MODELING LONG-RUN LOYALTY

Philip Stern*
Kathy Hammond

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Philip Stern is a lecturer in Marketing & Strategic Management at Warwick Business School; Kathy Hammond is a Research Fellow in the Centre for Marketing, and Director of the Future Media Research Programme, London Business School. They would like to thank TN AGB who provided the 5-year UK detergent panel data, and ISIS Research for providing access to their Jigsaw panel of General Practitioners.

*Correspondence to: Dr Philip Stern, Warwick Business School, Coventry CV4 7AL, UK
Tel: +44(0) 1203522148; Fax: 44(0) 1203524628; Email: P. Stern@warwick.ac.uk

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Modeling Long-run Loyalty

Abstract

Very little is known empirically about customer loyalty to frequently-bought goods over the long term (e.g. over five years or more, or over hundreds of purchases). In this paper we first explore patterns of long-run loyalty in two very different markets; a consumer market – laundry detergents, and a more frequently-used quasi-industrial market – the prescribing of antihypertensive drugs. Second, we determine how well long-run loyalty can be predicted from a stationary-market model – the Dirichlet. We use two measures of loyalty; portfolio size and the share of category requirements accounted for by the buyer’s favorite brand. We overcome problems of purchase incidence heterogeneity by repeatedly examining our loyalty measures for different usage rates rather than varying the analysis period as in traditional panel studies. We find that:

- Portfolio size increases with increasing category usage and then reaches a plateau largely independent of usage. This plateau for portfolio size is around 50% of available brands for laundry detergent, 77% for the prescribing of antihypertensive drugs.
- Share of category requirements given to the favorite brand declines as usage increases, but again reaches a minimum level (64% for laundry detergent, 30% for anti-hypertensive drugs) that is then largely independent of usage.
- The patterns of loyalty (if not the loyalty levels) are similar in both markets and both broadly predictable from an extension of the Dirichlet model. However, over the longer term there are small but consistent deviations from the model predictions.

Both the model fit and managerial implications are discussed.
1. Introduction

Loyalty remains close to the top of the marketing manager’s agenda although the focus of concern appears to be shifting. In the past, developing strong brands through marketing activities has been seen as the key to producing customer franchises that offer sustainable differential advantage. A number of events have challenged this approach. For example, in frequently-bought grocery markets, the growth of store brands (which have claimed large shares in some categories), Procter & Gamble’s attempted move towards low pricing of its brands, and ‘Marlboro Monday’ are indications that even brand leaders are not impregnable. For some commentators one solution is to develop loyalty through other routes such as networks and relationships, while still retaining a brand focus (e.g. Doyle 1995, Reichheld, 1996).

While such prescriptive approaches retain a great deal of intuitive appeal, there is a strand of research which has studied patterns of aggregate brand loyalty across many mature markets, and consistently found that, first, over the medium term (three months to a year), competing brands differ little in their levels of brand loyalty, second, using the NBD-Dirichlet model, a range of brand loyalty measures can be predicted solely from the market share of each brand (Ehrenberg 1988; Ehrenberg and Uncles 1997; Uncles et al 1994; Uncles, Ehrenberg and Hammond 1995). Recent work by Dekimpe et al. (1997) showing the absence of significant trends in aggregate brand loyalty over periods of up to two years, supports the view that, on average, most frequently-bought markets are stable, over the medium term.

1.1 Long-run Loyalty

In this paper we build on the Dirichlet approach to modeling brand loyalty but focus on exploring a hitherto under-researched area - brand loyalty over the longer term. By longer term we mean five years or more, or from 50 to 200+ purchases. While it has previously been noted that managers need to have better information on how buyers might behave over many years (e.g. Dekimpe et al. 1997), consumer panel data is not readily available over the longer term. We therefore adopt a three-pronged approach:
1. The calculation of loyalty measures over the longer term for one grocery product (laundry detergent), where we have access to five-year continuous consumer panel data.

2. The calculation of loyalty measures over five years, for one very frequently-used quasi-industrial product (doctors’ prescribing for heart disease – a very different type of product from groceries, but one which has been found empirically to conform in the medium term to Dirichlet-based assumptions that brand loyalty is mostly determined by market share, Stern 1994; Stern and Ehrenberg 1995). Because this category is so frequently-used (doctors on the panel averaged around one new prescription decision for heart disease per week), this category provides a base for exploring brand loyalty over many hundreds of purchase occasions, which can act as an analog for the very long run in frequently-bought consumer markets.

3. An extension of the Dirichlet model, using simulated data produced from model parameters calculated over the medium term, giving “norms” for long-run data, which can be compared to those observed in practice.

1.2 Loyalty Measures

For each product we focus on two measures of brand loyalty:

- **Portfolio size**: the number of different brands a consumer buys/(doctor prescribes) for a given level of category usage/(prescription rate).

- **Favorite brand share**: the share of purchase given to the favorite, or most frequently-purchased/(prescribed) brand, again for a given level of category use/(prescription rate).

Both these measures are calculated at the individual buyer level, and since we are interested here in the general level of brand loyalty in the long run rather than comparing loyalty between brands, we aggregate results across all users of leading brands in each category studied.

Favorite brand share is a commonly-used measure of brand loyalty, and our findings enable a comparison to be made with previous studies where this measure has been applied to the short term (e.g. across three purchases, Deighton, Henderson and Neslin (1994)), or to the medium
The use of portfolio size as a measure of loyalty has been used less often. This is partly a reflection of the limited time frame usually used in loyalty studies; for grocery brands portfolio size is typically two to three brands over three months and three to four brands over a year (Ehrenberg, 1988; Ehrenberg and Uncles, 1997).

Additionally, since we look at both measures across different weights of category purchase, this enables us to explore a number of potential sources of buyer heterogeneity in purchasing rates. Heterogeneity in purchasing presents a number of measurement problems. These problems fall into one or more of four types that are discussed below.

1.3 Heterogeneity in Purchase Frequency

*Light category buyers*

Infrequent or light category buyers typically raise loyalty measures based on share and portfolio size. This is because if a panelist makes only one purchase of the category in the time period being analyzed, then the share of purchases allocated to their favorite brand must be 100% and their portfolio size can only be one; if they buy the category twice, their favorite brand share can only fall to 50%, etc. The solution in the past has been to exclude these (often numerous) light category buyers from the analysis of loyalty and include only buyers who make at least three category purchases in the time period under study (Bhattacharya *et al.* 1996; Bhattacharya 1997; Fader and Schmittlein 1993).

*Heavy category buyers*

Panel data are rarely available for more than two or three years, therefore even for relatively frequently-purchased products, panels tend to contain only a few cases of very frequent (i.e. 50+) category buyers. It may be that behavioral loyalty observed over several hundred category purchases is substantially different from that seen over the short or medium term, the...
basis for calculating most measures of share loyalty (Bhattacharya et al. 1996; Fader and Schmittlein 1993; Tellis 1988).

**Small portfolio size**

In most grocery markets the number of category purchases varies between about 5 and 20 per year for the average buyer (Ehrenberg and Uncles, 1997). For this number of purchases brand portfolios tend to be four or less (Hauser and Wernerfelt, 1990). With average portfolios of only around four, the high proportion of product class purchases which an individual allocates to a favorite brand might indicate relatively strong loyalty, e.g. the value of around 65% observed by Brown (1952/53), Cunningham (1956) and Deighton, Henderson and Neslin (1994). We need to know whether portfolio sizes based on a year or less of purchasing are relevant to the longer term.

**Varying purchase frequency**

Even allowing for the previous problems, consumers still exhibit significant heterogeneity in terms of purchase incidence. This makes it difficult to compare loyalty levels across buyers with different usage rates. For instance, is increasing usage accompanied by variety seeking behavior which, in turn, increases portfolio size?

To illustrate this problem, consider two consumers who differ in just two observable ways. The first makes eight category purchases and uses a three brand portfolio during a year. The second makes ten purchases and uses a four brand portfolio, While they differ in both purchase rate and portfolio size, both consumers have the same favorite brand to which they devote as many purchases as possible given their portfolios. This is shown in Table 1.

<table>
<thead>
<tr>
<th>Table 1 Loyalty and Usage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Category Purchases</td>
</tr>
<tr>
<td>Buyer 1</td>
</tr>
<tr>
<td>Buyer 2</td>
</tr>
</tbody>
</table>
From Table 1 we see that buyer 1 shows higher percentage loyalty (75%) to their favorite brand (brand A) than does buyer 2 (70% to brand A). Despite buyer 2 showing lower loyalty to brand A this customer actually purchases more of brand A and is therefore, potentially, a more important marketing target. Ehrenberg (1988) provides one empirical example showing that heavy category buyers use more brands than less frequent buyers in one product field, but there has been little systematic study of this phenomenon.

Marketing managers and researchers who use brand loyalty as a measure of a company’s relationship with its customers need to know how the share of the favorite brand changes with level of use. If the share of the favorite brand decreases with category usage and the rate of change is constant, then the strength of the relationship will weaken in sales terms and require continuous reinforcement. If, however, it reaches an equilibrium point which is then largely independent of category usage, the relationship might need only periodic reinforcement.

1.4 Research Aims

In summary, the three main aims of the research are to:

- **Explore how loyalty changes with increasing category usage.** Based on previous research we hypothesize that *portfolio size* should *increase* with increasing category usage (Ehrenberg, 1988), while *share of the favorite brand* should *decline* with increasing category use, consistent with variety seeking behavior, Van Trijp, Hoyer and Inman, 1996).

- **Discover how similar our findings are in two very different markets.** We might expect lower brand loyalty in laundry detergent buying where the choice is relatively unimportant compared to decisions about drugs in critical illnesses such as heart disease. Previous research suggests, however, that loyalty measures produce similar results in very diverse markets (Ehrenberg and Uncles 1997), including drug therapy prescribing (Stern 1994; Stern and Ehrenberg 1995).

- **Determine how closely we can predict both loyalty measures.** In the medium term, we expect the measures to be predictable from the structural market parameters which
operationalise the Dirichlet model. The Dirichlet is a model formulated to describe category and brand purchase in a stationary market. In the longer run, even mature markets are not entirely stationary; any deviations from the model will increase our understanding of how loyalty manifests itself over time.

In the next section we describe the Data and Methodology. In Section 3 we present the Results. The Discussion makes a preliminary interpretation of these results in the light of other related research and also considers further research.

2. Data and Methodology

2.1 Data

The data sources were:

(i) Laundry detergent; a 5-year panel of 1,535 continuous category buyers in the UK, 1985-89 (TN AGB consumer panel). The top eleven brands were analyzed; these accounted for 92% of purchases by continuous category buyers over the five-year period. Twelve households had a minor brand (outside the top 11) as their favorite and were excluded from the analysis.

(ii) Drug panel data for antihypertensive prescriptions; a 5-year panel of 202 continuous medical prescribers in the UK, 1989-93 (“Jigsaw” panel, ISIS Research). The leading seventeen drugs were analyzed, these accounted for over 70% of total drugs prescribed in this treatment area. Antihypertensives are used to reduce blood pressure and prevent and treat heart disease. Prescribing heart drugs varies in one important way from purchases of detergents – the detergent buyer will subsequently use the product whereas the doctor merely writes the prescription but does not buy or use the drug. For the sake of clarity we will still refer to buyers and purchase rates when discussing prescribing. For further details of the construction, operation and validity of this panel see Stern (1994).1

1 Each week panel members complete a form which lists all new and changes of prescription they have written for their patients, they return this form to the panel operator and the data are aggregated and made available to companies in the industry. The panel members are selected by random probability sampling and the attrition rate is about 10% p.a. The sample closely matches the UK General Practitioner population in terms of practice size and geographical location.
2.2 Methodology

For each dataset we calculate two measures of loyalty – favorite brand share and portfolio size – for different weights of category purchasing, from a low of 5 for both data sets to a high of 200 for detergent purchases and 400 for antihypertension prescriptions. Analyzing category purchasing in this way enables us to address the issues, mentioned in Section 1.3, as described below.

Light category buyers

Prescribing can be described as extremely fast moving, doctors make new prescription decisions daily and in this particular therapy area they write almost one new prescription each week. Because the data cover a five-year period, only three out of 205 doctors on the panel wrote less than 5 prescriptions for the top 17 antihypertensive drugs in the five-year period (these doctors’ records were excluded from the analysis). Likewise for the detergent dataset there were a small number of very light buyers, and all continuous buyers made at least one purchase per year – so again the minimum purchase rate is 5 over five years.

Heavy category buyers

The length of the analysis period (5 years) means that there are large numbers of very frequent (50+) detergent buyers, and an even greater number of frequent prescribers; we are therefore able to see the extent to which loyalty changes as the purchase rate increases. The study of prescribing yields a further advantage. Given the high level of prescription incidence, it provides us with a potential analog for an extremely long-run consumer market. For the 5-year detergent data, we are able to analyze portfolio size and favorite brand shares up to 200 purchases; with the prescribing data used here we are able to analyze loyalty patterns up to 400 decisions (i.e. twice the ‘purchase weight’ of the detergent example).

Small portfolio size

The variation in portfolio size with usage can be directly observed as category purchasing increases to 200, therefore the problem of a possibly artificially-limited portfolio size and subsequent over-estimation of the level of brand loyalty is directly addressed.
**Varying purchase frequency**

The problem of purchase incidence heterogeneity is overcome by replicating the analyses of our two measures of loyalty at different purchase rates. For example, the analysis starts by examining the first 5 purchase/prescription decisions made, and is then repeated for increasing levels of category purchasing/prescribing (up to 200 for detergents and 400 for doctors’ prescribing).

On a related note, there have been concerns over the validity of the gamma/Poisson assumptions, which underlie the NBD part of the Dirichlet model (Morrison and Schmittlein 1988). By analyzing and modeling brand loyalty at varying purchase frequencies we are able to overcome many of the operational concerns over the size of the zero-buying class and the level of stationarity in a market.

Two separate analyses were conducted. First the number of different brands used (out of the available leading 11 brands for laundry detergents and 17 brands for doctors’ prescribing) was calculated for each purchaser/doctor at each level of category purchasing/prescribing over the 5 years.\(^2\) These portfolio sizes were then averaged across users for each category. The second analysis involved calculating the share of requirements accounted for by the most frequently purchased brand/prescribed drug used by each purchaser/doctor. These calculations were repeated at increasing levels of usage (from 5 to 200/400), and were also repeated for the average share of requirements satisfied by the second favorite and subsequent brands. These results are shown in Section 3.

The observed values for portfolio size and favorite brand share are compared with model predictions. Finally, we simulate and analyze a 10-year dataset for detergent buying. The model and method of deriving predictions from simulated data are described briefly below.

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\(^2\) It should be noted that “available brands” here and in the following results section refers to those brands analyzed, i.e. the 11 leading detergents brands which accounted for 92% of detergent purchases and the 17 leading antihypertensives which accounted for over 70% of all prescriptions written. A brand here refers to a top-level brand name, e.g. Ariel or Persil, and includes all brand variants that are marketed under that name. There were several remaining brands in each market but individually they accounted for a very small proportion of market share (resulting in too many missing values to be analyzed). These small brands were almost never a favorite brand.
The NBD-Dirichlet Model

The NBD-Dirichlet is a stochastic model of buyer behavior which was developed for the study of branded packaged goods in established competitive markets (Chatfield and Goodhardt 1975; Bass et al. 1976; Goodhardt et al. 1984; Ehrenberg 1988). The theory underlying the Dirichlet is that there is a small set of interrelated assumptions which describe and predict the patterns of purchase incidence and brand choice for any market which is approximately stationary, unsegmented and non-partitioned (see Appendix for a description of the model).

The aim here is not to test the predictive power of the Dirichlet model (this is well documented elsewhere, see, for example Uncles et al. 1995 or Ehrenberg and Uncles 1997), but to use it as a tool for providing a benchmark for loyalty in mature markets over the longer term. The usefulness of this model as a benchmark lies in the fact that it has successfully characterized the detailed structure of consumer markets over the medium term across a wide range of conditions (Ehrenberg and Uncles 1997), including for the pharmaceutical data used here (Stern 1994; Stern and Ehrenberg 1995). Stern found that the prescribing of drugs for both heart conditions and arthritis follows similar empirical patterns to those found in grocery purchase and that the Dirichlet successfully predicts a range of measures of prescribing behavior for periods up to a year. In this paper we use model parameters based on the medium-term (one year) to produce long-run “norms” for loyalty measures against which to compare our observed long-run results. The method used allows us to produce simulated data for any number of households, for any length of time period.

The Dirichlet model parameter (S) and the exponent of the NBD (K), are calculated from the raw panel data using the BUYER (1989) software package. For both datasets, these parameters are calculated from the average one-year penetration and purchase frequency figures for each brand in the market. Given the models assumptions that each buyer’s probability of buying a given brand is constant over time, once the mean rate of purchasing over different time periods (M) is known, simulated data can be generated for any time period. The S parameter gives an indication of the heterogeneity of buyers in the market. The K parameter

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3 The markets we study here are mature and, in the aggregate, stationary (i.e. brand shares change little from quarter to quarter) over the five years being studied.
reflects the extent to which overall purchasing differs from the mean, M. The other descriptive information needed to create a simulated panel is the market share for each named brand (we assume that market shares remain stable over time).

These data (S, K, M, plus market shares) are input into a purpose-written computer program which then generates brand choice probabilities and product and brand rates of buying (according to Gamma and Poisson distributions), and outputs brand frequencies for any number of “households”. For UK detergent buying, simulated data were created for the top eleven brands for 2000 households over a period of 10 years. For the doctor’s panel, the rate of prescribing for the top 17 drugs in the hypertensive therapy area was simulated, again over 10 years. As a check on the simulation procedure (and also a check on the stationary nature of these markets), Dirichlet values for all common measures (e.g. penetration, purchase frequency, etc.) were calculated over a five-year period for the simulated panels; and compared to the same data from which the model parameters were derived. The fit was found to be very close. For laundry detergents the correlation between the eleven observed and simulated brand penetrations was 0.947; the correlation for purchase frequency was 0.877. For hypertension prescribing the correlation between the seventeen observed and simulated brand penetrations was 0.913; for prescription frequency the correlation was 0.921.

3. Results
3.1 Laundry Detergent
In Table 2 we show the observed findings for portfolio size and share of category purchases accounted for by the favorite and second favorite brands for different weights of laundry detergent purchasing. The first column of Table 2 lists different number of category purchases from 5 (the minimum for continuous buyers over five years) to 200. In the second column we show a measure of portfolio size – how many different brands are used, on average, to satisfy category requirements at the different purchase weights. Column three shows the number of brands used as a percentage of the brands available. For example, in order to make 6 category purchases, an average household uses 2 brands out of the 11 brands available (18%) to satisfy
their purchase requirements; a household making 200 purchases uses, on average, just over half (52%) of the major brands available (5.7 out of 11 possible brands).

As rates of buying increase the number of buyers included in each row of table 2 declines. By definition all 1,523 buyers appear in the first row of the table, 804 households made 50 purchases and just 35 households made 200 purchases over the 5 year period.

One empirical finding of note is that a doubling of the category buying rate results in approximately an 0.6 increase in the number of brands used (e.g. as the category buying rate increases from 5 to 10 the number of brands used increases from 1.9 to 2.5, and as the category buying rate increases from 100 to 200, the number of brands used increases from 5.1 to 5.7). We can represent this as follows:

\[ N(k_2) = N(k_1) + \ln \left( \frac{k_2}{k_1} \right) \times 0.6 \]

Where \( N \) is the average number of brands used at category purchase rate \( k \).

The fourth and fifth columns of Table 2 give the share of category requirements satisfied by the favorite and second favorite brand. If we focus on the first row (a category purchasing rate of five), the favorite brand accounts for 78% of purchases, and the second favorite 27% of purchases. While at a category purchasing rate of 200 (the bottom row) the favorite share

<table>
<thead>
<tr>
<th>N. Category purchases</th>
<th>Portfolio size</th>
<th>Share of requirements satisfied by</th>
<th>Combined share of top 2 brands</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N. brands used</td>
<td>Favorite brand %</td>
<td>Second favorite %</td>
</tr>
<tr>
<td>5</td>
<td>1.9</td>
<td>17</td>
<td>78</td>
</tr>
<tr>
<td>6</td>
<td>2.0</td>
<td>18</td>
<td>77</td>
</tr>
<tr>
<td>7</td>
<td>2.2</td>
<td>20</td>
<td>76</td>
</tr>
<tr>
<td>8</td>
<td>2.3</td>
<td>21</td>
<td>75</td>
</tr>
<tr>
<td>9</td>
<td>2.4</td>
<td>22</td>
<td>74</td>
</tr>
<tr>
<td>10</td>
<td>2.5</td>
<td>23</td>
<td>74</td>
</tr>
<tr>
<td>50</td>
<td>4.4</td>
<td>40</td>
<td>66</td>
</tr>
<tr>
<td>100</td>
<td>5.1</td>
<td>47</td>
<td>64</td>
</tr>
<tr>
<td>150</td>
<td>5.6</td>
<td>51</td>
<td>64</td>
</tr>
<tr>
<td>200</td>
<td>5.7</td>
<td>52</td>
<td>64</td>
</tr>
</tbody>
</table>
falls to 64% and the second favorite share to 17%. The sales importance to consumers of the top 2 brands in their portfolios is shown in the last column of Table 2 – it declines from 94% to 82%, as purchases rise from 5 to 200.  

3.2 Light, Medium and Heavy Buyers: Discrete Segments

In Table 2 (above) the heaviest buyers are included in each row of table (i.e. those buying the category 200 times are included in all previous rows). While this is a very useful way of presenting the data, we need to discover if there is any difference between the loyalty of heavy, medium and light buyers when measured at the same purchase rates. (The Dirichlet explicitly assumes that there is no difference in loyalty at different rates of buying). To test if this assumption is correct, detergent buyers were divided into three discrete segments according their rate of category purchase. The same analysis as described in Section 3.1 was repeated for these discrete purchasing segments. The results are presented in Table 3.

<table>
<thead>
<tr>
<th>Category purchase over 5 years</th>
<th>Market Share (%)</th>
<th>Share of buyers (%)</th>
<th>Average portfolio size for 10 purchases</th>
<th>% share of favorite brand for 10 purchases</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light/Medium (5-50 purchases)</td>
<td>18</td>
<td>42</td>
<td>2.6</td>
<td>72</td>
</tr>
<tr>
<td>Heavy (51-100 purchases)</td>
<td>34</td>
<td>33</td>
<td>2.4</td>
<td>75</td>
</tr>
<tr>
<td>Very Heavy (&gt; 100 purchases)</td>
<td>49</td>
<td>25</td>
<td>2.4</td>
<td>75</td>
</tr>
</tbody>
</table>

The results of this analysis confirm that, at each rate of buying, the loyalty patterns are indeed very similar across the different segments. In Table 3 we illustrate this finding by showing the loyalty patterns for three discrete category segments (light/medium, heavy and very heavy buyers) at one particular buying rate (10 purchases). From Table 3 we see that average

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4 Third favorite, etc., brands have been omitted from this table as they make up a very small percentage of purchases (less than 10%). We are interested in the average share of category requirements satisfied by the favorite and second favorite brands, but for low levels of category buying some consumers use only one brand and so they are excluded from the averaging process for the second favorite. This means that the average share of the first and second favorite brands in table 2 sum to more than 100% at low buying levels.
portfolio size decreases slightly for the heavier buyers (from 2.6 to 2.4), and the percentage share given to the favorite increases slightly (from 72% to 75%). However in substantive terms there are no actionable differences here.

3.3 Model Predictions

Having analyzed the observed patterns of loyalty for a common grocery product (laundry detergent), we now compare the observed findings with model predictions. In Table 4 we show the model predictions for our two loyalty variables (this table is presented in the same manner as Table 2). If we focus on portfolio size, we see that for category purchasing rates of up to 10 (light buyers), the model accurately captures the trend in portfolio size and gives values close to (but always very slightly higher than) the observed values. For example, at a category buying rate of 5, the average number of available brands used is 1.9 or 17% compared with 2.2 or 20% predicted, at a category buying rate of 10, the number of available brands used is 23% compared with 25% predicted.

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td></td>
<td>No. of brands used</td>
<td>% available brands used</td>
<td>Favorite</td>
</tr>
<tr>
<td></td>
<td>Obs</td>
<td>Pred</td>
<td>Obs</td>
</tr>
<tr>
<td>5</td>
<td>1.9</td>
<td>2.2</td>
<td>17</td>
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<td>6</td>
<td>2.0</td>
<td>2.4</td>
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<td>7</td>
<td>2.2</td>
<td>2.5</td>
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<td>4.6</td>
<td>52</td>
</tr>
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</table>

When we turn to higher rates of category purchasing, we see that while the model again accurately captures the increasing trend in portfolio size, now it systematically under-predicts,
with this under-prediction becoming more noticeable as purchases increase, e.g. at a category buying rate of 50, the average portfolio size is 40% of available brands; whereas the model prediction is 35%; at a category buying rate of 200, the average portfolio size is 52% compared with a prediction of 42%. The relationship which we noted previously between observed portfolio growth and category usage, \( N(k_2) = N(k_1) + \ln \left( \frac{k_2}{k_1} \right) \left( \frac{0.6}{\ln 2} \right) \), is supported by the model predictions for light and medium purchasing. However, as the buying rate increases above 50, the model predicts that portfolios should grow more slowly than is observed.

Turning to the share of category purchases allocated to the favorite brand, from Table 4 we see that at a category purchase rate of 5, (first row), the favorite brand accounts for 78% of purchases (69% predicted), and the second favorite accounts for 27% of purchases (28% predicted). While at a category purchasing rate of 200 (last row) the favorite share falls to 64%, but is well-predicted (63%).

The sales importance of consumers’ top 2 brands is shown in the last two columns of Table 4 – the predicted share declines only slightly from 92% to 86% (the empirical observations are 94% to 82%) as category purchases rise from 5 to 200.

Below we summarize these findings:

**Loyalty at low-to-medium rates of purchasing for laundry detergent.** At low rates of category purchasing (5 to 10 purchases), consumers have an average portfolio of 2 to 2.5 brands or around 20% of the available brands. This model captures this well – with a very slight tendency to over-predict the size of the portfolio. At medium rates of purchasing (11 to 50 purchases) the average consumer portfolio is 3.5 brands or 32% of available brands. The model fit here is very close (3.3 brands or 30%).

For our second loyalty measure – share satisfied by the favorite brand – this averages 76% at low rates of purchasing; the model underpredicts here (67% predicted). As category purchasing rises to 50, the favorite brand share falls to 66% (64% predicted). The share given to the second favorite brand reaches 23% at ten category purchases (25% predicted) and 19% (23% predicted) at 50 category purchases. If we compare the observed and predicted results
for the share given to the favorite brand, at light levels of category buying (up to ten purchases), the model systematically under-predicts, while for the second favorite brand the fit is very close. For the favorite brand, this under-prediction is by 8 to 9 points at low usage rates. The combined share given to the top two brands in an individual household’s portfolio falls from 90% (89% predicted) at 10 category purchases to 84% (86% predicted) as the purchasing rate rises to 50.

**Predictions for high category purchase rates: laundry detergent.** As category buying rises to 100 purchases, average portfolio size increases to 5.1 brands or 47% of the main brands available. The model underpredicts this (predictions of 4.2 brands or 38%). There is only a small change in portfolio size as category purchase increases from 100 to 200 purchases. The share satisfied by the favorite brand stabilizes at around 50 purchases to 66% and declines very little after this. This is well-predicted (64% at 50 purchases) and again there is very little predicted change at higher rates of buying. The sales importance of the top two brands in the individual portfolio falls little from 50 to 200 purchases, averaging 83% (86% predicted).

### 3.4 Loyalty in Prescribing as Purchase Weight Increases to 400

We turn now to the panel for doctors’ prescribing of antihypertension drugs. The portfolio size for doctors’ prescribing increases with usage, in a similar manner to that seen in Tables 2 and 4 for detergents, so that from Table 5 we see that when doctors write 5 new prescriptions for hypertension they utilise just 18% of the main available drugs (21% predicted). This is very similar to the pattern seen, at low levels of category purchasing, for detergent buying. However, at a category purchasing rate of 100, detergent buyers use 47% of the available brands, while for this level of prescribing doctors use 62% of available drugs. As in the detergent example the model slightly but systematically over-predicts portfolio size at lower category usage rates and under-predicts it at higher rates. This is reflected in the measures of fit – for buying rates below 50 the correlation between the observed and predicted portfolio size is 0.9991, mean average deviation (MAD) 0.6, and for buying rates above 50 the correlation is 0.982, but the MAD = 8.
If we turn to the share of requirements satisfied by the favorite brand (i.e. most frequently) prescribed at the individual doctor level, again the model systematically underpredicts at low category prescribing levels (the correlation between the observed and predicted favorite brand’s share of requirements for buying rates below 50 is 0.98, MAD = 5.9). As in the detergent example, the fit improves for higher levels of category prescribing (for category prescribing of above 50 the correlation between observed and predicted is 0.98, MAD = 1.9).

In general the favorite drug accounts for a lower share of requirements than the favorite detergent brand but this difference is predicted. For the second favorite, the data for detergents and antihypertensives are very similar, but the combined share accounted for by the top two drugs used in hypertension declines much more markedly than the top two detergent brands, e.g. for 100 antihypertension prescriptions, a doctor’s “favorite” two drugs accounts for just over half (54%) of the total prescribed, whereas a household making 100 detergent purchases devotes 83% of their category purchases to their two favorite brands.

Below we summarize the findings for antihypertensive prescribing:

- **Loyalty for a quasi-industrial product – antihypertensive prescribing.** Over 5 to 10 prescriptions doctors have an average portfolio of 4 or 23% (26% predicted) of available

### Table 5: Doctors’ Prescribing of Antihypertensives: Loyalty for Different Levels of Category Prescribing

<table>
<thead>
<tr>
<th>N. Category prescriptions</th>
<th>Portfolio size % available brands used</th>
<th>Share of requirements satisfied by Favorite %</th>
<th>2nd Favorite %</th>
<th>Combined share of Top 2 brands %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Obs Pred</td>
<td>Obs Pred</td>
<td>Obs Pred</td>
<td>Obs Pred</td>
</tr>
<tr>
<td>5</td>
<td>18 21</td>
<td>53 46</td>
<td>26 24</td>
<td>78 70</td>
</tr>
<tr>
<td>6</td>
<td>20 23</td>
<td>50 44</td>
<td>25 24</td>
<td>74 67</td>
</tr>
<tr>
<td>7</td>
<td>22 25</td>
<td>48 42</td>
<td>25 24</td>
<td>73 66</td>
</tr>
<tr>
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<td>24 27</td>
<td>47 41</td>
<td>25 24</td>
<td>71 64</td>
</tr>
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<td>25 29</td>
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<td>70 63</td>
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<td>45 39</td>
<td>24 23</td>
<td>69 62</td>
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<td>51 53</td>
<td>37 34</td>
<td>22 21</td>
<td>58 54</td>
</tr>
<tr>
<td>100</td>
<td>62 60</td>
<td>34 33</td>
<td>21 20</td>
<td>54 53</td>
</tr>
<tr>
<td>150</td>
<td>71 67</td>
<td>31 33</td>
<td>21 20</td>
<td>52 53</td>
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<tr>
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<td>78 68</td>
<td>30 33</td>
<td>19 20</td>
<td>50 53</td>
</tr>
<tr>
<td>400</td>
<td>77 71</td>
<td>28 31</td>
<td>21 20</td>
<td>49 50</td>
</tr>
</tbody>
</table>
drugs – similar in percentage terms to detergent purchasing. As prescriptions increase to 50, portfolio size increases to 51% of available brands (53% predicted). At category prescribing rates of 200, the average portfolio size rises further to 78% (68% predicted) (compared with 52%, observed, 42% predicted for detergent purchasing). The share satisfied by the favorite drug falls from 45% (39% predicted) at 10 prescriptions to 37% (34% predicted) at 50 prescriptions and 30% (33% predicted) at 200 prescriptions.

- **Loyalty at very high levels of prescribing.** There is virtually no change in portfolio size or the share given to the favorite brands (observed or predicted) as category prescriptions increase from 200 to 400. After 200 prescriptions the combined share of the top two drugs also stabilizes and is, at around 50%, much lower than for laundry detergents (82%).

### 3.5 Simulating Loyalty Over 10 Years of Detergent Buying

Finally we show the results from a simulation of continuous detergent buyers over 10 years, with the number of category purchases rising to 450. The results shown in the bottom half of Table 6 indicate that, as category purchases increase to over 200, portfolio size and share of favorite and subsequent brands stabilizes (as seen for antihypertensive prescribing).

The model here provides an indication that consumers will, on average, only utilize up to 45% of the available detergent brands in their portfolios to satisfy up to 450 category purchases. Portfolio size is not expected to increase as category purchase rates rise above 200. However,

<table>
<thead>
<tr>
<th>N. Category purchases</th>
<th>Portfolio size used</th>
<th>Share of requirements satisfied by</th>
<th>Combined share of top 2 brands</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% available brands</td>
<td>Favorite %</td>
<td>2nd Favorite %</td>
</tr>
<tr>
<td>Obs</td>
<td>Pred</td>
<td>Obs Pred</td>
<td>Obs Pred</td>
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<tr>
<td>100</td>
<td>47 38</td>
<td>64 63</td>
<td>20 23</td>
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<td>150</td>
<td>51 41</td>
<td>64 63</td>
<td>19 23</td>
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<td>200</td>
<td>52 41</td>
<td>64 63</td>
<td>17 23</td>
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<td>Simulations</td>
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<td>300</td>
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<tr>
<td>450</td>
<td>45</td>
<td>61</td>
<td>21</td>
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</tbody>
</table>
we have evidence from the grocery panel and from the doctors’ prescribing panel that the model systematically underpredicts portfolio size for high rates of category purchasing, therefore a more realistic upper limit for this particular laundry detergent market might be 55%. The model also predicts that the share accounted for by the consumer’s favorite brand stabilizes at around 60% irrespective of usage rate, and that the top two brands dominate with a combined share of requirements of 85%. Based on evidence from the antihypertensive prescribing, we believe that the model provides close predictions of these share measures.

4. Discussion

This study has shown, first, what brand loyalty over several hundred category purchases looks like. We find that longrun loyalty develops in a regular way. Second, the Dirichlet model has been shown to be useful in predicting longrun loyalty, but there are some systematic deviations from the model. The main findings can be summarized as follows:

- Portfolio size in both markets increases as category usage increases. At low buying rates, portfolio size is slightly overpredicted by the model, while at higher buying rates it is underpredicted. At around 150-200 purchases the model predicts that portfolio size should stabilize at about 40% of available brands for a common grocery product, detergent, and 68% for a quasi-industrial product, doctors’ prescribing. In practice, we observe higher figures of 50% for detergent and 77% for prescribing.

- For both markets, the share of requirements satisfied by the favorite brand in the repertoire declines gradually as usage increases. For detergents, by 50 category purchases this share has reached a minimum (66%, 64% predicted) which is independent of subsequent buying rates. For doctors’ prescribing of antihypertensives the favorite share continues to decline as prescriptions increase to 400 (at which point the favorite share is around 30%, 33% predicted). For both markets, at low levels of buying the model underpredicts the share given to the favorite brand, at higher levels of buying the predictions are very close.
• In sales terms (especially for detergents), brands outside the core portfolio of two are generally unimportant.

• There are structural differences between the two markets (the rate of doctors’ prescribing antihypertensive drugs is twice that of households’ buying detergent). However, once these structural differences are taken into account, loyalty levels in these two very different markets are largely predictable from the basic model parameters (M, K and S).

4.1 Model Fit

We have shown how the Dirichlet model can be extended to provide predictions for two brand loyalty measures, and it does so (generally) very well. The approach adopted of looking at usage rates obviates some of the previous potential concerns with the model (i.e. Morrison and Schmittlein’s (1988) concerns over non-stationarity and the size of the zero class). In our study, however, there are discrepancies in the model fit that persist across the two very different markets analyzed. Portfolio size does grow with category usage, but in both categories the model over-predicts portfolio size and under predicts favourite share of category requirements at low usage levels. At high usage levels the model tends to under-predict portfolio size but closely predicts the favourite share of category requirements. It should be noted that the predictions are still very close in terms of managerial significance and therefore of considerable use in marketing planning.

4.2 Structural Differences

The main differences between the two datasets are the average category buying rates (73 over 5 years for laundry detergents versus 184 over 5 years for doctors’ prescribing) and the number of significant choice alternatives (11 for detergents, 17 for doctors’ prescribing). In terms of input parameters to the Dirichlet model, M and K differ as a result of the different rates of overall buying (as opposed to different proportions of buyers/non-buyers which is
usually the main difference between categories). The S parameter not only represents the
degree of different choices available, but also the propensity for buyers to avail themselves of
these choices.

These differences show up in portfolio sizes and the share of requirements of the favorite
brands. At the level of 100 purchases a detergent buyer will use under half (47%) of the main
brands on the market but a doctor will have prescribed 62% of the main antihypertensive
drugs. Again at the level of 100 purchases, the share of detergent purchases accounted for by
the favorite brand will be 64% but the favorite antihypertensive accounts for around half that
figure (34%). The detergent buyer’s two favorite brands will account for over 80% of their
category needs whereas the top two drugs will represent just over half of a doctor’s
antihypertensive requirements.

The dependence on a ‘core portfolio’ which shows little change as usage increases is
consistent with the fundamental Dirichlet assumption of fixed probabilities of making a
particular brand choice. If a grocery market with a similar competitive structure (in terms of
the model parameters and brand shares) to antihypertension prescribing was identified, we
would expect to see similar levels of loyalty as measured by our two variables.

Our results lead us to conclude that loyalty as measured by portfolio size and share of
requirements satisfied by the favorite brand is largely predictable and derived from the
structural parameters which define the market, rather than any intrinsic properties of the
buyers or the ‘bought’. Our results show how portfolio size (as a percentage of available
brands) varies with usage of a category, but varies little by product type. However, the
importance of the most frequently chosen item does not vary dramatically with usage rate
within the category but shows significant variation between our two very different markets
(and is therefore determined by the structural model parameters, i.e. the overall frequency of
buying or range of brand choice available).

The analysis of portfolio size confirm our expectation that in the very long run we would
expect an individual household’s brand choice probabilities to change (and to deviate from a
stationary model) – what is surprising here is that they change so little. Even more surprising
is the finding that the model predictions for the share of the favorite brand are closer in the long run than the short run.

4.4 Managerial Implications

One of the key tasks of marketing management is to increase the profitable sales within a category. In stationary (or near-stationary) markets this can only result from taking sales away from competitive offerings. The main finding from this research is that loyalty patterns do not change that much with usage (and hence, in a stationary market, with time). Marketing managers need to recognize this and take a much longer-term view of brand development. A brand which is not the category leader now, and has such an objective, requires both to increase the number of brand buyers and also the number of buyers for whom that brand is favorite. The finding that the average buyer has a core portfolio of just two brands in UK detergents, yields a rationale for the marketing strategies adopted by the main competitors in that market of providing multiple brands within the category, as well as the brand extension strategies which have proliferated in the UK detergent and other grocery markets. In contrast, antihypertensive prescribing relies on a wider portfolio size to capture the majority of prescribing needs. Despite this, pharmaceutical companies tend not to market multiple brands in the same way as detergent manufacturers do, however, they do implement brand extension strategies.

Here we do not analyze the effect of explanatory variables such as price, promotion and availability, or who buys versus who consumes. It is clear, however, that since brand loyalty is well-predicted by the parsimonious model adopted here, these additional explanatory variables can only improve the model at the margin.

Our finding that portfolio size remains limited even up to very high rates of category buying, suggests that, in these two markets, variety seeking is not a common behavior. It may be that in a food category we would see more variety seeking behavior. It is also possible that, particularly in the detergent market, there is switching between brand variants which is not
captured in this research. Analyzing brand variants and food products are extensions of this research that should be undertaken before we can be certain that the results generalize.

Our research provides a framework for managers to understand the loyalty patterns inherent in their market which are defined by structural parameters (the model inputs; S, K, M and brand shares) which are easily computed from commercially available sources.

Analysis across additional categories will show if there are stable patterns for long-run loyalty. There is a need to simulate a range of different markets that are characterized by varying structural parameters in order to explore the resulting changes in portfolio size and favorite brand shares. This will have a number of practical applications in helping marketing managers to understand how loyalty patterns in their markets change with both usage and time.
Appendix: Model Specification

For a population of $N$ consumers making purchases in a category with $g$ brands, the Dirichlet model specifies probabilistically how many purchases each buyer makes in a particular time period and which brand is bought on each purchase occasion. The model therefore combines purchase incidence and brand choice. The model specifies the probability vector of the $i$th shopper making any specific combination $\{r_j\}$ of purchases of the $j = 1$ to $g$ brands in a particular time period of length $T$. If we sum over the $j = 1$ to $g$ brands, $\Sigma r_j = n$, is the total number of purchases of the product made by the $i$th shopper in that period.

There are five distributional assumptions in the theoretical model: the incidence of product purchase is specified as a mixture of two distributions, brand choice is covered by another two distributions and there is a final assumption concerning how the product incidence and brand choice assumptions are related.

**Purchase Incidence**

(i) Successive purchases for each individual shopper ($i$) appear as if random, and are assumed to be independent with a constant mean rate $M_i$ in a particular time period. The number of purchases $n_i$ made in each of a succession of equal non-overlapping periods of relative length $T$ follows a Poisson distribution with mean $M_i T$.

(ii) Average purchasing rates are assumed to vary among consumers (consumer heterogeneity), i.e. there are light, medium and heavy category buyers. The average purchasing rates of individuals vary according to a Gamma distribution with the following density function:

$$e^{-\frac{\mu K}{M}} \frac{\mu^{K-1}}{(M/K)^K \Gamma(K)}$$
These two assumptions in combination give rise to a Negative-Binomial Distribution (NBD) for the number of purchases of the product made by all individuals in time period of length $T$, with mean $MT$ and exponent $K$. The indexing by time enables predictions to be made for different length time periods.

**Brand Choice**

In terms of brand choice each buyer is assumed to have a set of propensities for buying each of the available brands which are expressed as a set of probabilities. We assume a mixture of two probability distributions, to give the number of purchases, $r$, which an individual shopper makes of a particular brand, given that s/he makes $n$ purchases in total. The distributions are:

(iii) Each buyer’s probability, $(p_j)_i$, of buying brand $j$ from $j = 1, \ldots, g$ brands is constant over time and follows a multinomial distribution $M(r | p, n)$. Brand choices at successive purchases are assumed to be independent.

(iv) The distribution of such probabilities, $(p_j)_i$, among shoppers follows a “Dirichlet” type of multivariate Beta Binomial distribution $D(p | \alpha)$. Here $D(p | \alpha) = C \frac{\prod p_j^{\alpha_j-1}}{\prod_{j=1}^{g} p_j^{\alpha_j-1}}$ for $g$ brands, where the $\alpha$ are proportional to the brands’ market shares and where they sum to the parameter, $S$, and $C$ is a scaling coefficient which is a function of the $\alpha$.

Since the model is for unsegmented markets, it is assumed that the above statistical distributions are independent of each other, i.e. the average purchase frequency distributions and the brand choice probabilities over different consumers are distributed independently over the population.

By grouping terms we obtain the compound distribution for the number of purchases an individual makes of each of the $g$ brands in a period of time $T$:

$$[M(r | p, n) \hat{P} D(p | \alpha)] \hat{n} [P(n | \mu) \hat{\mu} G(\mu | MT, K)]$$
Where \( M, D, P \) and \( G \) denote the Multinomial, Dirichlet, Poisson and Gamma distributions, respectively. A further assumption is that all brands are able to compete with each other, i.e. all brands are equally available.

**Model Estimation**

The model is parsimonious in its input requirements. It is necessary to estimate the mean rate of purchasing, \( M \), and the exponent, \( K \), of the NBD. The Dirichlet component has one parameter, \( S \), (sometimes called the “switching” parameter). \( S \) is the sum of the values of \( \alpha \)s in the model \( S = \Sigma \alpha \), and reflects how diverse buyers are in their brand choices. This parameter can be estimated from: (a) two measures relating to the whole market: overall patronage or market penetration and the average shopping frequency of shoppers, and (b) these same two measures for each, or any, of the itemized brands. There are no closed algebraic formulae, so the estimation of the model’s parameters is essentially arithmetic. \( S \) is estimated iteratively for each brand and the brand \( S \) values are then combined to form an average (weighted by market share) for the category. The \( K \) parameter is calculated by fitting the NBD to the distribution of purchases for the total category. The \( S \) and \( K \) parameters can be calculated using *BUYER* (1989) software.
References


