Information Contagion and Inter-Bank Correlation in a Theory of Systemic Risk

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Abstract

Two aspects of systemic risk, the risk that banks fail together, are modeled and their interaction examined: First, the ex-post aspect, in which the failure of a bank brings down a surviving bank as well, and second, the ex-ante aspect, in which banks endogenously hold correlated portfolios increasing the likelihood of joint failure. When bank loan returns have a systematic factor, the failure of one bank conveys adverse information about this systematic factor and increases the cost of borrowing for the surviving banks. Such information contagion is thus costly to bank owners. Given their limited liability, banks herd ex-ante and undertake correlated investments to increase the likelihood of joint survival. If the depositors of a failed bank can migrate to the surviving bank, then herding incentives are partially mitigated and this gives rise to a pro-cyclical pattern in the correlation of bank loan returns. The direction of information contagion, the localized nature of contagion and herding, and the welfare properties, are also characterized.

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1 Introduction

The past two decades have been punctuated by a high incidence of financial crises in the world. In the period 1980–1996 itself, 133 out of 181 IMF member countries experienced significant banking problems, as documented by Lindgren, Garcia, and Saal (1996). Developed countries and emerging countries have been equally affected. These crises have been empirically shown to be associated with high real costs for the affected economies. Hoggarth, Reis, and Saporta (2001) document that the cumulative output losses have amounted to a whopping 15–20% of annual GDP in the banking crises of the past 25 years. The restructuring and output losses have been as high as 50–60% of annual GDP in some emerging-market banking crises.

Understanding bank failure risk, and especially systemic failure risk — the risk that most or all banks in an economy will collapse together — is considered the key to predicting and managing such financial crises. Indeed, the issue of systemic risk amongst banks has long been attributed as the raison d’etre for many aspects of bank regulation. Its causes, manifestations, and effects are however not yet fully understood. In this paper, we lay down a foundation that we hope will lead to an enhanced understanding of different forms of systemic risk.

In particular, we examine liability side contagion, asset side correlation, and their interactions. Liability side contagion arises when the failure of a bank leads to the failure of other banks due to a run by their depositors or a liquidation of their liabilities. Asset side correlation across banks arises if they lend to similar firms or industries. The paper’s goal is both positive as well as normative. On the positive side, we build a theoretical model whose assumptions and results are supported by empirical evidence. The normative aspects concern a welfare analysis of the costs and the benefits of systemic risk.

Recent models of contagion amongst banks include the work of Rochet and Tirole (1996), Kiyotaki and Moore (1997), Allen and Gale (2000), to cite a few. The primary focus of these studies is the characterization of contagion and financial fragility that arise due to the structure of inter-bank liabilities. By contrast, in our model there is no inter-bank linkage. Instead, we propose that systemic risk arises on the liability side of banks due to a revision in the cost of borrowing of surviving banks when some other banks have failed. Crucially, however, we also allow for systemic risk on the asset side of bank balance sheets. In particular, we show that banks choose a high correlation of returns on their investments by lending to firms in similar industries. The incentives for such action increase in the extent of systemic risk on the liability side. This interaction of liability side and asset side systemic risk is an important and novel contribution of this paper.

In our model, there are two periods and two banks with access to risky loans and deposits. The returns on each bank’s loans consist of a systematic component, say the overall state of the economy, and an idiosyncratic component. The nature of the ex-ante structure of each bank’s loan returns, specifically their exposure to systematic and idiosyncratic factors, is common knowledge; the ex-post performance of each bank’s loan returns is publicly observed. However, the exact realization of systematic and idiosyncratic components is not observed by the economic agents. Depositors in the economy are assumed fully rational, updating their beliefs about the prospects of the bank to which they lend based on the information received about not only that bank’s loan returns but also those of other banks. Ex-ante, banks choose whether to lend to similar industries and thereby maintain a high level of inter-bank correlation, or to lend to different industries.

When a bank’s loans incur losses, it may fail to pay its depositors their promised returns. Such failure conveys potential bad news about the overall state of the economy. Depositors of the surviving bank rationally update their priors and require a higher promised rate on their deposits. By contrast, if both banks experience good performance on their loans, then depositors rationally interpret it as good news about the overall state of the economy. Hence, they are willing to lend to banks at lower rates. The borrowing costs of banks are thus lower if they survive together than when one fails. This is an information spillover of one bank’s failure on the other bank’s borrowing costs, and in turn on its profits. Indeed, if the future profitability of loans is low, the surviving bank cannot afford to pay the revised borrowing rate and fails as well. An information contagion results.

How do banks respond to minimize the impact of such liability side contagion on their profits? We argue that the response of banks manifests itself in ex-ante investment choices. The greater the correlation between the loan returns of banks, the greater is the likelihood that they will survive together; in turn, the lower is their expected cost of borrowing in the future and higher are their expected profits. Consequently, banks lend to similar industries and increase the inter-bank correlation. In other words, banks herd.\textsuperscript{2} Intuitively, banks prefer to survive together rather than surviving individually. In the latter case, they face the risk of information contagion. By contrast, given their limited liability, bank owners view failing individually and failing together with other banks in a similar light. While information contagion sequentially transforms losses (or failure) at one bank into losses (or failure) at the other bank, greater inter-bank correlation increases the risk of simultaneous bank failure if the industries they lend to suffer a common shock.

We extend the model to allow the depositors of the failed bank to migrate to the surviving bank, if any exists. Intuitively, this captures a flight to quality phenomenon sometimes

\textsuperscript{2}Note that this form of \textit{ex-ante} herding is different from \textit{ex-post} or sequential herding that arises in typical information-based models of herd behavior. We elaborate on this difference in the Related Literature section.
observed upon bank failures. Such flight to quality enables surviving banks to gain from the failure of another bank by scaling up their own operations. In this sense, flight to quality counteracts herding incentives by reducing the costs of banks from information contagion. Nevertheless, if the future profitability of loans is expected to be low, depositors may rationally choose not to lend even to the surviving bank. Formally, in the presence of flight to quality, the extent of ex-ante herding measured through inter-bank correlation is decreasing in the expected profitability of loans tomorrow. If the expected profitability of loans tomorrow is high, inter-bank correlation is low, and vice versa. Thus, we call this phenomenon the procyclicality of herding. Competition amongst banks for loans, whereby banks earn lower returns on loans if they lend to the same industry, gives rise to similar effects as flight to quality. Numerical examples illustrate the effect on procyclicality of the extent of systematic risk in bank loans and the relative likelihoods of good and bad states of the economy.

Next, we introduce a “foreign” bank in the model to study the direction and the scope of information contagion and herding. The foreign bank’s loan returns are assumed to be affected by a systematic factor that is different from the one affecting the loan returns of domestic banks. We argue that information contagion and herding are likely to be localized phenomena. The failure of a domestic bank affects other domestic banks more than it affects the foreign bank. Conversely, the failure of a foreign bank has little information spillover to the domestic banks. By implication, the incentives of banks to herd with each other are stronger within the class of domestic banks than between domestic and foreign banks. This localization could be interpreted as purely geographic in nature, or as a metaphor for some richer heterogeneity amongst banks in their specialization, for example, due to wholesale vs. retail focus, small business lending vs. large business lending, etc.

Finally, we conduct a welfare analysis. To do so, we allow for the possibility that banks can earn better returns by lending to some industries. In this setting, a potential welfare cost of herding arises when loans to more profitable industries are passed up in favor of loans correlated with other banks. Compared to the first-best investments, herding can sometimes produce investments in firms and industries that are less profitable. Similarly, while flight to quality mitigates herding, it can sometimes be inefficient relative to the first-best: it gives banks competitive incentives to lend to different industries, even if a particular industry in the economy is more profitable for all banks.

In the context of our model, however, it is difficult to argue that herding is constrained inefficient. Herding is undertaken ex-ante to mitigate the ex-post costs that bank owners face from information contagion. Furthermore, these ex-post costs comprise social costs for the planner charged with maximizing the value of banking sector in the economy, specifically the sum of the values of bank equity values and deposits. Thus, taking financial intermediation as given, herding occurs in equilibrium only when it is also socially (constrained) efficient. In turn, the systemic risk arising from herding is also (constrained) efficient in our model. This
is an interesting result since it is in contrast to the inefficiency that arises in other herding models. We suggest possible mechanisms via which our result on the constrained efficiency of herding may be overturned. The regulatory assessment of systemic risk must thus take careful account of its different manifestations and delineate the social costs of systemic risk that exceed the costs to bank owners.

Section 2 discusses the related literature. Sections 3 and 4 present the model. Section 5 derives the information contagion. Section 6 demonstrates the herding behavior in response to information contagion and incorporates flight to quality. Section 8 presents the welfare analysis. Sections 9 and 10, respectively, discuss the robustness of the model to extensions and the incorporation of bank regulation. Section 11 concludes. Throughout the paper, empirical evidence is provided to support the theoretical results. All proofs are in the Appendix.

2 Related Literature

De Bandt and Hartmann (2000) provide a comprehensive survey of the literature on systemic risk. Below we summarize the literature that is most relevant to this paper.

Several aspects of our model have roots in the documented empirical facts about banking crises. In models such as Diamond and Dybvig (1983), bank runs occur as sunspot phenomena. By contrast, banks in our model fail when depositors rationally update bank prospects with information gleaned from the realization of returns on bank loans. Gorton (1988), Calomiris and Gorton (1991) provide evidence that banking crises in the U.S. during the pre-Federal Reserve era, that is pre-1914, were preceded by shocks to the real sector and were not based purely on panic. The information spillover to other banks from a bank’s failure is documented (Gorton, 1985, Gorton and Mullineaux, 1987) as the formative reason for the commercial-bank clearinghouses in the U.S., and eventually for the Federal Reserve. Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988) also model information-based bank runs. In their models, a depositor’s decision to run on a bank leads to an information spillover on the decision of other depositors to run, either on the same bank or on others.

The empirical studies on bank contagion test whether bad news, such as a bank failure, the announcement of an unexpected increase in loan-loss reserves, bank seasoned stock issue announcements, etc., adversely affect the other banks.3 These studies have concentrated on various indicators of contagion, such as the intertemporal correlation of bank failures (Hasan

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3If the effect is negative, the empirical literature calls it the “contagion effect.” The overall finding is that the contagion effect is stronger for highly leveraged firms (banks being typically more levered than other industries) and is stronger for firms with similar cash flows. If the effect is positive, it is termed the “competitive effect.” The intuition is that demand for the surviving competitors’ products (deposits, in the case of banks) can increase. Overall, this effect is found to be stronger when the industry is less competitive.

Most empirical investigations of bank contagion are event studies of bank stock price reactions in response to bad news. These studies estimate a market model for bank returns in a historical period before the event conveying bad news. Then the predicted value from the regression is compared with the actual value for a window surrounding the day of the event. Significant negative abnormal returns are regarded as evidence for contagion. These studies generally conclude that such reactions are rational investor choices in response to newly revealed information, rather than purely panic-based contagion.

Our model of information contagion has similarities to the recent papers of Chen (1999) and Kodres and Pritsker (2002). Chen (1999) extends the Diamond-Dybvig model to multiple banks and allows for interim revelation of information about some banks. With Bayesian-updating depositors, a sufficient number of interim bank failures results in pessimistic expectations about the general state of the economy, and leads to runs on the remaining banks. These results are similar to our first result on information contagion. But in our model, the information spillover shows up in both increased borrowing rates and also in runs (if the spillover is large enough). This aspect of our model relates better to the empirical evidence.

Kodres and Pritsker (2002) allow for different channels for financial markets contagion including the correlated information channel. The main focus of their paper is however on the cross-market rebalancing channel wherein investors can transmit idiosyncratic shocks from one market to the others by adjusting their portfolio exposures to shared macroeconomic risks. They show how contagion can occur between markets in the absence of correlated information and liquidity shocks. By contrast, contagion in our paper results necessarily from the correlated information channel. Furthermore, these papers do not model the endogenous choice of correlation of banks’ investments. On this front, our paper is closest in spirit to Acharya (2000) who examines the choice of ex-ante inter-bank correlation in response to financial externalities that arise upon bank failures and in response to “too-many-to-fail” regulatory guarantees. The channel of information spillover that we examine however complements the channels examined in Acharya (2000).

The herding aspect of our paper is related to the vast literature on herding surveyed in Devenow and Welch (1996). In this literature, herding is often an outcome of sequential

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decisions, with the decision of one agent conveying information about some underlying economic variable to the next set of decision-makers. Herding, however, need not always be the outcome of such an informational cascade. It can also arise from a coordination game. In our paper (as also in Rajan, 1994), herding is a simultaneous ex-ante decision of banks to coordinate correlated investments (disclosures of losses). Finally, the welfare costs of herding relative to the first-best arise in our analysis from bypassing superior projects by bank owners in a spirit similar to the welfare analysis in Scharfstein and Stein (1990), Rajan (1994).

Comprehensive empirical evidence on asset correlations of banks has not yet been undertaken. In a recent study, Nicolo and Kwast (2001) find that the creation of very large and complex banking organizations increases the extent of diversification at the individual level and decreases the individual firm’s risk. However, this increased similarity introduces systemic risk. They use correlations of bank stock returns as an indicator of systemic risk potential, concluding their paper with the following: “[W]e know no studies of indirect interdependency, such as any tendency for loan portfolios to be correlated across banks.” Documentation of the correlations in loan portfolios of banks could provide potentially valuable information about the extent of systemic risk in a banking sector.

3 Model

We build a simple model that captures simultaneously (i) information spillover arising from bank failures, (ii) endogenous choice of correlation of bank returns, and (iii) flight to quality.

First, we provide a general overview of the model. In our model, each bank has access to a risky investment, the return from which has a systematic and an idiosyncratic component. Only banks can invest in the risky assets. Banks make investments twice, that is, at two different times. Depending upon the realization of past bank profits, depositors assess the profitability of the risky asset of their bank and incorporate that information in the return they demand on their deposits. Depositors regard the failure of a bank as bad news about the systematic component of bank asset returns. As a result, the surviving banks must promise a higher return to the depositors. This negative effect constitutes an information spillover arising from a bank failure, which, in our model, affects the ex-ante choice of correlation in bank loan portfolios.

Formally, there are two banks in the economy, Bank A and Bank B, and three dates, \( t = 0, 1, 2 \). The timeline in Figure 1 details the sequence of events in the economy. There is a

\[\text{Specifically, Nicolo and Kwast (2001) find that stock prices of the biggest 22 U.S. banking organizations tended to increasingly move in lockstep during 1989–1999. The degree of correlation in stock price movements increased from 0.41 in 1989 to 0.56 during 1996–1999. They suggest on basis of this evidence that “Troubles at a single bank could easily generate investor perceptions of similar troubles at other big banks.”}\]
single consumption good at each date. Each bank can borrow from a continuum of risk-averse
depositors of measure 1. Depositors consume their each-period payoff (say, $w$) and obtain
time-additive utility $u(w)$, with $u'(w) > 0$, $u''(w) < 0$, $\forall w > 0$, and $u(0) = 0$. Depositors
have one unit of the consumption good at $t = 0$ and $t = 1$. Banks are owned by financial
intermediaries, henceforth referred to as bank owners. Bank owners are risk-neutral and also
consume their each-period payoff.

All agents have access to a storage technology that transforms one unit of the consumption
good at date $t$ to one unit at date $t+1$. In each period, that is at date $t = 0$ and $t = 1$,
depositors choose to keep their good in storage or to invest it in their bank. Deposits
take the form of a simple debt contract with maturity of one period. In particular, the
promised deposit rate is not contingent on realized bank returns. Furthermore, since bank
investment decisions are assumed to be made after deposits are borrowed, the promised
deposit rate cannot be contingent on these investment decisions. Finally, the dispersed nature
of depositors is assumed to lead to a collective-action problem, resulting in a run on a bank
that fails to pay the promised return to its depositors. In other words, the contract is “hard”
and cannot be renegotiated.

Banks choose to invest the borrowed goods in storage or in a risky asset. The risky asset
is to be thought of as a portfolio of loans to different industries in the corporate sector, real-
estate investments, etc. Investment by a bank in its risky asset at date $t$ produces a random
payoff $\tilde{R}_t$ at date $t+1$. The payoff is realized at the beginning of date $t+1$ before any
decisions are taken by banks and depositors at date $t+1$. The quantity $\tilde{R}_t$ takes on values
of $R_t$ or 0.

$$\tilde{R}_t = \begin{cases} R_t & \text{for } t = 0, 1. \\ 0 & \text{for } t = 0, 1. \end{cases}$$

The realization of $\tilde{R}_t$ depends on a systematic component, the overall state of the economy,
and an idiosyncratic component. The overall state of the economy can be $Good(G)$ or $Bad(B)$.
The prior probability that the state is $G$ for the risky asset is $p$.

$$State = \begin{cases} Good(G) & \text{with probability } p \\ Bad(B) & \text{with probability } 1 - p. \end{cases}$$

Even if the overall state of the economy is good (bad), the return on the risky asset can
be low (high) due to the idiosyncratic component. The probability of a high return when the
state is good is $q > \frac{1}{2}$: when the state is good, it is more likely, although not certain, that
the return on bank investments will be high. The probability that the return is high when
the state is bad is $(1 - q) < \frac{1}{2}$. Therefore, the probability distributions of returns in different
states are symmetric. To summarize,
The resulting joint probabilities of the states and bank returns are given in Table 1. For simplicity, we assume that, conditional on the state of the economy, the realizations of returns in the first and second period are independent.

Crucially, banks can choose the level of correlation of returns between their respective investments. We discuss this next. In order to focus exclusively on the choice of inter-bank correlation, we abstract from the much-studied choice of the absolute level of risk by banks.

### 3.1 Correlation of Bank Returns

Banks can choose the level of correlation between the returns from their respective investments by choosing the composition of loans that compose their respective portfolios. We will refer to this correlation as “inter-bank correlation.” To model this in a simple and parsimonious manner, we allow banks to choose a continuous parameter $c$ that is positively related to inter-bank correlation and thus affects the joint distribution of their returns. This is a joint choice of the banks which could be interpreted as the outcome of a co-operative game between banks. In our model, this joint choice of inter-bank correlation is identical to the one that arises from the Nash equilibrium choice of industries by banks playing a coordination game.

For example, suppose that there are two possible industries in which banks can invest, denoted as 1 and 2. Bank $A$ ($B$) can lend to firms $A_1$ and $A_2$ ($B_1$ and $B_2$) in industries 1 and 2, respectively. If in Nash equilibrium banks choose to lend to firms in the same industry, specifically they either lend to $A_1$ and $B_1$, or they lend to $A_2$ and $B_2$, then they are perfectly correlated. However, if they choose different industries, then their returns are less than perfectly correlated, say independent. Allowing for a choice between several industries in the coordination game can produce a spectrum of possible inter-bank correlations (without affecting the total risk of each bank’s portfolio). We do not adopt this modeling strategy for most of our exposition since it sacrifices parsimony. Instead, we directly consider the joint choice of inter-bank correlation by banks. In the welfare analysis (Section 8), we do employ the coordination game formulation with only two industries, which by implication gives rise to two possible values for inter-bank correlation.

The precise joint distribution of bank returns in different states of the economy as a
Table 2: Joint distribution of bank returns in the good state.

<table>
<thead>
<tr>
<th>A \ B</th>
<th>High</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>$c$</td>
<td>$q - c$</td>
</tr>
<tr>
<td>Low</td>
<td>$q - c$</td>
<td>$1 - 2q + c$</td>
</tr>
</tbody>
</table>

Table 3: Joint distribution of bank returns in the bad state.

<table>
<thead>
<tr>
<th>A \ B</th>
<th>High</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>$1 - 2q + c$</td>
<td>$q - c$</td>
</tr>
<tr>
<td>Low</td>
<td>$q - c$</td>
<td>$c$</td>
</tr>
</tbody>
</table>

function of the inter-bank correlation parameter $c$ is given in Tables 2 and 3. As can be verified from these tables, the probability of a high return for an individual bank remains the same in each state: $q$ in good state, and $(1 - q)$ in bad state. However, the joint probabilities vary with the correlation parameter $c$. Indeed, the joint distribution representation in Tables 2 and 3 is the only assumption which is consistent with the probabilities of high and low returns for an individual bank, and which is also symmetric, that is it ensures that the probability of both banks having a high return in the good state of the economy is the same as the probability of both banks having a low return in the bad state of the economy. This probability, denoted as $c$, is thus a sufficient statistic for the choice of inter-bank correlation. Replacing $c$ in the joint distribution of returns in Tables 2 and 3 by a function $f(c) \in [2q - 1, q]$, $f'(c) > 0$ produces identical results. Thus, we have chosen the linear specification $f(c) = c$, which produces the most transparent statement of our results.

The maximum value of the correlation parameter $c$, denoted $c_{\text{max}}$, is $q$; the minimum value of $c$, denoted $c_{\text{min}}$, is $(2q - 1)$. Restricting $c$ to the range $[c_{\text{min}}, c_{\text{max}}]$ ensures that all probabilities are non-negative and not greater than one. The covariance, $\sigma_{ab}$, and the variances, $\sigma_a^2$ and $\sigma_b^2$, of bank returns can be shown to be

\[
\sigma_{ab} = R^2 \left[ c(1-q)^2 - 2(q-c)(1-q)q + (1-2q+c)q^2 \right],
\]

\[
\sigma_a^2 = \sigma_b^2 = q(1-q)R^2,
\]

where the time subscript has been suppressed. Hence, the correlation of bank returns is

\[
\rho = \frac{\sigma_{ab}}{\sigma_a\sigma_b} = \frac{c(1-q)^2 - 2(q-c)(1-q)q + (1-2q+c)q^2}{q(1-q)}.
\]

It follows that $\frac{\partial \rho}{\partial c} = \frac{1}{q(1-q)} > 0$, consistent with our reference to parameter $c$ as the inter-bank
correlation. In particular, the levels of inter-bank correlation for some specific values of $c$ are:

$$
\rho = \begin{cases} 
1 & \text{when } c = q \\
0 & \text{when } c = q^2 \\
1 - \frac{1}{q} & \text{when } c = (2q - 1).
\end{cases}
$$

For example, in the welfare analysis in Section 8, we employ the two-industry example discussed above and restrict the choice of inter-bank correlation to $c = q$ when banks lend to the same industry, or $c = q^2$ when banks lend to different (independent) industries.

4 Investments

While the choice of inter-bank correlation is determined by backwards induction, it is easier for sake of exposition to first examine the investment problem at date 0. At date 0, both banks exist. By contrast, at date 1, depending upon the first-period return realizations, one or both banks might have failed.

4.1 First Investment Problem (date 0)

In the first period, both banks are identical. Hence, we consider a representative bank. Since depositors have access to the storage technology, their individual rationality requires that the bank offers a promised return $r_0$ that gives depositors their reservation utility $u(1)$, assumed to be 1. Since $r_0 \geq 1$, it is straightforward to show that it is never optimal for banks to invest in the safe asset. Given their limited liability, banks maximize their equity “option” by investing all borrowed goods in the risky asset.\(^6\)

Thus, depositors are paid the promised return $r_0$ only if the return on bank loans is high, that is $R_0$. Because of the limited liability of banks, depositors get nothing when the return on bank loans is low. The probability of a high return on bank loans, denoted as $\alpha_0$, is

$$\alpha_0 = \Pr(G) \Pr(R_0|G) + \Pr(B) \Pr(R_0|B)$$

(4.1)

$$= pq + (1-p)(1-q).$$

(4.2)

The promised return $r_0$ that satisfies depositors’ individual rationality is thus given by

$$\alpha_0 u(r_0) = 1.$$  

(4.3)

\(^6\)Formally, suppose the bank invests $\theta$ units in the safe asset and $(1-\theta)$ units in the risky asset. Then the payoff in the next period is $\theta + (1-\theta)\bar{R}$. In the low state, this payoff equals $\theta$ which is lower than $r_0$, the promised amount to depositors. Hence, bank owners receive no payoff in this case. In the high state, the bank's payoff is $\theta + (1-\theta)R$. Hence, bank owners receive $\max[R - r_0 - \theta(R - 1), 0]$, which is decreasing in $\theta$. Hence, banks always choose $\theta = 0$. That is, they invest all borrowed goods into the risky asset.
Thus, it follows that the promised rate of return $r_0$ is

$$ r_0 = u^{-1}(1/\alpha_0). \tag{4.4} $$

We assume that $R_0 > r_0$, as otherwise the problem at hand is rendered uninteresting. The payoff to the bank at date 1 from the first period investment, denoted as $\pi_1$, is thus given by

$$ \pi_1 = \begin{cases} R_0 - r_0 & \text{if } \tilde{R}_0 = R_0 \\ 0 & \text{if } \tilde{R}_0 = 0 \end{cases}. \tag{4.5} $$

The expected payoff to the bank at date 0 from its first-period investment, $E(\pi_1)$, is thus

$$ E(\pi_1) = \alpha_0(R_0 - r_0). \tag{4.6} $$

Note that this expected payoff in the first period is independent of the choice of inter-bank correlation. Therefore, when banks choose the level of correlation, they examine the expected payoffs in different states of the world in the second period.

### 4.2 Second Investment Problem (date 1)

We assume that if the return from the first period investment is low, then there is a run on the bank, it is liquidated and it cannot operate any further. If the return is high, then bank owners make the second-period investment. Therefore the possible cases at date 1 are given as follows, where $S$ indicates survival and $F$, failure:

- **SS**: Both banks had the high return, and they operate in the second period.
- **SF**: Bank $A$ had the high return, while Bank $B$ had the low return. Only Bank $A$ operates in the second period. Bank $B$ depositors invest their second-period goods in storage.
- **FS**: This is the symmetric version of state SF.
- **FF**: Both banks failed. No bank operates in the second period.

The possible cases are summarized in Table 4. Recall our simplifying assumption that realizations of returns in the first and second periods are independent, conditional on the true
state of the economy. However, depositors have more information at \( t = 1 \) than they had at \( t = 0 \) to judge the profitability of the risky asset in which their bank invests: they have the realizations of the returns in the previous period for \textit{both} banks. Depositors thus rationally update their beliefs about the profitability of the risky asset their bank invests according to the information revealed by these returns.

Although a bank can have a high return in both states of the economy, that is in the good state as well as in the bad state, there is a systematic component in the probabilities of returns. Thus, the other bank’s return is relevant information to assess the profitability of the risky asset of a given bank. Therefore, the cases \( SS \) (bank \( B \) survives) and \( SF \) (bank \( B \) fails) will have different continuation payoffs for bank \( A \). In the next section, we compute the continuation payoffs of bank \( A \) for the case \( SS \), and thereafter for the case \( SF \).

4.2.1 Both banks survived (\( SS \))

In this case, both banks operate for another period. Armed with the information of the survival of both banks in the first period, depositors can update the probabilities about the overall state of the economy using Table 2 and Table 3, to obtain

\[
\Pr(G|SS) = \frac{\Pr(G \text{ and } SS)}{\Pr(SS|G) \Pr(G) + \Pr(SS|B) \Pr(B)}
\]

\[
= \frac{pc}{pc + (1-p)(1-2q+c)}
\]

\[
= \frac{pc}{(1-2q)(1-p)+c}, \text{ and}
\]

\[
\Pr(B|SS) = \frac{(1-p)(1-2q+c)}{(1-2q)(1-p)+c}.
\]

Using these, depositors can calculate the probability of a high return for their bank in the second period, denoted as \( \alpha_1 \), as

\[
\alpha_1 = \Pr(\hat{R}_1 = R_1|SS)
\]

\[
= \Pr(G|SS) \Pr(\hat{R}_1 = R_1|G) + \Pr(B|SS) \Pr(\hat{R}_1 = R_1|B)
\]

\[
= \frac{pc}{(1-2q)(1-p)+c} q + \frac{(1-p)(1-2q+c)}{(1-2q)(1-p)+c} (1-q)
\]

\[
= \frac{pcq + (1-p)(1-q)(1-2q+c)}{(1-2q)(1-p)+c}.
\]

As argued in the first-period investment, the individual rationality of depositors implies that the promised return, \( r_1^{ss} \), should satisfy

\[
\alpha_1 u(r_1^{ss}) = 1.
\]
Therefore, we obtain that
\[ r_{1}^{ss} = u^{-1}(1/\alpha_1). \]  
(4.16)

Since \( \alpha_1 \) depends on the inter-bank correlation \( c \), we denote this borrowing rate as \( r_{1}^{ss}(c) \).

Again, because of limited liability, banks honor their promises to depositors only when they have the high return. Thus, in this case the payoff to each bank at date 2 from the second period investment, denoted as \( \pi_2^{ss} \), is given by
\[ \pi_2^{ss} = \begin{cases} 
R_1 - r_{1}^{ss} & \text{if } \tilde{R}_1 = R_1 \text{ and } R_1 > r_{1}^{ss} \\
0 & \text{otherwise}
\end{cases}. \]  
(4.17)

Note that if \( R_1 < r_{1}^{ss} \), then it is individually rational for depositors not to lend their goods to banks. Storage is preferred to deposits, since the highest return on loans is insufficient to compensate depositors for the risk of bank failure.

### 4.2.2 Only one bank survived (SF or FS)

This is the case where one bank had a high return, while the other had a low return and has been liquidated. Without loss of generality, we concentrate on the case SF where Bank A had a high return. From the symmetry of the joint probabilities in different states and using Tables 2 and 3, we obtain
\[ \Pr(G|SF) = p. \]  
(4.18)

Essentially, the good news about the economy from the performance of bank A is annulled by the bad news from the failure of bank B. Excepting the possibility that \( R_1 \neq R_0 \) in general, this case is the same as the first-investment problem where the only information was the prior belief. Therefore,
\[ r_{1}^{sf} = u^{-1}(1/\alpha_0) = r_0. \]  
(4.19)

Observe that while the level of inter-bank correlation \( c \) affects the cost of borrowing in the joint survival state, it does not affect the cost of borrowing in the individual survival state. Thus, in this case the payoff to the bank at date 2 from the second period investment, denoted \( \pi_2^{sf} \), is given by
\[ \pi_2^{sf} = \begin{cases} 
R_1 - r_0 & \text{if } \tilde{R}_1 = R_1 \text{ and } R_1 > r_0 \\
0 & \text{otherwise}
\end{cases}. \]  
(4.20)

We have assumed here that depositors of the failed bank cannot migrate to the surviving bank. This assumption will be relaxed later and its implications explored fully.
5 Information Contagion

We can now characterize the spillover from the failure of a bank on the surviving bank. First, the surviving bank’s cost of borrowing rises relative to the state where both banks survive. This is a negative spillover of a bank’s failure; or, put another way, the survival of a bank results in a positive spillover on other surviving banks by lowering the cost of borrowing. In general, this reduces the profits of banks in states where they survive but their peers fail. In particular, if the profitability of the surviving bank’s investments is low, the increased borrowing cost also renders the surviving bank unviable: depositors find it better to invest in the storage technology than lend to their bank. In other words, there is a “run” on the surviving bank induced by an updating of the state of the economy by depositors in response to one bank’s failure. The result is an “information contagion.”

Proposition 5.1 (Information Contagion) \( \forall p, q, \text{ and } c, \)

\[
\begin{align*}
(i) & \quad r_1^{ss} < r_1^{sf} = r_0. \\
(ii) & \quad \pi_2^{ss} > \pi_2^{sf}, \forall R_1 > r_1^{ss}. \\
(iii) & \quad \text{Bank A is viable in the joint survival state } SS, \text{ but is unviable in the individual survival state } SF, \forall R_1 \in (r_1^{ss}, r_0].
\end{align*}
\]

Much empirical evidence exists to support such rational updating by depositors and the resulting information spillover on bank values (see Section 2). We focus below on a few representative papers.

Slovin, Sushka, and Polonchek (1992) examined share-price reactions to the announcements of seasoned stock issues by commercial banks. They found negative effects (significant -0.6%) on rival commercial and investment banks. In another study, Slovin, Sushka, and Polonchek (1999) investigated 62 dividend reductions and 61 regulatory enforcement action announcements over the period 1975–1992. They found that actions against money center banks had negative contagion-type externality for other money center banks.

In a more direct evidence, Lang and Stulz (1992) investigated the effect of bankruptcy announcements on the equity value of the bankrupt firm’s competitors. They found that, on average, bankruptcy announcements decrease the value of a value-weighted portfolio of competitors by 1%. This they attributed to a contagion effect. The effect was stronger.

\[7\text{It is plausible that banks increase their lending rates when faced by an increased borrowing cost. However, this would ration the bank’s borrowers with project returns that are lower than the lending rate offered by the bank. Providing that a bank cannot undo completely the decrease in its profits from increased borrowing rates by increasing its lending rates, this result on information contagion holds. We consider this scenario reasonable, given the typical diminishing returns to scale faced by banks on lending side. See ample empirical evidence in the discussion following Proposition 5.1 that supports the information contagion story.}\]
for highly leveraged industries (banks being the primary candidate) and for firms exhibiting substantial similarities.

Rajan (1994) looked at the effects of an announcement on December 15, 1989, that Bank of New England was hurt from the poor performance of the real estate sector and that it would boost its reserves to cover bad loans. He found significant negative abnormal returns (-2.4%) for all banks, and the effect was stronger for banks with headquarters in New England (-8%). He also found significant negative abnormal returns for the real estate firms in general, whereas the negative effect is stronger for real estate firms with holdings in New England. This suggests that the announcement revealed information about the real estate sector and more so about the real estate sector in New England, and that this information was rationally taken into account by investors in their updating process.

Finally, Schumacher (2000) examined the 1995 banking crisis in Argentina triggered by the 1994 Mexican devaluation. She showed that the failed banks had to pay significantly higher interest rates than the surviving banks, to attract depositors during a period from 3 years before the crisis, until the crisis. She interprets this as a rational updating by depositors of their priors about a bank’s balance sheet.

In the next section, we explore the consequences of such information contagion for the endogenous choice of inter-bank correlation at date 0. To do so, the following computation of the expected payoff of banks from their second-period investment is required.

### 5.1 Expected Payoff from Second-Period Investment

To calculate the expected payoff to the banks in the second period, we use the superscripts $a$ and $b$ to represent the returns on investments of banks $A$ and $B$, respectively. Denote $(\tilde{R}_1^a = R_1, \tilde{R}_0^a = R_0, \tilde{R}_0^b = R_0)$ as $(R_1, R_0, R_0)$ and $(\tilde{R}_1^a = R, \tilde{R}_0^a = R_0, \tilde{R}_0^b = 0)$ as $(R_1, R_0, 0)$. We can calculate the ex-ante expected second-period return of bank $A$ (and by symmetry, of bank $B$) as

$$E(\pi_2(c)) = \Pr(R_1, R_0, R_0) (R_1 - r_1^{ss})^+ + \Pr(R_1, R_0, 0) (R_1 - r_1^{sf})^+$$

(5.1)

where $x^+ = \max(x, 0)$. Furthermore, we obtain

$$\Pr(R_1, R_0, 0) = \Pr(G) \Pr(R_1, R_0, 0|G) + \Pr(B) \Pr(R_1, R_0, 0|B)$$

(5.2)

$$= p(q - c)q + (1 - p)(q - c)(1 - q)$$

(5.3)

and

$$\Pr(R_1, R_0, R_0) = \Pr(G) \Pr(R_1, R_0, R_0|G) + \Pr(B) \Pr(R_1, R_0, R_0|B)$$

(5.5)

$$= pcq + (1 - p)(1 - 2q + c)(1 - q).$$

(5.6)
Substituting these in the expression for $E(\pi_2(c))$, we obtain

$$E(\pi_2(c)) = [pcq + (1-p)(1-2q+c)(1-q)] (R_1 - r^{ss}_1(c))^+ +$$

$$ (q-c) [pq + (1-p)(1-q)] (R_1 - r_0)^+ .$$

We assume henceforth that $R_1 > r^{ss}_1(q)$, which ensures that banks are viable in the state $SS$, $\forall c$. This follows because $r^{ss}_1(c)$ is increasing in $c$, as shown in Lemma A.1 in the Appendix. That is, the joint survival of highly correlated banks does not convey good news about the overall economy to the degree conveyed by banks’ simultaneous survival in a state of lower correlation.

Thus, if $R_1 \in (r^{ss}_1(q), r_0]$, then

$$E(\pi_2(c)) = [pcq + (1-p)(1-2q+c)(1-q)] (R_1 - r^{ss}_1(c))$$

and if $R_1 \geq r_0$, then

$$E(\pi_2(c)) = [pq^2 + (1-p)(1-q)^2] [R_1 - (\lambda(c)r^{ss}_1(c) + (1-\lambda(c))r_0)] ,$$

$$\lambda(c) = \frac{pcq + (1-p)(1-q)(1-2q+c)}{pq^2 + (1-p)(1-q)^2} .$$

In particular, if $R_1 \geq r_0$, then expected second-period profits are the expected return on bank loans in the second period minus the expected borrowing cost in the second period. This expected borrowing cost is a weighted average of the costs of borrowing in the states $SS$ and $SF$, that is, $r^{ss}_1(c)$ and $r_0$, with the respective weights being $\lambda(c)$ and $(1-\lambda(c))$. These weights, up to a constant, are simply the probabilities of being in the states $SS$ and $SF$, respectively. Thus, these expressions make it clear that the level of inter-bank correlation enters the expected return of a bank through the promised interest rates and through the probabilities of joint and individual survival states.

6 Choice of Inter-Bank Correlation

In this section, we show that banks choose to be perfectly correlated at date 0 in response to the anticipated information spillover at date 1 when banks fail. If banks survive together, they subsidize each other’s borrowing costs. To capitalize on this, banks prefer to invest in assets correlated with those of other banks by lending, for example, to similar industries or geographic regions.

The objective of each bank is to find the level of inter-bank correlation $c$ that maximizes

$$E(\pi_1) + E(\pi_2(c))$$

(6.1)
where discounting has been ignored since it does not affect any of the results. With first-period profits, $E(\pi_1)$, unaffected by inter-bank correlation, it is the second-period profits, $E(\pi_2(c))$, that determine the preference of banks for correlation.

Consider first the case where $R_1 \in (r^{ss}_1(q), r_0]$. In this case, banks would choose to be perfectly correlated, specifically $c = q$, provided $E(\pi_2(c))$ in equation (5.9) is increasing in $c$, $\forall c \in [2q−1, q)$. This always holds (see the Appendix). Next, consider the second case where $R_1 \geq r_0$. Again, banks would choose to be perfectly correlated provided $E(\pi_2(c))$ in equation (5.10) is increasing in $c$, $\forall c \in [2q−1, q)$. For the economy studied thus far, this result is always valid as well (see the Appendix). That is, the expected cost of attracting depositors is minimized when banks are perfectly correlated. The following result on ex-ante herding amongst banks formalizes this intuition.

**Proposition 6.1 (Herding)** The expected second period profits, $E(\pi_2(c))$, increase in inter-bank correlation $c$. In equilibrium, banks choose to be perfectly correlated, that is, they choose $c = c_{max} = q$.

The limited liability of banks plays a crucial role here. The information spillover of a bank’s failure makes it less attractive for a bank to survive in an environment where the other bank fails than to survive when the other bank also survives. To capitalize on this relative benefit from surviving with the other bank, each bank seeks to increase inter-bank correlation, which increases the likelihood of joint survival (state $SS$) relative to the likelihood of individual survival (state $SF$). In so doing, however, the likelihood of joint failure (state $FF$) also increases relative to the likelihood of individual failure (state $FS$). Since banks have limited liability in failure, this latter shift in probabilities does not affect bank owners’ welfare. Hence, the interaction of limited liability of banks and the information spillover of bank failures leads to ex-ante herding by banks. This intuition is captured in the expected second-period profits of bank $A$ under different first-period outcomes, shown in Table 5.

<table>
<thead>
<tr>
<th>Bank A \ Bank B</th>
<th>High</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>$\pi^{ss}_2 &gt; \pi^F_2$</td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5: Bank $A$’s expected second-period profits based on the first-period outcomes.

Furthermore, the risk-aversion of depositors plays a crucial role. On the one hand, increasing inter-bank correlation helps banks benefit from more frequent joint survival. However, 

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8If banks’ choice is over which industry to lend to, then Proposition 6.1 would imply that banks lend to the same industry producing the highest possible correlation in their returns.

9Acharya (2000) refers to such behavior of banks as “systemic risk-shifting,” since banks collectively maximize the value of their equity options by holding correlated portfolios.
conditional upon joint survival, the cost of borrowing is \( r_1^{ss}(c) \), which is increasing in inter-bank correlation \( c \): survival of both banks is not as good news about state of the economy if banks are more correlated as when they are less correlated. Formally, relative bank profits between joint survival and individual survival states, \([\pi_2^{sf}(c) - \pi_2^{sf}]\), are a decreasing function of \( c \), because \( \pi_2^{sf} \) is independent of \( c \). At first blush, this might suggest that banks would resist choosing the highest possible level of inter-bank correlation. The proof in the Appendix, however, shows that as long as depositors are risk-averse, i.e., \( u''(\cdot) < 0 \), the decrease in relative profits \([\pi_2^{sf}(c) - \pi_2^{sf}]\) as \( c \) increases is more than offset by the corresponding increase in the relative likelihood of joint survival state. Hence, herding takes the extreme form of \( c = c_{max} \) whenever depositors are risk-averse.

Formally, expected bank profits are equal to expected loan returns minus the weighted average cost of borrowing in states \( SS \) and \( SF \), the weights being the probabilities of these states, \( \lambda(c) \) and \( (1 - \lambda(c)) \), respectively (up to a multiplicative constant). With risk-neutrality, this weighted average of \( r_1^{sf}(= r_0) \) and \( r_1^{ss}(c) \) is independent of \( c \), and as a result, banks remain indifferent between alternate choices of inter-bank correlation. That is,

\[
\lambda(c)r_1^{ss}(c) + (1 - \lambda(c))r_0 = r_1^{ss}(c_{max}), \quad \forall c, \text{ where } r_1^{ss} = \frac{1}{\alpha_1}, \text{ and } r_0 = \frac{1}{\alpha_0}. \quad (6.2)
\]

These facts imply that \( \lambda(c) = (\frac{1}{\alpha_1(c_{max})} - \frac{1}{\alpha_0})/(\frac{1}{\alpha_1(c)} - \frac{1}{\alpha_0}) \).

With risk-averse depositors, banks have to pay an extra premium for the risk-aversion of the depositors. This makes \( r_0 \) high enough that the weighed average cost of borrowing is minimized when the inter-bank correlation is highest. Let \( u^{-1} = v \). Then, it follows that \( v(\cdot) \) is convex, and \( r_1^{ss}(c) = v(\frac{1}{\alpha_1(c)}) \), and \( r_0 = v(\frac{1}{\alpha_0}) \). We know that \( \alpha_1(c) < \alpha_1(c_{max}) < \alpha_0 \). These are simply the facts that (i) \( r_1^{ss}(c) \) is increasing in inter-bank correlation \( c \), and (ii) there is information spillover. With risk-aversion, the average borrowing cost employs the same weight \( \lambda(c) \) as in the case of risk-neutrality. The weight \( \lambda(c) \) is determined by the distribution of the joint returns of banks and is independent of depositors’ utility function. It follows now that \( \forall c \),

\[
\lambda(c)r_1^{ss}(c) + (1 - \lambda(c))r_0 = \lambda(c) v\left(\frac{1}{\alpha_1(c)}\right) + (1 - \lambda(c)) v\left(\frac{1}{\alpha_0}\right) > v\left(\frac{1}{\alpha_1(c_{max})}\right) = r_1^{ss}(c_{max}), \quad (6.3)
\]

since the convexity of \( v(\cdot) \) implies that

\[
\lambda(c) = \frac{\left(\frac{1}{\alpha_1(c_{max})} - \frac{1}{\alpha_0}\right)}{\left(\frac{1}{\alpha_1(c)} - \frac{1}{\alpha_0}\right)} > \frac{\left[v\left(\frac{1}{\alpha_1(c_{max})}\right) - v\left(\frac{1}{\alpha_0}\right)\right]}{\left[v\left(\frac{1}{\alpha_1(c)}\right) - v\left(\frac{1}{\alpha_0}\right)\right]}, \quad (6.5)
\]
Under our assumed two-point return distribution for each bank, the information spillover arises precisely when a bank fails. We might, however, consider the implications of assuming a continuous return distribution. In this case, the information event that leads depositors to update their beliefs about the state of the economy need not only be bank failures. In fact, any combination of realizations of bank profits leads to rational updating by depositors. The overall spillover nevertheless remains qualitatively similar. The bank with superior performance always suffers some information spillover due to the relatively inferior performance of the other bank. To summarize, date 1 in our model could be considered simply an “information event” that leads to rational updating by depositors. The resulting revision of borrowing costs would affect bank profits as long as banks require additional financing.

In the next section, we show that if depositors of the failed bank choose rationally between lending to the surviving bank and investing in risk-free technology, then banks do not always choose to be perfectly correlated. That is, herding incentives are mitigated.

### 6.1 Flight to Quality

We relax the assumption that depositors of the failed bank simply keep their goods in storage. Suppose in state $SF$, the depositors of the failed bank migrate to the surviving bank. Clearly, such a migration is individually rational for depositors only if the surviving bank is viable, that is, if it has profitable opportunities whose returns exceed the promised deposit rate in some states of the world. We call such migration “flight to quality.” The effect of such flight to quality is essentially to increase the scale of the surviving bank: the surviving bank receives total deposits of two units when it is the only surviving bank, rather than its previous allocation of one unit. This increases the attractiveness of state $SF$ compared to the situation without flight to quality. In turn, it mitigates the herding behavior of banks.

In the presence of flight to quality, the expected second-period profits of banks, denoted as $E(\pi_{2}^{FQ})$, are given as:

\[
E(\pi_{2}^{FQ}(c)) = [pcq + (1 - p)(1 - 2q + c)(1 - q)](R_1 - r_{1}^{ss}(c))^+ + 2(q - c)[pq + (1 - p)(1 - q)](R_1 - r_{0})^+ \\
= E(\pi_{2}) + [(q - c)(pq + (1 - p)(1 - q))](R_1 - r_{0})^+. 
\]  

(6.6)  
(6.7)  
(6.8)

The expected profits in the absence of depositor migration are augmented by the increase in scale of the surviving bank, provided depositors migrate, that is, if $R_1 > r_{0}.$

To examine the choice of inter-bank correlation, we consider the behavior of $E(\pi_{2}^{FQ}(c))$ as a function of $c$. It follows that

\[
\frac{\partial E(\pi_{2}^{FQ})}{\partial c} = \frac{\partial E(\pi_{2})}{\partial c} - (pq + (1 - p)(1 - q))(R_1 - r_{0})^+. 
\]  

(6.9)
The first term on the right hand side of equation (6.9) is positive, as shown in Proposition 6.1, and induces banks to correlate with other banks. However, the prospect of increased profits conferred by survival in an environment of failure of the other bank induces a countervailing incentive. The effect of flight to quality is thus to weaken the herding incentives. In fact, if the attractiveness of second-period investments, measured by $R_1$, is sufficiently high, then increasing the scale of the bank dominates any induced spillover. Thus, banks choose to be minimally correlated at date 0. For intermediate values of $R_1$, banks choose an interior level of correlation, which is decreasing in the profitability of the second-period investment, $R_1$.\textsuperscript{10}

Proposition 6.2 (Flight to Quality and Pro-Cyclicality of Herding) In the presence of flight to quality, $\forall p$ and $q$, $\exists R_1^* > r_0$ and $\exists R_1^{**} \geq R_1^*$ such that

(i) $\forall R_1 \in (r_1^{**}(q), R_1^*)$, banks choose to be perfectly correlated, that is, they choose $c = c_{max} = q$;

(ii) $\forall R_1 \in [R_1^*, R_1^{**})$, banks choose an interior level of correlation $c^*(R_1) \in (c_{min}, c_{max}) = (2q - 1, q)$ such that $c^*(R_1)$ is decreasing in $R_1$; and

(iii) $\forall R_1 \geq R_1^{**}$, banks choose the lowest level of correlation, that is, they choose $c = c_{min} = 2q - 1$.

There exist parameter values for which $R_1^{**} = R_1^*$, so that the choice of inter-bank correlation switches directly from $c_{max}$ to $c_{min}$ as $R_1$ increases. The numerical examples in Section 6.2 show, however, that there also exist robust sets of parameterizations such that $R_1^{**} > R_1^*$. The result is a choice by banks for an interior level of correlation over the range $R_1 \in [R_1^*, R_1^{**})$.

Empirical evidence supports the migration of survivors of failed banks to surviving banks, while also indicating that when information contagion is sufficiently severe investors flee the banking sector as a whole, taking their deposits with them.

Saunders and Wilson (1996), for example, examined deposit flows in 163 failed and 229 surviving banks over the Depression era of 1929–1933 in the U.S. They found evidence for flight to quality for years 1929 and 1933: withdrawals from the failed banks during these years were associated with deposit increases in surviving banks. However, for the period 1930–1932, deposits in failed banks as well as surviving banks decreased, which the authors interpreted as evidence for contagion. Importantly, the deposit decrease in the failed banks exceeded those at the surviving banks, most likely a manifestation of rational updating of beliefs about bank prospects by informed depositors. In another study, Saunders (1987) studied the effects on

\textsuperscript{10}If each bank chooses from one of two possible industries to lend to, then Proposition 6.2 would imply that there is a critical value of $R_1$, the future profitability of loans, such that below this critical value, banks choose to lend to the same industry, and above this critical value, banks choose to lend to different industries.
the other banks’ deposits due to two announcements regarding an individual bank in April and May 1984. While the first announcement did not have a significant effect, the second one, made by the U.S. Office of the Comptroller of the Currency, resulted in a flight to quality.

More broadly, we interpret the result in Proposition 6.2 as the “pro-cyclicality” of herding.

**Pro-cyclicality of herding:** Historical evidence on bank lending and its fluctuations suggests that herding is pro-cyclical: lending to some industry surges in the economy at peaks in the cycle affecting that industry, and a sharp contraction ensues at troughs of the cycle.

The present analysis provides a possible rationale for such pro-cyclical lending behavior. At business cycle peaks, the expected future return on bank investments is lower (lower $R_1$), for example, due to a possible slow-down in the economy. Thus, the expected benefit to banks in differentiating from other banks is not large. Simply stated, there is not much business for banks in the forthcoming periods. Such an economic state causes herding incentives to dominate and banks to continue to lend to a common industry. By contrast, at business-cycle troughs, the future profits from bank investments are attractive (higher $R_1$). The consequent expected benefit of survival when other banks fail, for example, through an increase in the scale of business, are sufficient to overcome the benefits of herding. The result is that banks differentiate at the troughs and lending to a common industry is retrenched.

Furthermore, if returns on bank investments indeed exhibit such cyclical behavior, then aggregate bank lending to a particular industry must show a “trend-chasing” behavior. Indeed, Mei and Saunders (1997) demonstrated that investments in real-estate by U.S. financial institutions tended to be greater precisely in those times when the real-estate sector looked less attractive from an ex-ante standpoint. Interpreting such behavior at the level of an individual bank or institution may perhaps suggest a behavioral inefficiency on part of the loan officers: banks appear to increase their lending to an industry when its expected returns are falling and reduce their lending when its expected returns are rising. However, when viewed in the context of the herding incentives of banks, this is exactly the lending behavior one should anticipate from profit-maximizing loan officers.11

In fact, the findings of Mei and Saunders provide a possible means to distinguish our results from those of herding models that are based on considerations of managerial reputation. We discuss two of these models, Scharfstein and Stein (1990) and Rajan (1994), in some detail in Section 9. Scharfstein and Stein’s sequential model of herding is quiet about the variation in herding behavior over the business cycle. Rajan’s simultaneous herding model more closely resembles the model of the present paper. In Rajan’s model, banks coordinate and hide

---

11The pro-cyclicality of bank lending has also been documented by Berger and Udell (2002) and the references therein. These studies examine the overall level of bank lending and its fluctuations through the business cycle. We focus our discussion around the evidence of Mei and Saunders (1997) since they examine lending only to the real-estate which relates more directly to correlated lending and its pro-cyclicality.
their losses in business cycle peaks when public information about the poor performance of the corporate sector has not become available. This leads to excessive lending in these periods. However, in business cycle troughs when the corporate sector performance is public knowledge, banks announce their losses and take profit-maximizing lending decisions. This latter result contradicts the finding of Mei and Saunders that banks act in a trend-chasing behavior in both business cycle peaks and troughs. By contrast, our model is able to explain this evidence consistently in peaks as well as troughs.

6.2 Numerical Examples

We provide a numerical example to illustrate these results. Suppose \( u(w) = \sqrt{w} \), \( p = \frac{1}{2} \), and \( q = \frac{3}{4} \). Then \( \alpha_0 = \frac{1}{2} \) and \( r_0 = u^{-1} (\frac{1}{\alpha_0}) = 4 \). We also obtain that \( r^*_{1s}(c) = (1/\alpha_1)^2 \), where \( \alpha_1 \) is a function of \( c \) given by equation (4.14). Substituting for \( \alpha_1(c) \), we obtain \( r^*_{1s}(c) = 16 \left( \frac{4c - 1}{8c - 1} \right)^2 \).

Since, \( c_{\text{max}} = q = \frac{3}{4} \) and \( c_{\text{min}} = 2q - 1 = \frac{1}{2} \), we have that \( r^*_{1s}(c) \leq \frac{64}{25} < 4 = r_0, \forall c \).

Next, we calculate the expected second-period profit of banks assuming no flight to quality. We assume that \( R_0 \) and \( R_1 \) are greater than 4 so that the surviving bank is viable in states \( SF \) and \( FS \). Then, from equations (4.6) and (5.10), we obtain

\[
E(\pi_1) = \frac{1}{2}(R_0 - 4), \quad (6.10)
\]

\[
E(\pi_2(c)) = \frac{5R_1}{16} - \frac{(12c - 1)}{2(8c - 1)}. \quad (6.11)
\]

Then, \( \frac{\partial}{\partial c} [E(\pi_1) + E(\pi_2(c))] = \frac{2}{(8c - 1)^2} > 0, \forall c, R_0, R_1. \)

With flight to quality, the total expected profits of each bank (equation 6.9) are

\[
E(\pi_1) + E(\pi_2^{\text{FQ}}(c)) = \frac{R_0}{2} - 2 + \left( \frac{11 - 8c}{16} \right) R_1 + \left( \frac{16c^2 - 20c + 2}{8c - 1} \right). \quad (6.12)
\]

These profits may be increasing, U-shaped, or decreasing as a function of \( c \).

In Figure 2, we assume \( R_0 = 6 \) and consider two possible values for \( R_1 \): \( R_1 = 4.3 \) (low) and \( R_1 = 8 \) (high). The expected profits in absence of flight to quality are plotted in dashed lines, and those with flight to quality are plotted in solid lines. The figure illustrates two important features. First, in absence of flight to quality (NoFQ), the expected bank profits are increasing in \( c \), the level of inter-bank correlation. Thus, banks herd and pick a correlation of \( c_{\text{max}} = \frac{3}{4} \). Second, with flight to quality (FQ), when \( R_1 \) is low, herding is only partially mitigated. Expected profits are U-shaped in \( c \), reaching a maximum near \( c = 0.58 \). By contrast, at the high value of \( R_1 \), the expected profits are always declining in \( c \) and herding is completely eliminated: banks pick the lowest inter-bank correlation of \( c_{\text{min}} = \frac{1}{2} \).
In Figure 3, we assume that $p = \frac{1}{2}$ and plot the choice of inter-bank correlation $c^*$ as a function of $R_1$ for three different values of $q$: 0.55, 0.75, and 0.95. In each case, $c^*$ equals $c_{max}$ for low $R_1$, and it decreases to $c_{min}$ as $R_1$ rises. The range of $R_1$ over which herding is ameliorated, that is, over which $c^* < c_{max}$, decreases as $q$ is increased. Recall that $q$ is inversely related to the extent of idiosyncratic risk of returns on bank loans. Intuitively, as idiosyncratic risk of bank loans decreases, information contagion worsens and herding is ameliorated by flight to quality only if the profitability of future loans is extremely high.

In Figure 4, the critical levels $R^*_1$ and $R^{**}_1$ are plotted as functions of $q$, again with $p = \frac{1}{2}$. In the region below $R^*_1$, flight to quality cannot mitigate herding and $c^* = c_{max}$. In the region $(R^*_1, R^{**}_1)$, herding is partially mitigated and $c^* \in (c_{min}, c_{max})$. If idiosyncratic risk is very low, that is, if $q$ is high, then herding incentives dominate competitive incentives for a large range of $R_1$ values. However, when the competitive incentives dominate, they do so sharply and cause $R^*_1 \approx R^{**}_1$. The same behavior occurs at high idiosyncratic risk, that is, low $q$, resulting in $R^*_1 \approx R^{**}_1$, but at a much lower value of $R_1$ since competitive incentives are now stronger. For moderate idiosyncratic risk, the trade-off between herding and competitive incentives is more gradual, and $R^*_1$ and $R^{**}_1$ differ substantially. In particular, if $q = \frac{3}{4}$ then we see that $R^*_1 = 6.72$ and $R^{**}_1 = 8$ (consistent with Figure 3).\textsuperscript{12}

In Figure 5, the choice of inter-bank correlation $c^*$ is plotted as a function of $R_1$ for three different values of $p$ (0.25, 0.50, and 0.90) with $q = \frac{3}{4}$. Recall that $p$ measures the unconditional likelihood of the good state of the economy. As this likelihood decreases, herding occurs over a greater range of $R_1$ values. The intuition for this behavior is identical to that for Figure 3. Finally, in Figure 6, $R^*_1$ and $R^{**}_1$ are plotted as functions of $p$ for $q = \frac{3}{4}$. In contrast to Figure 4, $(R^{**}_1 - R^*_1)$ decreases monotonically as $p$ increases. The trade-off between herding incentives and competitive incentives is gradual. When the likelihood of the bad state of the economy is high, that is, $p$ is low, herding incentives dominate. Their effect, however, weakens monotonically as $p$ increases.

7 Effect of Heterogeneity and More Than Two Banks

In this section, we extend the basic model to derive conclusions regarding properties of banks for which information contagion, and by implication the herding incentives, are likely to be

\textsuperscript{12}A possible interpretation of this finding concerns the effect of the failure of large banks whose portfolios are typically well-diversified, and in turn, contain little idiosyncratic risk. The information spillover arising from the failure of such a bank is severe since it primarily conveys information about the state of the economy. Consistent with this interpretation, Slovin, Sushka, and Polonchek (1999) found that regulatory actions against money center banks (big banks, well diversified, higher $q$) had negative contagion-type effects on other money center banks. By contrast, the actions against regional banks (less diversified, more idiosyncratic risk, low $q$) had positive competitive effects on geographical rivals.
stronger. For simplicity, we do not state our results as formal propositions.

Suppose there are three banks in the model, but one of the banks is a “foreign” bank. The foreign bank also has access to a set of depositors with a unit of consumption good each period. The foreign bank is affected by a foreign systematic risk factor, whereas the two “domestic” banks are affected by the same domestic systematic risk factor. Suppose that the probabilities of the good state and the high return are the same as before for each bank. That is, the probabilities have the same value ex-ante for all banks, the return realizations are drawn from the same distribution for the two domestic banks, but they are drawn from a different distribution for the foreign bank.

Under this structure, the realization of domestic banks’ returns conveys no information about the foreign systematic risk factor, and vice versa. The updating by depositors and the promised rates are thus the same for the two domestic banks and identical to the values derived in the analysis thus far. However, this is not the case for the foreign bank. Suppose that the foreign bank had the high return in the first period. Then, denoting \( G_f \) and \( B_f \) as the Good and the Bad states for the foreign systematic risk factor, we obtain

\[
\Pr(G_f|R_0) = \frac{pq}{pq + (1-p)(1-q)}, \tag{7.13}
\]

\[
\Pr(B_f|R_0) = \frac{(1-p)(1-q)}{pq + (1-p)(1-q)}. \tag{7.14}
\]

Using these, the probability of a high return for the foreign bank in the second period, given the foreign bank had a high return in the first period, is calculated as

\[
\alpha_f^1 = \Pr(R_1|R_0) = \frac{pq}{pq + (1-p)(1-q)} q + \frac{(1-p)(1-q)}{pq + (1-p)(1-q)} (1-q) \tag{7.15}
\]

\[
= \frac{pq^2 + (1-p)(1-q)^2}{pq + (1-p)(1-q)}. \tag{7.16}
\]

The promised return to the foreign bank’s depositors at date 1, \( r_f^1 \), thus satisfies

\[
\alpha_f^1 u(r_f^1) = 1. \tag{7.17}
\]

Since \( \alpha_f^1 = \alpha_1(c_{\text{max}}) > \alpha_0 \), it follows that \( r_f^1 = u^{-1}(1/\alpha_f^1) < r_0 \). Therefore, when one of the domestic banks fails, the foreign bank (if it has survived) can attract depositors by offering a lower promised rate than the surviving domestic bank.

More generally, suppose that due to geographic segmentation and/or due to diseconomies of scope, one set of banks is different from the other set in terms of loan portfolio exposure. Then, upon the failure of some banks, deposits would rationally “fly” to banks that are
considered different from, and hence deemed safer than, the failed banks. As a result, the banks considered to be similar to the failed ones will observe a decrease in their deposit base even though they have not failed. To summarize, the benefit from flight to quality to dissimilar banks could result in the accentuation of information contagion for similar banks.

Consistent with this, Kaufman’s (1994) survey on systemic risk documented abnormal negative returns of other banks when a given bank fails, only if the surviving banks also invested in the same product or market area. In another piece of supporting evidence, Schumacher (2000) examined the 1995 banking crisis in Argentina triggered by the 1994 Mexican devaluation. Her analysis showed evidence of bank-specific contagion and flight to quality, rather than a contagion to the whole system. More specifically, after the failure of some domestic wholesale banks, surviving domestic wholesale banks suffered significant deposit losses. The suspension of retail bank operations increased the withdrawals from surviving small retail banks. But the foreign retail banks, and to some extent large domestic banks, had significant increases in their deposits during the crisis.

Consider next the failure of the foreign bank. Since the foreign bank’s loan returns are affected by a systematic factor that is different from the one affecting domestic banks’ loans, it follows that its failure has no impact on the cost of borrowing for the domestic banks. We conclude that information contagion is more likely to arise from the failure of banks whose portfolio returns are anticipated to be more correlated with the overall state of economy. Furthermore, the information contagion is likely to be localized, affecting those banks the most whose portfolio returns are also anticipated to be highly correlated with the overall economy. Banks affected largely by factors that are not related to factors affecting the failed banks may in fact benefit due to migration of depositors. By implication, herding amongst banks should be a localized phenomenon as well.

8 Welfare Analysis

Two matters point to the undesirability of herding by banks. One concerns the fact that when banks herd, they may ignore the profitability of firms and industries they can lend to. This effect is similar to that considered in the reputation-based herding models of Scharfstein and Stein (1990), Rajan (1994). To embed the welfare costs that arise from this effect, we revert to the two-bank case but relax the model to permit the possibility of differential returns to banks from the firms and industries to which they can lend.

The loss in diversification faced by investors constitutes the second reason why herding by banks can be socially undesirable. Depositors or public equity holders hold less diversified portfolios when banks herd, provided these investors have access to multiple banks. This effect generates excessive aggregate risk in the economy and leads to welfare costs that are
different from those described above. A complete analysis embedding such welfare costs requires additional machinery and is beyond the scope of the current paper. In our model, depositors lend to only one bank, and there are no public equity holders. Hence, we focus on the welfare costs that arise from the passing up of profitable projects by banks.

Suppose there are two possible industries in which banks can invest, denoted 1 and 2. Bank $A$ ($B$) can lend to firms $A_1$ and $A_2$ ($B_1$ and $B_2$) in industries 1 and 2, respectively. We assume for simplicity that all firms are equally profitable in the first period, i.e., they have identical return $R_0$ in high state in the first period. However, firms may have different levels of profitability in the second period. Bank owners receive signals at date 0 about the profitability of all firms in the second period. In particular, they receive a signal whether the return on a firm in the high state in the second period is $R_1$ or $(R_1 + \epsilon)$, $\epsilon > 0$. One of the two firms available to each bank has a return of $R_1$, whereas the other firm has a return of $(R_1 + \epsilon)$. After receiving these signals, bank owners decide which firm to lend to.\footnote{Implicitly, we assume here that if banks lend to a firm in the first period, they continue to lend to the same firm in the second period. It is straightforward to relax this assumption and allow banks to receive private signals about $\tilde{R}_0$, the first-period profitability of firms. Furthermore, if we allow banks to receive signals only about the firms they can lend to and not about firms that other banks can lend to, then the analysis is more involved. With private signals, bank actions potentially reveal their information to other banks. We wish to consider a coordination game between banks and hence assume that each bank knows profitability of all firms in the economy.} For simplicity, the signals are perfect: bank owners know the true return on each firm with certainty. A coordination game results if banks are assumed to play a Nash strategy in their choice of industry. The outcome of this game determines the inter-bank correlation.

If banks choose to lend to firms in the same industry, specifically they either lend to $A_1$ and $B_1$, or they lend to $A_2$ and $B_2$, then they are perfectly correlated ($c = q$). However, if they choose different sectors, then their returns are imperfectly correlated, say independent ($c = q^2$). With only a few additional assumptions, this two-industry structure maps onto our model that permits choice of inter-bank correlation, but limits that choice to two values, $c = q$ or $c = q^2$. These assumptions are as follows: First, bank owners make their lending decisions simultaneously. Second, we assume that $R_1 > r_0$, so that bank $A$ ($B$) is viable in the state $SS$ as well as in the states $SF$ ($FS$). Finally, there is no flight to quality upon bank failures at date 1. The implications of relaxing these assumptions are discussed later.

By symmetry, it suffices to analyze the two possible information structures that arise as shown in Tables 6 and 7. In structure $(A_1, B_1)$, firms $A_1$ and $B_1$ have the higher return $(R_1 + \epsilon)$. In structure $(A_1, B_2)$, firms $A_1$ and $B_2$ have the higher return. We wish to compare bank lending decisions under these information structures to the first-best investments and to the constrained efficient investments. Under the first-best investments, financial intermediation is not required: funds in the economy can be directly lent to firms in different industries.
and transfers made across economic agents. In particular, funds are lent to the most profitable firms available. By contrast, under constrained efficient investments, a financial intermediation structure that includes the deposit contract, is taken as given. Constrained efficient investment choices maximize the total expected profits of banks in the economy, because depositors in our economy are always guaranteed a fixed reservation utility in both periods.

From Table 6, it follows that the first-best investments under the information structure \((A_1, B_1)\) involve lending funds of depositors of bank \(A\) to firm \(A_1\) and funds of depositors of bank \(B\) to firm \(B_1\). Similarly, the first-best investments under the information structure \((A_1, B_2)\) involve lending to firms \(A_1\) and \(B_2\).

The following proposition establishes that under the information structure \((A_1, B_1)\), bank investments, and by implication inter-bank correlation, are first-best efficient (under the Pareto-dominating equilibrium). However, under the information structure \((A_1, B_2)\), bank investments are in general not first-best efficient. The first-best requires banks to invest in different industries in this case, but banks herd and lend to firms in the same industry as long as the differential return between the more profitable firms of the two industries is not too large. The intuition is that if the benefits of differentiating from other banks is not large enough, then banks herd to reduce expected losses tomorrow from information spillovers. Consequently, inter-bank correlation is greater than the socially optimal level and thus generates excessive systemic risk.

**Proposition 8.1 (Herding and First-Best Investments)** Assume there is no flight to quality upon bank failures at date 1.

(i) Under the information structure \((A_1, B_1)\), two Nash equilibria exist. The equilibrium where bank \(A\) lends to firm \(A_1\) and bank \(B\) lends to firm \(B_1\) is the Pareto-dominating equilibrium and gives rise to first-best investments.\(^{14}\)

\(^{14}\)We focus our analysis on pure strategy equilibria. There does exist a mixed-strategy equilibrium but it is also Pareto-dominated.
(ii) Under the information structure \((A_1, B_2)\), \(\exists \epsilon^*\), a critical level of differential return between the firms, such that

(a) if \(\epsilon < \epsilon^*\), then two Nash equilibria exist that are not Pareto-ranked and neither of which lead to first-best efficient investments: bank \(A\) lends to firm \(A_1\) \((A_2)\) and bank \(B\) lends to firm \(B_1\) \((B_2)\), whereas the first-best investment leads to lending to firms \(A_1\) and \(B_2\).

(b) if \(\epsilon \geq \epsilon^*\), bank \(A\) lends to firm \(A_1\) and bank \(B\) lends to firm \(B_2\) in the unique Nash equilibrium giving rise to first-best investments.

Thus, passing up of superior investments in favor of investments correlated with other banks gives rise to welfare costs of herding relative to the first-best. With the possibility of flight to quality at date 1, the nature of welfare costs under the information structure \((A_1, B_2)\) remains qualitatively similar. Formally, herding leads to welfare costs under the information structure \((A_1, B_2)\) if and only if \(\epsilon < \epsilon^{**}\), where \(\epsilon^{**} < \epsilon^*\). Interestingly however, flight to quality may itself generate welfare costs. Consider the information structure \((A_1, B_1)\). In this case, the first-best investments involve lending all goods to industry 1. However, the prospect of capturing profits at date 1 through flight to quality gives incentives to banks to differentiate by lending to less profitable industries. This incentive prevails if both industries are sufficiently profitable, that is, \(R_1\) is sufficiently high (as in Proposition 6.2), but industry 1 is not too profitable compared to industry 2, that is, \(\epsilon\) is sufficiently small.

A formal statement of this welfare analysis is rather messy, since different equilibria arise depending upon the magnitudes of \(R_1\) and \(\epsilon\). We simply summarize it with the following counter-intuitive observation. To conclude that flight to quality is a panacea that always mitigates the welfare costs of herding is not correct in general. It may be socially optimal for banks to lend to the same industry if that industry is highly profitable compared to other industries. Flight to quality however provides incentives to differentiate even in these cases.

First-best investments may however be too strict a welfare criterion. If the planner must take the nature of intermediation, in particular the separation of bank owners and depositors, as given, then the information spillover is in fact ex-post optimal from the planner’s standpoint as well. When the planner is thus constrained, we obtain the following result in our model. Investment choices made by banks are the same as those of the constrained planner.

**Proposition 8.2 (Herding and Constrained Efficiency)** Lending decisions of banks and the resulting inter-bank correlation are constrained efficient under the information structures \((A_1, B_1)\) and \((A_1, B_2)\).

The intuition is as follows. Promised interest rates in our model guarantee the depositors their reservation utility in both periods. Hence, the constrained planner makes lending decisions that maximize the welfare of banks. Since bank owners are risk-neutral, their welfare
is identical to their expected profits. Due to symmetry, this in turn is exactly identical to the objective of each set of bank owners and gives rise to the constrained efficiency of their investments. Importantly, this result holds even if (i) we allow for flight to quality upon bank failures at date 1, and (ii) we relax the assumption that $R_1 > r_0$ whereby information contagion can render an otherwise viable bank unviable (Proposition 5.1). Flight to quality leads to benefits for banks in the second period. If $R_1 < r_0$, then the costs of information spillover are greater. If banks herd, they reduce the impact of the spillover but give up second-period profits to be made from flight to quality. The equilibrium lending decisions of banks trade off these two effects in precisely the same manner as would the constrained planner.

This is an interesting result in itself since it is in contrast to the inefficiency that arises in other herding models. In the context of our model, banks fully internalize the costs of information contagion and minimize these costs by herding. Systemic risk arising from such herding is thus socially (constrained) efficient. An important implication of this result is that the presence of herding and the attendant increase in the joint bank failure risk are not sufficient to warrant a regulatory intervention.

We do not imply though that herding will always be constrained efficient. Instead, we interpret the result as suggesting that bank herding in response to the information contagion constitutes an inefficiency only if herding and/or contagion lead to costs over and above the costs to bank owners. One conclusion is that, given the costs of ex-post contagion to bank owners, it is difficult to isolate the extent of ex-ante herding that is socially undesirable. A welfare analysis that takes into account the micro-motives for existence of banks may, however, suggest the nature of the additional costs of herding and contagion beyond those to bank owners. Such costs arise for example in models that incorporate the effects of bank failures on the real sector (Bernanke, 1983), the effect of bank failures on the risk-free rates of interest in the economy (Acharya, 2000), and the liquidity provision role of banks on both the asset and liability sides (Diamond and Rajan, 2001a, 2001b).

9 Robustness Issues and Extensions

9.1 Competition Amongst Banks

The flight to quality phenomenon can be interpreted as a mechanism of competition amongst banks. Direct competition amongst banks for loans would suggest that if banks correlate their

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15The recent work of Diamond and Rajan (2001a, 2001b) suggests that modeling the liquidity provision role of banks on both the asset and liability sides leads to interesting interactions of bank solvency and bank liquidity at both the individual bank level and in the aggregate. Incorporating ex-ante portfolio choices in such micro-models appears to be a fruitful goal that to the best of our knowledge has not yet been pursued.
investment portfolios, for example by lending to firms in the same industry, then they reduce each other’s profit margins. Lang and Stulz (1992) found that the competitors of failed firms benefitted from the failed firms’ bankruptcies for industries with high concentration and low leverage. This is another possible explanation for the finding of Slovin, Sushka, and Polonchek (1999) that regulatory actions against money center banks (MCBs) in the U.S. had negative contagion-type effects for other MCBs, whereas the actions against regional banks had positive competitive effects on geographic rivals.

A fully structural analysis of this effect is beyond the scope of this paper. Nevertheless, the likely effect can be well understood by positing a reduced-form return on bank loans as a decreasing function of inter-bank correlation. Let $R_0(c) = R_1(c) = (R - \delta c)$. Assume for simplicity that there is no flight to quality. It can be shown that in this case herding incentives are weakened. Formally, the choice of inter-bank correlation is given by a function $c^*(\delta)$ which is decreasing in $\delta$: the greater the competition in lending markets, the lower is the propensity of banks to lend to similar industries.

### 9.2 Inter-Bank Linkages

Rochet and Tirole (1996), Allen and Gale (2000), and Dasgupta (2000), to cite a few, consider contagion arising from inter-bank linkages such as inter-bank deposits that provide liquidity insurance to banks. Such linkages do not affect the main thrust of our herding results. If inter-bank deposits are priced fairly in each state of the world, then their usage does not affect expected profits for the lender bank. However, the borrower bank’s profits increase as it can potentially avoid default by drawing down inter-bank deposits when its loan returns are insufficient to meet deposit payments. This reduces the information spillover and mitigates herding incentives. However, for the borrower bank to be able to pay back the lender bank as well as the depositors, the second period return ($R_1$) has to be sufficiently high. Otherwise, either the lender bank or the depositors (depending on the seniority) would not lend.

If the provision of liquidity insurance is mutual and entails no ex-ante cost but only an ex-post to banks, then the (ex-post) lender bank faces an additional withdrawal when the (ex-post) borrower bank accesses the insurance. In the context of our model, bank A’s profits in state $FS$ are not zero as it can access liquidity insurance from bank $B$ and possibly avoid failure. In state $SF$, however, bank A’s profits are reduced as bank $B$ uses insurance provided by bank $A$. Given symmetry, this is simply a transfer of profits of bank $A$ between states $FS$ and $SF$, and does not affect the qualitative nature of ex-ante herding incentives.

More generally, we consider the information channel of contagion to be an important, complementary channel to the one of inter-bank linkages. In fact, empirical evidence has found it hard to attribute the magnitude of contagion effects purely to inter-bank linkages.
Kaufman (1994) claims that there is little direct evidence to suggest that inter-bank exposure has served to transmit shocks from failing banks to solvent banks. He presents the case of Continental Illinois Bank, the seventh largest bank in the U.S. when it failed in 1984, as providing evidence against a significant role being played by inter-bank linkages in contagion.\textsuperscript{16}

### 9.3 Herding Linked to Managerial Reputation

Scharfstein and Stein (1990), Rajan (1994) modeled herding behavior based on managerial concerns for reputation.\textsuperscript{17} Scharfstein and Stein’s model involves sequential herding in investments. Rajan’s model involves simultaneous herding by bank managers as they announce losses on their loan portfolios and adjust their credit policies when faced with short-term horizons.\textsuperscript{18} In these models, it is privately optimal for managers to fail when other managers fail so as to “share the blame.” This leads to a preference for correlated investments or correlated announcements of losses. By contrast, in our model, it is privately optimal for bank owners to succeed when other managers succeed since each bank’s success subsidizes the other banks’ borrowing costs. Hence, the channel of information contagion in our paper is complementary to that of Scharfstein and Stein, and Rajan.

Furthermore, these papers discuss the managerial concern for profits as a countervailing force to herding behavior. For example, Rajan (1994) adds profits to the objective function of managers and demonstrates its countervailing effect over a set of parameters. In our paper, managers maximize bank profits, and yet there is herding. This suggests that aligning managerial objectives with maximization of bank (firm) profits may not be sufficient to ameliorate herding behavior. Put even more strongly, maximization of profits may not be a countervail-

\textsuperscript{16}Shortly before its failure, 2299 banks had exposure to Continental Illinois in terms of deposits or funds (U.S. Congress, 1984). 976 of these banks had exposures more than the government insured amount of $100,000, only 65 banks had uninsured exposures in excess of 100\% of their capital, while 101 banks had uninsured exposures of between 50\% and 100\% of their capital. Kaufman (1985, 1994) estimate that if the losses in Continental Illinois bankruptcy were as large as 60 cents on the dollar, then only 27 of the 2299 banks would have suffered uninsured losses in excess of their capital and become legally insolvent, these losses amounting to a total of $137 million. Another 56 banks would have suffered losses between 50\% and 100\% of their capital and these losses would have totaled $237 million. If the losses in Continental were 10 cents on the dollar, then no banks would have suffered losses more than their capital, while only 2 banks would have suffered losses more than 50\% of their capital with these losses amounting in total to only $1 million. The actual losses in Continental were less than 5 cents on the dollar and as estimated by the mentioned studies, no correspondent bank experienced threatening losses.

\textsuperscript{17}Other incentives for managerial herding (“conservatism”) have also been discussed in the literature. See, for example, Zwiebel (1995) and the survey article by Devenow and Welch (1996).

\textsuperscript{18}Rajan (1994) also provides evidence for his theory based on the New England Banking Crisis of 1990. He shows that the banks “bunch” their provisioning and charge-off decisions: the quarterly bank loan loss provisions (charge-offs) in New England are significantly related to the quarterly loan loss provisions (charge-offs) of other banks in New England.
ing force to herding behavior at all. Managerial reputation and profit maximization can both generate herding, which as a result may be a robust economic outcome.

Finally, another relevant factor is the reward to managers for relative ability, that is, the reward for being successful when others fail. These papers do not model such relative rewards, but instead argue that they counteract herding incentives. Such relative rewards correspond closely to the flight to quality aspect in our model.

9.4 A Further Extension of Our Model

In Scharfstein and Stein (1990) and Rajan (1994), the distribution of outcomes is assumed to be asymmetric over states. This is the main factor driving the herding results in these papers. To summarize their distribution assumptions, there is only one way for managers to be correct, but several different ways for them to be incorrect. By contrast, we assume a symmetric distribution for joint success and failure. In our model, asymmetry arises in the payoffs of bank owners due to the interaction of limited liability and information contagion. Furthermore, in the presence of flight to quality, allowing an asymmetric distribution of outcomes in our model strengthens the herding incentives as we now demonstrate.

Thus far, the probability of high return ($R_t$) in the good state ($G$) and the probability of low return (0) in the bad state ($B$) were both assumed to be equal to $q$. Suppose we allow for different values of $q$ in each state. For example, suppose that

$$\Pr(R_t|G) = q_1 \quad \text{and} \quad \Pr(0|B) = q_2, \quad q_2 > q_1.$$  \hfill (9.1)

This means that outcomes are more correlated with the state of economy when it is the Bad state: low outcome in the Bad state is more likely than high outcome in the Good state. In this case, when one bank survives but the other bank failed (state $SF$ or $FS$), the posterior probability of the high state is lower than the prior of $p$. Formally,

$$\Pr(G|SF) = \frac{p(q_1 - c)}{p(q_1 - c) + (1 - p)(q_2 - c)} < p.$$  \hfill (9.2)

Therefore, $r_{sf}^1$ exceeds $r_0$ whereas they were equal in the model thus far. In turn, even if all depositors fly to the surviving bank, the bank has to pay a higher rate and the benefit from flight to quality is reduced. In the extreme case, $r_{sf}^1$ exceeds $R_1$ and all funds escape the banking system. Ex-ante, this strengthens the herding incentives of banks.
10 Bank Regulation and Policy Implications

Our model abstracted from issues concerning bank regulation. This is justified in our setting since we obtain constrained efficiency of bank decisions. Also, the results we derived are potentially relevant for unregulated industries. Nevertheless, taking bank regulation as given, we consider its implications for our results.

10.1 Deposit Insurance

Most banking systems have deposit insurance. Full deposit insurance renders deposit rates insensitive to fluctuations in bank’s health and reduces information contagion of the sort we described. However, the provision of deposit insurance is only partial in most countries. Furthermore, banks also have subordinated debt in their capital structures that is typically not insured and is often the marginal source of funds. Interest rates on both uninsured deposits and subordinated debt should respond to information pertaining to the bank’s health. Thus, in a world with deposit insurance, information contagion would still arise, but its effect on bank profits would be restricted to the amount of uninsured and subordinated debt.

To this extent, deposit insurance only partially mitigates ex-post information contagion and in turn ex-ante herding. While this role of deposit insurance as a provision for confidence in the banking system has been well recognized, its effect on herding has not been documented before. Any welfare analysis of deposit insurance must also account for an implicit cost: the lack of incentives for insured depositors to differentiate between sound and unsound banks reduces the competitive (flight to quality) benefits to banks.

10.2 Bank Bailouts

When information contagion is severe in our model, banks that would otherwise survive are also rendered unviable. In the absence of flight to quality, such failures can lead to welfare losses. Bank bailouts in this situation mitigate the ex-post costs of information contagion for these banks and the economy at large. Hence, they also mitigate bank incentives to herd ex-ante. Such forbearance is, however, typically time-inconsistent, as studied by Mailath and Mester (1994): exercised forbearance levels exceed the optimal ex-ante level. Furthermore, in models where regulators are captured by the interests of bank owners, discretion over such forbearance leads to a “too-many-to-fail” guarantee where banks rationally anticipate a greater likelihood of failure if they fail together. This in turn generates a collective moral hazard that gives rise to herding incentives, as demonstrated in Acharya (2000).
10.3 Bank Capital

Consider first the private choice of bank capital. Capital buffers a bank from failure by lowering the critical return on loans below which the bank defaults (as would be the case in a richer model with continuous returns on bank loans). Hence, banks can employ capital as a strategic device to reduce the ex-post impact of information contagion from other banks. Note that herding is also a strategic device that serves a similar purpose. While issuance of capital may entail dilution costs to bank owners, as in the models of Gorton and Winton (1999), Bolton and Freixas (2000), herding entails costs to bank owners as well due to passing up of profitable investments. The role of bank capital as a substitute to herding is thus quite sensitive to the relative magnitude of these costs.

The extent of information contagion is linked to the extent of systematic and idiosyncratic risks of bank loans (Figures 2–5). Thus, the levels of bank capital should vary cross-sectionally and in time-series as a function of these components of bank portfolio risk. To our knowledge, this empirical issue has not been investigated in detail. Finally, if dilution costs of capital issuance were mere transfers in the economy, then capital issuance might be less costly from a social standpoint than from the bank owners’ standpoint. This should give rise to a role for minimum capital requirements as a device to pre-commit banks to reduce herding.

10.4 Release of Bank-Specific Information

The release of bank-specific information can mitigate information contagion, since depositors would know the realization of systematic and idiosyncratic shocks in causing the bank failure. This in turn can mitigate the herding incentives. Gorton (1985), Gorton and Mullineaux (1987), and Park (1991) discuss the private clearinghouse arrangements adopted by banks to mitigate information spillovers. Park (1991) argues that banks simply pooled their resources in the earliest such arrangement in 1857. This, however, resulted with a run on all these pooled resources, and the following banking crisis turned out to be quite severe. Banks collectively restored confidence and mitigated disaster in later crises by first investigating the bank under attack. If they found out that it was sound, then they provided assistance to it. Such assistance was a signal that the bank under attack was sound. This prevented further runs on the banking sector.

11 Conclusion

We have undertaken an ex-ante analysis of systemic risk amongst banks. Information contagion generates costs for banks that mitigate its impact on their profits by herding. While
most studies of systemic risk and financial fragility are concerned with the ex-post effects of 
bank failures, our paper demonstrates that analyzing the ex-ante response of banks to these 
effects is important. The extent of herding, the ex-ante aspect of systemic risk, affects the 
likelihood of joint failure of banks, but also affects and is affected by the extent of information 
contagion, which is the ex-post aspect of systemic risk.

An important implication of our paper is the pro-cyclical nature of herding amongst banks, 
and it conforms well with empirical evidence. A complete empirical analysis demonstrating 
the reciprocal causality between inter-bank correlations and information contagion is called 
for. We are currently formulating such an empirical test. We plan to measure herding and 
spillover using the Merton-style inversion of the correlation of bank equity returns into the 
correlation of bank asset values over “ex-ante” and “ex-post” periods.

The interaction between the two forms of systemic risk renders the identification of its 
welfare costs somewhat tricky. In future theoretical work, we plan to introduce firms, that 
is, the real sector, into the model. This would enable a richer welfare analysis of herding 
and contagion. Furthermore, we also plan to introduce a market for risk-free bonds, whereby 
the model can study the effect of flight to quality and bank failures on the spread between 
bank borrowing rates and risk-free rates. The optimal design of prudential bank regulation to 
mitigate the welfare costs of herding amongst banks and a comparison of this optimal design 
with current regulation also appears to be a fruitful objective to pursue.

A Proofs

The following lemmas are employed towards proving the main propositions of the paper.

**Lemma A.1** The probability of a high return, given that both banks had high returns from 
their first investments, is decreasing in $c$. Formally, $\frac{\partial \alpha_1}{\partial c} < 0$, $\forall p, q$. In turn, the cost of 
borrowing is increasing in $c$, i.e., $\frac{\partial r_{ss}^1}{\partial c} < 0$, $\forall p, q$.

**Proof:** From equation (4.14), we obtain

$$
\frac{\partial \alpha_1}{\partial c} = \frac{[pq + (1 - p)(1 - q)] \left[(1 - 2q)(1 - p) + c\right] - [pcq + (1 - p)(1 - q)(1 - 2q + c)]}{\left[(1 - 2q)(1 - p) + c\right]^2} (A.1)
$$

which is negative, since the numerator equals $-p(1 - p)(2q - 1)^2$, which is always less than 
zero. From equation (4.16) for $r_{ss}^1$ and the fact that $u'(\cdot) > 0$, it follows that $\frac{\partial r_{ss}^1}{\partial c} < 0$. ♦

**Lemma A.2** When banks are perfectly correlated, the probability of a high return, given that 
both banks had high returns from their first investments, is greater than the prior probability 
of a high return. Formally, $\alpha_1(c_{max}) > \alpha_0$, $\forall p, q$. 

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Proof: From equations (4.2) and (4.14), the claim reduces to showing that

\[
\alpha_1(q) = \frac{pq^2 + (1-p)(1-q)^2}{pq + (1-p)(1-q)} > \alpha_0 = pq + (1-p)(1-q).
\]  

(A.2)

This holds since \(pq^2 + (1-p)(1-q)^2 - [pq + (1-p)(1-q)]^2 = p(1-p)(2q-1)^2 > 0\). \(\diamondsuit\)

These lemmas and the maintained assumption that \(u'(\cdot) > 0\) imply the following result.

Lemma A.3 \(\forall p, q, \text{ and } c, \alpha_1(c) > \alpha_0 \text{ and } r_{1s}^+(c) < r_0.\)

Proof of Proposition 5.1: The three parts of the proposition are a consequence of Lemma A.3 and the expressions for second-period profits in states SS and SF (equations 4.17 and 4.20, respectively). \(\diamondsuit\)

Proof of Proposition 6.1: We need to consider two cases as discussed in Section 6. First, if \(R_1 \in (r_{1s}^+(q), r_0]\), then from equation (5.9), the expected second period profits are given by

\[
E(\pi_2(c)) = [pcq + (1-p)(1-2q+c)(1-q)](R_1 - r_{1s}^+(c)).
\]  

(A.3)

In this case, taking the partial derivative with respect to \(c\), we obtain

\[
\frac{\partial E(\pi_2(c))}{\partial c} = (1-p)(1-q)(R_1 - r_{1s}^+(c)) - [pcq + (1-p)(1-2q+c)(1-q)] \frac{\partial r_{1s}^+(c)}{\partial c}
\]

which is greater than zero, since from Lemma A.1, \(\frac{\partial r_{1s}^+(c)}{\partial c} < 0\).

Second, if \(R_1 \geq r_0\), then from equation (5.10), the expected second-period profits are

\[
E(\pi_2(c)) = [pq^2 + (1-p)(1-q)^2] [R_1 - (\lambda(c)r_{1s}^+(c) + (1-\lambda(c))r_0)].
\]  

(A.4)

We wish to show that

\[
\frac{\partial E(\pi_2(c))}{\partial c} = -[pq^2 + (1-p)(1-q)^2] \left[ \frac{\partial \lambda(c)}{\partial c}(r_{1s}^+(c) - r_0) + \lambda(c) \frac{\partial r_{1s}^+(c)}{\partial c} \right]
\]

(A.5)

is greater than zero. Since \([pq^2 + (1-p)(1-q)^2] > 0\), the sign of the expression above is determined by the sign of the expression inside the second set of square brackets on the right hand side of equation (A.5). We prove this in steps:

(i) From the expression for \(\lambda(c)\) in equation (5.11), it follows that

\[
\frac{\partial \lambda(c)}{\partial c} = \frac{pq + (1-p)(1-q)}{pq^2 + (1-p)(1-q)^2}.
\]  

(A.6)
(ii) From equations (4.14) and (4.15), it follows that

\[ u(r_{1s}^s(c)) = \frac{(1 - 2q)(1 - p) + c}{pcq + (1 - p)(1 - q)(1 - 2q + c)}. \]  

(A.7)

Taking the partial derivative of both sides with respect to \( c \), we get

\[ u'(\cdot) \frac{\partial r_{1s}^s(c)}{\partial c} = \frac{pcq + (1 - p)(1 - q)(1 - 2q + c) - [(1 - 2q)(1 - p) + c][pq + (1 - p)(1 - q)]}{[pcq + (1 - p)(1 - q)(1 - 2q + c)]^2} \]

\[ = \frac{p(1 - p)(1 - 2q)^2}{[pcq + (1 - p)(1 - q)(1 - 2q + c)]^2}. \]

Therefore, we obtain that

\[ \lambda(c) \frac{\partial r_{1s}^s(c)}{\partial c} = \frac{1}{u'(r_{1s}^s(c))} \frac{p(1 - p)(1 - 2q)^2}{pq^2 + (1 - p)(1 - q)^2 [pcq + (1 - p)(1 - q)(1 - 2q + c)]} \]

(iii) Finally, from equations (4.2), (4.14) and (5.11), we obtain

\[ \frac{\partial \lambda(c)}{\partial c} (r_{1s}^s(c) - r_0) = \frac{pq + (1 - p)(1 - q)}{pq^2 + (1 - p)(1 - q)^2} \left[ u^{-1}\left(\frac{1}{\alpha_1}\right) - u^{-1}\left(\frac{1}{\alpha_0}\right) \right]. \]  

(A.8)

Let \( v = u^{-1} \). Since \( u(\cdot) \) is increasing and concave, \( v'(\cdot) > 0 \) and \( v''(\cdot) > 0 \). Thus,

\[ \left[ v\left(\frac{1}{\alpha_1}\right) - v\left(\frac{1}{\alpha_0}\right) \right] < v\left(\frac{1}{\alpha_1}\right) \left(\frac{1}{\alpha_1} - \frac{1}{\alpha_0}\right). \]  

(A.9)

Substituting for \( \alpha_0 \) and \( \alpha_1 \) from equations (4.2) and (4.14),

\[ v\left(\frac{1}{\alpha_1}\right) \left(\frac{1}{\alpha_1} - \frac{1}{\alpha_0}\right) = \frac{1}{u'(r_{1s}^s(c))} \left[ \frac{(1 - 2q)(1 - p) + c}{pcq + (1 - p)(1 - q)(1 - 2q + c)} - \frac{1}{pq + (1 - p)(1 - q)} \right] \]

\[ = - \left[ \frac{1}{u'(r_{1s}^s(c))} \left(\frac{p(1 - p)(1 - 2q)^2}{pcq + (1 - p)(1 - q)(1 - 2q + c)} \right) \right]. \]

Therefore,

\[ \frac{\partial \lambda(c)}{\partial c} (r_{1s}^s(c) - r_0) < - \frac{1}{u'(r_{1s}^s(c))} \left[ \frac{1}{pq^2 + (1 - p)(1 - q)^2} \right] \left[ \frac{p(1 - p)(1 - 2q)^2}{pcq + (1 - p)(1 - q)(1 - 2q + c)} \right]. \]

Using steps (i), (ii) and (iii) above, it follows that

\[ \frac{\partial \lambda(c)}{\partial c} (r_{1s}^s(c) - r_0) + \lambda(c) \frac{\partial r_{1s}^s(c)}{\partial c} < 0. \]  

(A.10)
It follows that in either of the cases, \( R_1 \in (r_1^{ss}(q), r_0] \) or \( R_1 \geq r_0 \), \( E(\pi_2(c)) \) is increasing in \( c \), and thus banks pick the highest possible inter-bank correlation \( c_{max} = q \).

Proof of Proposition 6.2: For the case with flight to quality, the first-order condition for the choice of inter-bank correlation (from equation 6.9) is

\[
\frac{\partial E(\pi_2^{FQ})}{\partial c} = \frac{\partial E(\pi_2)}{\partial c} - \alpha_0 (R_1 - r_0)^+.
\]  

(A.11)

We prove the proposition in parts.

(i) From Proposition 6.1, we know that \( \frac{\partial E(\pi_2)}{\partial c} > 0 \). Thus, for values of \( R_1 \leq r_0 \), \( \frac{\partial E(\pi_2^{FQ})}{\partial c} = \frac{\partial E(\pi_2)}{\partial c} > 0 \). That is, banks choose the highest possible inter-bank correlation \( c_{max} = q \).

The rest of the proof deals with values of \( R_1 > r_0 \), the case where the surviving bank is viable in states SF and FS.

(ii) Let \( \frac{\partial E(\pi_2)}{\partial c}(c_{max}) \) denote the value of the partial derivative of the expected second period profit (in absence of flight to quality) with respect to \( c \), at \( c_{max} = q \). Define

\[
R_1^* = \left[ \frac{\partial E(\pi_2)}{\partial c}(c_{max}) / \alpha_0 \right] + r_0.
\]

(A.12)

Then, at \( R_1^* \), \( \frac{\partial E(\pi_2^{FQ})}{\partial c}(c_{max}) = 0 \), and, for \( R_1 \leq R_1^* \), \( \frac{\partial E(\pi_2^{FQ})}{\partial c}(c_{max}) > 0 \). Thus, banks choose the highest possible inter-bank correlation \( c_{max} = q \) for \( R_1 \leq R_1^* \).

(iii) Similarly, for \( R_1 > R_1^* \), \( \frac{\partial E(\pi_2^{FQ})}{\partial c}(q) < 0 \), so that banks choose a level of \( c \) that is less than \( c_{max} \). Now, define

\[
R_1^{**} = \left[ \left( \max_c \frac{\partial E(\pi_2)}{\partial c} \right) / \alpha_0 \right] + r_0.
\]

(A.13)

Note that \( R_1^* \leq R_1^{**} \). For values of \( R_1 > R_1^{**} \), \( \frac{\partial E(\pi_2^{FQ})}{\partial c} < 0 \) for all possible \( c \), so that banks choose the lowest possible inter-bank correlation \( c_{min} = 2q - 1 \).

(iv) Finally, for \( R_1 \in (R_1^*, R_1^{**}) \), banks choose an interior level of correlation. To see this, note that \( \exists k_1 > 0 \) such that for \( R_1 \in (R_1^*, R_1^* + k_1) \), \( \frac{\partial E(\pi_2^{FQ})}{\partial c}(c_{min}) > 0 \) and \( \frac{\partial E(\pi_2^{FQ})}{\partial c}(c_{max}) < 0 \).

By the continuity of \( \frac{\partial E(\pi_2^{FQ})}{\partial c} \) and the intermediate-value theorem, it follows that \( \exists c^*(R_1) \in (c_{min}, c_{max}) \) such that \( \frac{\partial E(\pi_2^{FQ})}{\partial c}(c^*) = 0 \).

Similarly, \( \exists k_2 > 0 \) such that, for \( R_1 \in (R_1^{**} - k_2, R_1^{**}) \), \( \frac{\partial E(\pi_2^{FQ})}{\partial c}(c_{min}) < 0 \) and \( \frac{\partial E(\pi_2^{FQ})}{\partial c}(c_{max}) > 0 \). Since \( \frac{\partial E(\pi_2^{FQ})}{\partial c} \) is continuous in \( c \), the intermediate-value theorem implies that \( \exists c^*(R_1) \in (c_{min}, c_{max}) \) such that \( \frac{\partial E(\pi_2^{FQ})}{\partial c}(c^*) = 0 \).
The above facts follow from the definitions of $R_1^*$ and $R_1^{**}$. Next, observe that, from the envelope theorem for $c^*(R_1)$ to maximize $E(\pi_2^{FQ})$, we obtain

$$\text{sign} \left( \frac{\partial c^*}{\partial R_1} \right) = \text{sign} \left( \frac{\partial^2 E(\pi_2^{FQ})}{\partial c \partial R_1} \right) = -\alpha_0 < 0$$

(A.14)

since from Proposition 6.1, $\frac{\partial E(\pi_2)}{\partial c}$ is independent of $R_1$. It follows that $c^*(R_1)$ is monotonically decreasing in $R_1$ and hence, for all $R_1 \in (R_1^*, R_1^{**})$, $c^*(R_1) \in (c_{\min}, c_{\max})$.

Steps (i)–(iv) together prove all parts of the proposition. ♦

**Proof of Proposition 8.1:** We prove Part (ii) of the proposition which is less direct than Part (i). Part (i) follows along similar lines. We focus below only on pure strategy equilibria. Consider the information structure $(A_1, B_2)$ described in Table 7. Suppose bank $A$ lends to firm $A_1$. Consider the best-response of bank $B$. If bank $B$ lends to firm $B_1$, then inter-bank correlation is $c = q$, and its loan return in high state is $R_1$. By lending to firm $B_2$, the inter-bank correlation is $c = q^2$, and bank $B$’s loan return in high state is $(R_1 + \epsilon)$. Since the first-period profits are constant, the best-response of bank $B$ is determined by its expected second-period profits under the two cases. From equation (5.10), these profits are, respectively,

$$E(\pi_2(q)) = \left[ pq^2 + (1 - p)(1 - q)^2 \right] (R_1 - h(q)), \quad \text{and}$$

$$E(\pi_2(q^2)) = \left[ pq^2 + (1 - p)(1 - q)^2 \right] (R_1 + \epsilon - h(q^2)),$$

where

$$h(c) = \lambda(c)r_1^{**}(c) + (1 - \lambda(c))r_0$$ (A.17)

and $\lambda(c)$ is as defined in equation (5.11). In the proof of Proposition 6.1, we showed in equation (A.8) that $h'(c) < 0$. Hence, $h(q^2) > h(q)$, and it follows that

$$E(\pi_2(q)) - E(\pi_2(q^2)) = \left[ pq^2 + (1 - p)(1 - q)^2 \right] (h(q^2) - h(q) - \epsilon)$$

(A.18)

is greater than zero iff $\epsilon < \epsilon^* \equiv h(q^2) - h(q)$.

Thus, if $\epsilon < \epsilon^*$, the best-response of bank $B$ is to lend to firm $B_1$ and correlate its lending with bank $A$. By contrast, if $\epsilon > \epsilon^*$, the best-response of bank $B$ is to lend to firm $B_2$ and differentiate its lending from bank $A$. Continuing this exercise, one can derive the best-responses for both banks as given in Tables 8 and 9.

It can be verified now that if $\epsilon < \epsilon^*$, then bank $A$ lending to firm $A_1$ and bank $B$ lending to firm $B_1$ is a Nash equilibrium, and, similarly, bank $A$ lending to firm $A_2$ and bank $B$ lending to firm $B_2$ is also a Nash equilibrium. Under the first equilibrium of this coordination problem, bank $A$ makes greater expected profits than does bank $B$. The exact converse holds
under the second equilibrium. Hence, these equilibria are not Pareto-ranked. If \( \epsilon > \epsilon^* \), there is exactly one Nash equilibrium in which bank A lends to firm \( A_1 \) and bank B lends to firm \( B_2 \). This proves Part (ii) of the proposition.

Constructing the best-responses of both banks under the information structure \((A_1, B_1)\) reveals that if \( \epsilon > \epsilon^* \), then there exists only one Nash equilibrium where bank A lends to firm \( A_1 \) and bank B lends to firm \( B_1 \). If \( \epsilon < \epsilon^* \), then in addition to this equilibrium there is another Nash equilibrium where bank A lends to firm \( A_2 \) and bank B lends to firm \( B_2 \). However, this latter equilibrium is Pareto-dominated by the first equilibrium where banks derive benefit from herding as well as from the superior returns on the firms they lend to.

\[ \textbf{Proof of Proposition 8.2:} \] Consider the objective function of the constrained planner. The planner puts optimally designed weights on the welfare of depositors and bank owners. However, if the deposit contract is taken as given by the planner, then in our model depositors earn simply their reservation utility of 1 in each period. It follows that for all possible weights in the planner’s objective on the welfare of depositors and bank owners, the planner’s choice of inter-bank correlation \( c \) maximizes the expected profits of bank owners, which are equal to \([E(\pi_1) + E(\pi_2(c))]\) summed over both banks. By symmetry, this is tantamount to the objective function

\[
\max_c [E(\pi_1) + E(\pi_2(c))]
\]

which in turn is the objective function of each bank’s owners. It follows thus that the investment choices of banks are constrained efficient in our model.

\[ \textbf{References} \]


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Figure 1: Sequence of events.

<table>
<thead>
<tr>
<th>$t = 0$</th>
<th>$t = 1$</th>
<th>$t = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nature chooses the overall state of the economy (<em>Good</em> or <em>Bad</em>)</td>
<td>Returns on first-period investments are realized</td>
<td>Returns on second-period investments are realized</td>
</tr>
<tr>
<td>Banks choose the level of correlation ($c$)</td>
<td>Banks with the low return are liquidated</td>
<td>Overall state of the economy is realized</td>
</tr>
<tr>
<td></td>
<td>Banks with the high return invest again</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2: Profits as a function of $c$ for low $R_1 (=4.3)$ and high $R_1 (=8)$
Figure 3: Choice of inter-bank correlation $c^*$ (with flight to quality) as a function of $R_1$ for different $q$ (with $p = 0.5$)

Figure 4: $R_1^*$ and $R_1^{**}$ as a function of $q$ (with $p = 0.5$)
Figure 5: Choice of inter-bank correlation $c^*$ (with flight to quality) as a function of $R_1$ for different $p$ (with $q = 0.75$)

Figure 6: $R_1^*$ and $R_1^{**}$ as a function of $p$ (with $q = 0.75$)