is also sensitive to the amount of time remaining (unlike the quantity effects in the last section). Uncertainty increases $q/q_0$ more as $T$ grows. Or, equivalently, the bubble naturally deflates as $t$ approaches $T$. Figure 9 shows some typical adjustment paths of investment and $q$. Here the true value of $\alpha$ is 1.2 and simulations are shown for $m_0 = 1.0, 1.2$, and 1.4, with $v_0 = .20^2$ for each case. (The corresponding paths of the state variables for these simulations are plotted in Figure 10.) Looking at the $q$ plot, after
Figure 7: Price-Capital Ratios

The figure shows lines of constant posterior standard deviation for the price-capital ratio \( q \) as a function of the posterior mean. The lines correspond to values of \( \sqrt{v} \) running from 0 to 0.2. The case shown is log utility with 10 years remaining. Other parameter settings are as in Table 1.

The first few periods investment is stable and asset prices decay smoothly. The initial periods though are characterized by a steeper fall as the inflation in \( q \) due to uncertainty evaporates. The model thus suggests a two-phase deflationary dynamic, with a “crash” followed by a longer “hangover”.\(^{21}\)

The calculations in this section demonstrate that the learning problem modeled here may account for at least part of the overinvestment and inflated asset prices that seem to characterize the early years of important new industries. At this point it is hard to assess exactly how realistic the numbers are, because our empirical knowledge of such “revolutions” is very limited. Even aside from the infrequency of major innovations, it is extremely difficult in any industry at any time to measure the “correct” level of

\(^{21}\)I make this more precise in the next section.
Figure 8: The Uncertainty Effect in $q$

The figure shows lines of constant posterior standard deviation for the price-capital ratio $q$ as a function of the posterior mean. The $q$ values are standardized by the corresponding perfect-information level $q_0$. The case shown is log utility with 10 years remaining. Other parameter settings are as in Table 1.

investment or prices, with respect to which excesses can be compared. As the introduction argued, most of our knowledge, such as it is, about bubbles comes through evidence on realized returns to investments in “glamour” assets, in particular ones with high measured $qs$. The next section works out the model’s implications for predicted patterns of returns, and investigates their relation to the price-capital ratio.

4 Returns

In drawing comparisons with financial market experience, this section will necessarily have to stretch the interpretation of the basic one-asset model somewhat. I will proceed by analogy with three distinct asset pricing results, starting from the most directly
The figure shows some simulated adjustment paths of the price-capital ratio (top panel) and investment (bottom panel). The true value of $\alpha$ is 1.2 and the three paths used $m_q = 1.0, 1.2,$ and $1.4,$ corresponding to the paths with successively higher values at $t = 1$. The case shown is log utility. Parameter settings are as in Table 1.

analogous and then venturing progressively farther. Where ever the line of plausibility is drawn, however, the basic message is the same: to the extent that the data capture the link between new industries and expected returns, the model appears consistent with the observed patterns and magnitudes.

Expected returns are straightforward to compute, via numerical integration, from the price and quantity solutions already derived. A subtle issue arises, however, in specifying the relevant probability measure. Prices in the economy are set by the agent assessing expected cash-flows and marginal utility with respect to his own, subjective, information set (as described in Section 2). And, in fact, to him, expected returns obey the standard consumption-based model (CCAPM). This means that the subjective excess expected returns are will be proportional (with a negative constant) to the covariance of dividends.
The figure shows paths of the state variables for the simulations plotted in Figure 9. These are, from top to bottom, the returns to scale estimate \( m \), its standard deviation \( \sqrt{v} \), and wealth \( W \).

with marginal utility. Since optimal consumption rises and marginal utility falls when output shocks are positive, the required subjective excess returns will be positive.

In measuring expected returns with realized actual returns, on the other hand, we as econometricians are assessing *objective* excess returns, or true long-run frequencies. With respect to this *ex post* measure, it is certainly not true that the CCAPM need hold. The difference between the two probability measures is parameter risk. With respect to the objective measure, log output at time \( t + 1 \) is simply distributed normally with mean \( a + \alpha x(K_t) \) and variance \( \sigma^2 \). Whatever the bias, the objective variance is lower than the subjective one (c.f. equation (2.1.5)) which will usually translate into lower expected payoffs due to Jensen’s inequality. To fully specify the objective measure I also need to take a stand on the true \( \alpha \). As in the last section, I set \( \alpha = m \) when making static comparisons in order to isolate patterns that have nothing to so with the bias that may
prevail at any one time.\textsuperscript{22}

As the introduction discussed, the closest sample to the model’s idealized new technology is probably that of initial public offerings (IPOs). IPOs tend to cluster in time, which, although it raises some tricky econometric issues, means that the data do approximate the emergence of entire industries. These industries are not always genuinely new and are seldom revolutionary. But they are likely to be characterized by the particular type of uncertainty modeled here: uncertainty about their ultimate scale.

While the measurement issues are somewhat contentious, typical conclusions in the empirical literature broadly corroborate the initial findings in Ritter (1991) of abnormal excess buy-and-hold returns on the order of -5\% to -10\% per year for the first three to five years. Figure 11 shows the objective excess returns in my model for the log utility case with ten years remaining.\textsuperscript{23} For values of $m$ around 1.2 with initial uncertainty of 0.20, the annual excess expected return is about -10\%. This expected underperformance is entirely an information effect. It vanishes as knowledge about $\alpha$ improves. According to the model, new firms in well-understood businesses should have no underperformance.\textsuperscript{24}

In terms of the dynamics of adjustment, Figure 12 shows the average expected returns across 1000 simulated paths with $\alpha = m_0 = 1.2$. (For this exercise, the objective expectations on any given path do involve bias since $\alpha$ is fixed.) This plot makes explicit the “crash” plus “hangover” pattern noted in the last section. The underperformance does not persist as strongly in the simulations as in IPO returns, perhaps because in real-life the time scale of the learning process is longer than the one year period in the model.

Another calculations these simulations suggest is of the time-series relation between measures of market “fundamentals” and expected returns. This is the second, and somewhat less apt, application of the model’s predictions. The asset-pricing results I have in

\textsuperscript{22}Note that, in computing excess returns, it is not also necessary distinguish between a subjective and objective risk-free rate. Even though this rate is stochastic, it is observed at time $t$, and hence is the same under any time-$t$ measure.

\textsuperscript{23}I use the log utility case throughout this section. There is nothing qualitatively special about this case. As in the last section, the learning effects are smaller with more risk aversion and bigger with less. There are also no qualitative differences between the CARA and CRRA cases.

\textsuperscript{24}The fact that expected excess returns are nearly all negative is partially due to the low risk aversion in this one case, but, more generally comes from the smoothness of consumption, which is characteristic of production-based models. Consumption would be more volatile, of course, if the rest of the economy were not modelled as riskless.
The figure shows expected annual excess returns for the risky asset as a function of the mean and standard deviation of the distribution of the returns to scale parameter. Expectations are evaluated with respect to the unbiased objective measure as discussed in the text. The case shown is log utility with 10 years remaining. Other parameter settings are as in Table 1.

mind here are those of e.g. Campbell and Shiller (1998) showing that when prices appear inflated based on such measures, indeed, expected returns over the intermediate term are low. Since periods of “new economy” mania do see such inflation (Shiller (2000)), this can be read as evidence of the conventional wisdom view linking innovation to financial excess. The reason the model is not fully able to analyze this relation is that it only has a very primitive depiction of the non-new part of the economy. A richer model would be needed to capture any spillover effects, for example, from the new technology to the prices of other assets. Since I have emphasized that the new investment in the model should be viewed as a relatively small part of the economy, it is somewhat inconsistent to view its predictions as applying to market aggregates.
Figure 12: Expected Excess Returns

The figure shows expected annual excess returns for the risky asset as a function of time remaining. The individual years’ numbers are averages across 1000 time-series simulations starting from $m_0 = 1.2$ and $\sqrt{v_0} = 0.20$. Expectations are evaluated with respect to the objective measure with $\alpha = 1.2$. The case shown is log utility. Other parameter settings are as in Table 1.

Nevertheless, since the model does provide a natural measure of asset price inflation in $q$, it is interesting to quantify the relation it predicts between this measure and expected returns. In Figure 13, I plot the two quantities for the time-series simulations used above. As in the actual time-series data, a strong negative relationship is evident. The clustering of the data onto diagonal lines reflects the functional relationship that pertains for each time-period. (The nine lines correspond, from left to right, with $t = 9, 8, \ldots 1$. For $t = 10$ all the simulations plot at the single point with $q = 1.5$ and expected return $= -0.10$.) So the graph also tells us that a strong negative relation exists conditional on any choice of $t$. The magnitude of the effects here seem, if anything, too large relative to

\footnote{Of course, in real data one does not observe expected returns as can be done here.}
the somewhat tenuous relations usually estimated empirically. But, again, what is being simulated here are the values of the single new sector, not those of the entire economy.

Figure 13: Expected Excess Returns vs Price/Capital Ratio

The figure shows expected annual excess returns for the risky asset as a function of time remaining. The individual years’ numbers are averages across 1000 time-series simulations starting from \( m_0 = 1.2 \) and \( \sqrt{V_0} = 0.20 \). Expectations are evaluated with respect to the objective measure with \( \alpha = 1.2 \). The case shown is log utility. Other parameter settings are as in Table 1.

As a final point of comparison, I consider the cross-sectional relation between expected returns and characteristics of the risky technology. Since I have a good proxy for the asset’s book-to-market ratio in \( q \), and another one for its size in \( P \) (the market value of its equity), I can ask whether the model throws any light on the well-known relation between these characteristics and expected returns. To the extent that these traits capture “value” or “glamour” (Lakonishok, Schleifer, and Vishny 1994), the empirical evidence supports the notion of systematic bias in assessing growth prospects, which is one explanation for the linkage between technological revolutions and bubbles. The question is: does the
rational experimentation story here provide an alternative explanation?

This is, again, not an issue the model can speak to, rigorously. There is no cross-section of risky assets in the model. Correctly speaking, all one can compare are characteristics and expected returns in a cross-section of *economies*, each with a single new technology. This is the actual exercise undertaken.

Since, in the model, the characteristics are functions of the state, to make the desired comparison, I need to specify some cross-sectional distribution over these states. A straightforward choice is just to take uniform draws in the $m \times \sqrt{v}$ plane over the range depicted in 11. (This is the log utility case with wealth fixed at 100 and $T = 10$.) I do this 10000 times, yielding a population which I then sort into quintiles of $P$ and $q$. Table 2 shows the average expected excess return per cell, with no bias in the expectations. The pattern is, in fact, that of Fama and French (1992). Returns decline with market value, and, holding size fixed, returns are still inversely proportional to $q$. The difference between the extreme cells of about 5.5% a year corresponds to 45 basis points per month. Fama and French (1992) find about 70 basis point a month. Of course, in any real cross-section there is important variation in other characteristics (leverage, output risk, etc.) which is not captured here. Also missing is any variation due to bias. At any given time, a given sample of firms will include ones whose recent history contained good random shocks. These will have higher current $P$ and $q$ and lower objective expected returns. Likewise firms with recent unlucky shocks will have $P$ and $q$ biased down, and positive expected returns. Hence bias in the model goes the same way as the pure information effect, and would reinforce the cross-sectional spread shown in the table.

The goal of this section has been to show that, to the extent that there is empirical support for the linkage of technological revolutions and economic excess, the model is at least consistent with that evidence. I have focused on financial patterns (returns) because of the lack of any consensus measures of real overinvestment. It is perhaps worthwhile to stress, at this point, that the numbers presented here are not the result of a calibration exercise. I have not fully explored the flexibility afforded by the model’s free parameters (e.g. $\sigma$, $b$, $T$). So despite its obvious shortcomings as a depiction of the emergence of a new industry, it seems reasonable to conjecture that the findings here can be preserved in the context of a richer model.
Table 2: \textit{Pseudo Cross-Section of Expected Returns}

<table>
<thead>
<tr>
<th></th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size ((P)):</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small</td>
<td>0.0000</td>
<td>0.0000</td>
<td>-0.0001</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>2</td>
<td>-0.0005</td>
<td>-0.0003</td>
<td>-0.0005</td>
<td>-0.0007</td>
<td>-0.0021</td>
</tr>
<tr>
<td>3</td>
<td>-0.0200</td>
<td>-0.0084</td>
<td>-0.0055</td>
<td>-0.0035</td>
<td>-0.0015</td>
</tr>
<tr>
<td>4</td>
<td>-0.0557</td>
<td>-0.0234</td>
<td>-0.0175</td>
<td>-0.0171</td>
<td>-0.0048</td>
</tr>
<tr>
<td>Large</td>
<td>-0.0319</td>
<td>-0.0231</td>
<td>-0.0266</td>
<td>-0.0188</td>
<td>-0.0062</td>
</tr>
</tbody>
</table>

The table shows expected annual excess returns, computed under the unbiased objective measure, for a pseudo cross-section of 10000 risky assets sorted by size and book-to-market. The risky assets are generated by drawing a state vector \(\{m, \sqrt{v}\}\) uniformly over the range depicted in 11 and evaluating \(\hat{P}\) and \(\hat{q}\) for the log utility case with wealth fixed at 100 and \(T = 10\).

5 Conclusion

Looking at the history of human invention, it is hard not to be intrigued by the repeated pattern of speculative excess accompanying new technologies. This paper takes this anecdotal linkage at face value, and asks whether more than unrelenting folly may be at work. The answer is yes. In fact, the efficient way to discover how much to invest when returns-to-scale are unknown is to start by overshooting. Numerical solutions to the adaptive control problem of an agent facing an innovative production opportunity can reproduce many of the stylized traits of new industry bubbles.

Other models might well be able to do so too. I have concentrated on irrationality as an alternative hypothesis mainly because of the starkly different conclusions it suggests about efficiency and welfare, and the role of capital markets. Moreover the behavioral story seems natural, and a growing body of literature, both popular and academic, takes for granted that no further explanation is needed.

An important question, therefore, is whether the two explanations can be disentangled empirically. This paper’s model implies that the driving factor behind apparent excesses is the degree of uncertainty about the curvature of the production function. This might be related to measures of dispersion in beliefs about industry growth rates, since both implicitly forecast a likely long-run level of investment. (There are no explicit growth rates in the model, which is frictionless with stationary shocks. So this conjectured
relationship needs to be formalized.) A better proxy might be a measure of the genuine novelty of a sector, such as research intensity or patent activity. In behavioral models, it is bias, not uncertainty, that drives bubble effects. So the distinguishing prediction of the adaptive control model is that these effects would not be observed after controlling for technological uncertainty.

The competing explanations are not mutually exclusive, of course. And, indeed, bias plays an important (though idiosyncratic) role in the dynamics of the model developed here. The issue is whether the bias is always in the same direction, and hence should be prevented. Deciding this question before the next wave of innovation should be a high priority for economic research.
References


Easley, David, and Nicholas M. Kiefer, 1988, Controlling a stochastic process with un-


Gomes, Joao, Leonid Kogan, and Lu Zhang, 2002, Equilibrium cross section of returns,

Goodwin, G. C., and K. S. Sin, 1984, *Adaptive Filtering, Prediction and Control*

approach to the production of information and learning by doing, *Review of Economic
Studies* 44, 533–547.

International Monetary Fund, October 1, 2001, *World Economic Outlook*.


Jovanovic, Boyan, and Yaw Nyarko, 1996, Learning by doing and the choice of technology,
*Econometrica* 64, 1299–1310.

Kiefer, Nicholas M., and Yaw Nyarko, 1989, Optimal control of an unknown linear process

Lakonishok, Josef, Andrei Schleifer, and Robert W. Vishny, 1994, Contrarian investment,

Mazzola, Joseph B., and Kevin F. McCardle, 1996, A bayesian approach to managing

Mirman, Leonard J., Larry Samuelson, and Amparo Urbano, 1993, Monopoly experi-

Prescott, Edward C., 1972, The multiperiod control problem under uncertainty, *Econo-
metrica* 7, 331–347.


