Stage Financing and the
Role of Convertible Securities*

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Abstract

Venture capital financing is characterized by extensive use of convertible securities and stage financing. In a model where an entrepreneur obtains funding for a project from a venture capitalist, we illustrate an advantage of convertible debt over a mixture of debt and equity in stage financing situations. Essentially, when the venture capitalist retains the option to abandon the project, the entrepreneur has an incentive to engage in window dressing and bias positively the short-term performance of the project, reducing the probability that it will be liquidated. An appropriately designed convertible debt contract prevents such short-termistic behavior since window dressing also increases the probability that the venture capitalist will convert debt into equity.

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1 Introduction

Projects backed by venture capitalists are characterized by great uncertainty and high failure risk. As such, they find it difficult to obtain financing from traditional sources, such as banks. Lerner and Gompers (1999) provide an overview for the reasons why traditional sources of financing are not suitable for such projects, and stress that the tools employed by venture investors should be seen as responses to their special nature. In this paper, we focus on two such tools—stage financing and convertible securities.

A widely used financing technique by venture capital companies is the infusion of capital over time. The venture capitalist who provides the funds retains the option to abandon the venture at any stage whenever the forward looking net present value of the project is negative. Financing rounds are usually related to significant stages in the development process such as completion of design, pilot production, first profitability results, or the introduction of a second product. At every stage, new information about the venture is released (Sahlman 1990).1 Also widely used in venture capital backed projects are convertible securities issued by entrepreneurs in exchange for funds.2 Our goal is to provide a rationale for the combined use of stage financing and convertible securities in venture capital financing.

Stage financing is appealing to venture capitalists for two reasons. First, the option to abandon is essential because an entrepreneur will almost never stop investing in a failing project as long as others are providing capital (Admati and Pfleiderer 1994). Second, the threat to abandon creates incentives for the entrepreneur to maximize value and meet goals. But this threat has the potential drawback that it might induce the entrepreneur to focus only on meeting the immediate hurdle of the next stage.3 To illustrate, the entrepreneur can make the conditions under which a project will be evaluated more favorable, whether it is the test of a prototype or a market test, increasing the likelihood of good interim performance. This phenomenon is commonly described as “window dressing.” The venture capitalist then has to decide whether

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1 See also Barry (1994), Lerner (1995), and Bernhardt, Hollifield, and Hughson (1996). Gompers (1995) provides evidence that the number of stage financing rounds increases with the fraction of intangible assets. Egli, Ongena, and Smith (2000) study a model where the choice of stage financing arises endogenously. Neher (1999) studies a model where the number of financing rounds and the duration of the project are determined endogenously. Black and Gilson (1998), Table 4, report the percentage of capital infused on average at various venture capital financing rounds in the US and Germany.

2 Kaplan and Strömbäck (2001) provide a comprehensive characterization of the contracts used in venture capital financing and document the extensive use of convertible securities by venture capitalists.

3 Sahlman (1988, p.32) observes that having a “fume date” may serve as a motivating factor, but “may create incentives to aim for short-term success rather than long-term value creation.”
to continue investing in the project on the basis of interim performance that has been artificially improved. Stage financing thus creates a conflict of interest between the venture capitalist, who provides the funds, and the entrepreneur, who wants to continue with the project.

We model window dressing as follows. The venture capitalist and the entrepreneur commonly observe the interim performance of the project. For brevity, we call interim performance the “signal.” Before the signal is realized, the entrepreneur can manipulate its distribution so that a good signal realization is more likely to appear, without affecting the actual quality of the project. We show that this change in the distribution of the signal should not be thought of as “adding noise” to the signal (signal jamming), nor as reducing the amount of noise in the signal. The reason is that neither type of signal manipulation constitutes a conflict of interest: increasing the amount of noise in the signal is undesirable to both the venture capitalist and the entrepreneur while reducing the amount of noise in the signal is desirable to both. To capture the conflict of interest that arises from stage financing, window dressing must be modeled as a shift of probability mass from low to high realizations of the signal rendering low realizations less likely to appear.

It is often hard to specify fully and unequivocally what is meant by “good interim performance.” For instance, when evaluating whether a “working prototype” indeed works, or when evaluating the responses of consumers to a market test, a great deal of subjective judgement must be exercised. To capture this feature, we assume that although the interim performance signal is commonly observed by the venture capitalist and the entrepreneur, it is nonverifiable.4

Window dressing reduces the venture capitalist’s payoff because his refinancing/liquidation decision is based on lower quality information. In some cases, this may render stage financing non-viable so that a profitable project will not be financed altogether. Yet, we will show that with debt-equity financing (debt, equity, or a combination of both) the entrepreneur will always window dress. Moreover, we will show that convertible debt financing can be designed so that the advantage to the entrepreneur from reducing the likelihood of liquidation is more than offset by the increased likelihood of debt conversion (conditional on refinancing). This is because although window dressing renders low quality projects harder to identify (reducing the probability of liquidation), it renders high quality projects easier to identify. This, in turn, increases the probability that in the event of refinancing the venture capitalist will exercise

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4Nevertheless, we devote a section to the version of the model where the signal is both commonly observed and verifiable since there are real world situations where interim performance can be measured objectively (e.g., sales revenue).
the debt conversion option becoming the owner of a substantial fraction of the venture and appropriating much of the project’s value. If the terms of conversion are set in advance to be sufficiently favorable to the venture capitalist, this effect dominates in terms of payoffs, and the entrepreneur will not engage in short-term window dressing. A central element of our analysis is the determination of an appropriate conversion ratio and a suitable amount of convertible debt that will ensure that the project is financed, but at the same time will deter the entrepreneur from engaging in window dressing.

Warrants and convertible preferred equity are also commonly used in venture capital financing. We will show that our analysis is virtually unchanged if the entrepreneur issues a combination of convertible debt and warrants. Convertible preferred equity has similar features to convertible debt, but is special in several respects. Until conversion, a convertible preferred stock promises a fixed dividend (and hence is similar to convertible debt). Unlike debt, failure to pay the dividend does not trigger liquidation; rather, the unpaid dividends accrue and must be paid before any dividends are paid out to common stock holders. Since we do not consider payoffs at interim stages, our model applies to convertible preferred equity as well.5

We first analyze the model abstracting from the possibility of renegotiation and then consider renegotiation explicitly, as follows. First, we show that convertible debt financing prevents window dressing equally well even if renegotiation is allowed. Then, we ask whether convertible debt financing is necessary for preventing window dressing if debt-equity contracts can be renegotiated. We distinguish between spontaneous renegotiation and designed renegotiation in the sense of Aghion, Dewatripont, and Rey (1994).6 We explain why debt-equity financing cum spontaneous renegotiation cannot prevent window dressing. Then, we turn to designed renegotiation adapting Aghion, Dewatripont, and Rey’s (1994) definition of “simple renegotiation” to the context of our model. We show that debt-equity financing combined with simple renegotiation indeed prevents window dressing. The point is that debt-equity financing cum simple renegotiation is “isomorphic” to convertible debt financing. In this sense, convertible debt financing can be interpreted as a simple way to design renegotiation and introduce it in a contract.

5Other major advantages of preferred shares are the favorable tax treatment and the fact that preferred shares carry votes. Since our model abstracts from taxes and control rights, it would make little difference if we used convertible preferred equity rather than convertible debt. For an analysis of transfers of control in venture capital financing situations see, for example, Chan, Siegel, and Thakor (1990).

6Aghion, Dewatripont, and Rey use the term “voluntary” rather than “spontaneous.”
Explanations for the use of convertible securities are often centered on ex-ante asymmetry of information between managers and investors. Stein (1992) focuses on ex-ante asymmetry of information regarding the quality of the issuing firm. Brennan and Schwartz (1988) focus on ex-ante asymmetry of information regarding the risk of the issuing firm. Since the value of the option embedded in the convertible security increases with the volatility of the underlying security, offering a conversion option is less “costly” for low risk firms—by issuing convertible debt these firms signal to investors that they are of low risk.

In venture capital financing it is often the case that ex-ante, at the time of initial financing, the entrepreneur and the financier are equally informed regarding the project’s chances of success, and the true quality is gradually revealed to both parties. The main conflict of interest in such situations is the asymmetry of information regarding the future actions of the entrepreneur. Our model stresses this aspect of venture capital financing rather than ex-ante information asymmetry. Green (1984) also focuses on asymmetric information regarding the entrepreneur’s actions. In his model, a mix of convertible securities and debt is shown to be superior to straight debt since the conversion option reduces the inclination of the entrepreneur to engage in risky projects. In terms of economic intuition, the difference between the two models is that while Green (1984) describes a situation in which the entrepreneur reduces the value of the project by taking excessive risk, the window dressing modeled in our paper describes a situation in which the entrepreneur is excessively committed to a project, even when it is not profitable. Moreover, with a sufficiently high proportion of equity financing, the danger of excessive risk taking (“asset substitution”) would not arise in Green’s model. In our model, the conflict of interest between the entrepreneur and the venture capitalist arises whether the project is financed with debt, equity, or any combination of both.

In our model, interim performance is commonly observed by the venture capitalist and the entrepreneur. Therefore, the entrepreneur cannot engage in mis-reporting of information (“cooking the books”), but can manipulate the “technology” that generates the signal. In models such as Milgrom and Roberts (1986) and Shin (1994), this technology is fixed but the signal is observed by only one party that has an incentive to mis-report its private information ex-post. If we introduced asymmetry of information in our model, for example because the entrepreneur obtains information about the project’s quality that the venture capitalist does not have, the danger of mis-reporting would arise.

Some recent papers provide other explanations for the use of convertible debt in venture
capital situations. Repullo and Suarez (1998) focus on the advisory role of the venture capitalist and look at the optimal contractual arrangement when a double sided moral hazard problem arises. Schmidt (2000) also focuses on double sided moral hazard and shows that convertible debt can induce both parties to invest efficiently. Hellmann (2000) focuses on convertibility at the IPO stage. Biais, Bisière, and Décamps (1998) analyze a situation where the type of securities used in financing a project (e.g., convertible debt) is affected by the trade-off between anticipated costs of financial distress and the need to provide incentives to managers. Berglöf (1994) looks at the role of convertible securities in mitigating the distributional conflicts associated with a future sale of the firm. None of these papers, however, looks at the combined use of convertible securities and staged infusion of capital which are so common in venture capital financing.

Several other papers are concerned with venture capital financing but not with convertible securities. Hellmann (1998) focuses on transfers of control and provides an explanation for why an entrepreneur might agree to be replaced by the venture capitalist. Marx (1998) argues that a mix of debt and equity dominates only equity and only debt by generating the right incentives for the venture capitalist to intervene in the project as a response to poor performance. Bergemann and Hege (1998) study a model with venture capital financing, focusing on the optimal compensation of the entrepreneur in order to provide him with the right incentives for the allocation of funds. Ueda (2000) asks why start-up firms typically prefer venture capital rather than bank financing, and stresses the importance of intellectual property rights protection. Finally, von Thadden (1995) shows that a contract that resembles a long-term credit line can reduce short-termistic behavior in stage financing situations.

In the next section we present the model under the assumption that contracts cannot be renegotiated, and show how the introduction of convertible debt generates an improvement for both parties. In Section 3 we allow for contract renegotiation, in Section 4 we allow for convertible debt cum warrants financing, and in Section 5 we analyze the model under the assumption that the signal is verifiable. In Section 6 we further extend the model (with nonverifiable sig-

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7 As mentioned, Gompers and Lerner (1999) offer reasons for why banks typically do not finance high risk and “high-tech” projects. First, banks insist on collateral which such projects do not have since they are not intensive in tradable physical assets such as plants and equipment. Second, banks are perceived as having comparative advantage in monitoring through long-term relationships with borrowers, whereas in the “high-tech” industry change is very rapid, and effective monitoring requires skills that banks do not specialize in. Third, in the United States, the Glass-Steagall Act prohibited banks from owning the equity of non-financial firms. This prevented banks from providing financing via convertible securities in general, and financing “high-tech” ventures in particular. It is still early to tell whether the recent repeal of Glass-Steagall in 1999 will induce entry of banks into this market niche.

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nal) by introducing an additional decision for the entrepreneur—whether to exert effort that increases long-term profits. The basic results are unaffected by this generalization. In Section 7 we study other specifications of signal manipulation. Section 8 concludes.

2 The model

2.1 The Basic Set-Up Without Signal Manipulation

At time 0 an entrepreneur considers undertaking a project with uncertain returns. There are two possible states of nature: with probability 1/2 the state is “good”, which we denote $h$, and with probability 1/2 the state is “bad,” $l$. The project generates a (stochastic) output, $\pi_i$, distributed exponentially on the interval $[0, \infty)$, with density $\lambda_i e^{-\lambda_i \pi_i}$, where $1/\lambda_i$ is the mean of $\pi_i$ for $i = h, l$. Let

$$\frac{1}{\lambda_l} > \frac{1}{\lambda_h} > \frac{1}{\lambda_l}$$

(1)

i.e., the difference between the expectation of output in the two states of nature is not “too large.”

The entrepreneur has no capital and, therefore, will ask a venture capitalist for the required funds. The total amount of money necessary for the project is $I_1 + I_2$, of which $I_1$ must be invested at time 0, and $I_2$ can be delayed until time 1, provided that the project is not liquidated.

Stage financing

Stage financing means that $I_1$ is invested at time 0, and $I_2$ is delayed. At time 1, a signal, $x$, about the project’s quality (the state of nature) is realized and is observed by the entrepreneur and the venture capitalist. For example, the signal can be the short-term performance of the project, and both parties update their expectations about the project’s quality. We assume that $x$ is nonverifiable. After observing the signal, the venture capitalist decides whether to refinance the project and supply $I_2$ to the entrepreneur, or liquidate. We assume that the payoff to both parties after liquidation is zero.\(^8\) If the project is refinanced, the output is realized at time 2 and is shared by the entrepreneur and the venture capitalist according to the contract that has

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\(^8\)For our purposes, this is an innocuous assumption. In general, liquidation value can be of great importance. In Neher (1999), the central economic force is the gradual embodiment of the entrepreneur’s human capital in the project’s physical capital, and as a consequence, the project’s liquidation value increases over time. Neher derives the optimal number of financing rounds and project duration.
been agreed upon.

In principle, the venture capitalist can commit at time 0 to supply both amounts $I_1$ and $I_2$ irrespectively of the signal, but to underline the problem created by window dressing we assume that

\[
\frac{1}{2} \frac{1}{\lambda_l} + \frac{1}{2} \frac{1}{\lambda_h} - (I_1 + I_2) < 0 ,
\]

which implies that committing $I_1 + I_2$ at time 0 is not a viable solution to the window dressing problem. This assumption allows us to illustrate our point in a much sharper manner, and to say that if the venture capitalist does not succeed in preventing window dressing (by means of an appropriately designed contract) the project will not be financed, underlining the loss of surplus. But assumption (2) is not necessary at all: even if profits without stage financing were positive, the venture capitalist would still prefer to finance the project in stages since he thus keeps the valuable option to abandon the project after the realization of the signal, and the danger of window dressing would arise.

**Ex-post payoffs with debt-equity financing**

Suppose the venture capitalist can provide financing to the entrepreneur in the form of debt, equity, or a combination of the two. We denote a debt-equity contract by $(d, s)$, where $d$ is the amount of debt that the entrepreneur owes the venture capitalist, and $s$ is the fraction of the enterprise’s equity owned by the venture capitalist. In this section, we abstract from the possible renegotiation of debt-equity contracts at time 1.

Consider the debt-equity contract $(d, s)$. The magnitudes $d$ and $s$ are determined at time 0 and do not change throughout the life-time of the venture. At time 0, $I_1$ is supplied. At time 1, if the project is not liquidated after observing the signal, $I_2$ is supplied. At time 2, the output is divided according to the sharing rule induced by the debt-equity contract $(d, s)$. The promised amount $d$ may be larger or smaller than $I_1 + I_2$, the total amount of financing.\(^9\)

\(^9\)For example, $d > I_1 + I_2$ reflects a positive interest rate. Kaplan and Strömberg (2001) report that in more than 98 percent of the cases in their sample the liquidation rights of the venture capitalist (which correspond to $d$) are larger than his cumulative investment.

\(^{10}\)Both parties are likely to take into account the entire loan $I_1 + I_2$ when specifying $d$ and $s$ at time 0. If not, one of them may be held up at time 1. The intuition is as follows. If the entrepreneur has enough bargaining power, then at date 1 the venture capitalist may be “locked in.” For example, if the entrepreneur has all the bargaining power, then at time 1 he will offer a new debt-equity contract which guarantees the venture capitalist just enough (expected) revenue to cover $I_2$. The venture capitalist will accept the offer since $I_1$ is sunk. Foreseeing this, the venture capitalist will prefer to have $d$ and $s$ specified at time 0. A similar logic applies to the entrepreneur if his participation in the project requires an arbitrarily small amount of effort at time 0 (as we will assume in
We assume $s \in [0, 1)$, i.e. the entrepreneur must own some shares. This is easily justified on the grounds that otherwise the entrepreneur will not make effort for the project to succeed in the long-run.\footnote{In Section 6 we extend the model allowing for entrepreneurial effort that affects long-run (time 2) profits. We show that, indeed, $s < 1$ is a necessary condition for inducing the entrepreneur to exert such effort.} Since the signal about the project’s quality (observed by both parties at time 1) is nonverifiable, it is not possible to write in the contract at time 0 that the provision of $I_2$ (refinancing) will depend on the realized value of the signal.

Assuming limited liability and zero bankruptcy costs, the ex-post payoffs (after the state of nature is revealed but before profits are realized) to the venture capitalist and the entrepreneur from a debt-equity contract, $(d, s)$, for $i = l, h$, are:

\[
\begin{align*}
\mathcal{V}^{\text{DE}}_i &= \int_0^d \pi_i \lambda_i e^{-\lambda_i \pi_i} d\pi_i + \int_d^\infty [d + s(\pi_i - d)] \lambda_i e^{-\lambda_i \pi_i} d\pi_i \\
&= \frac{1}{\lambda_i} [1 - e^{-\lambda d}(1 - s)],
\end{align*}
\]

\[
\begin{align*}
\mathcal{E}^{\text{DE}}_i &= \int_d^\infty (1 - s)(\pi_i - d) \lambda_i e^{-\lambda_i \pi_i} d\pi_i \\
&= \frac{1}{\lambda_i} e^{-\lambda d}(1 - s).
\end{align*}
\]

**Ex-post payoffs with convertible debt financing**

Alternatively, the venture capitalist and the entrepreneur can sign a convertible debt contract which we characterize in the following way. At time 0, the initial debt and equity holdings by the venture capitalist are $d_0$ and $s_0$. Suppose $d_0 > 0$, and let part of the debt be convertible into equity at the conversion ratio $\gamma$ so that $d = d_0 - \gamma(s - s_0)$, where $d \geq 0$ and $s \in [s_0, 1)$ are the post-conversion debt and equity positions after all the convertible debt is converted, and $s - s_0$ is the fraction of the equity purchased. A special case is 100 percent initial debt financing ($s_0 = 0$). Another special case is that all the debt is convertible ($d = 0$). A contract with convertible debt is, therefore, characterized by $(d_0, s_0, \gamma, s)$.

The ex-post payoffs to the venture capitalist and the entrepreneur from a convertible debt
contract, \((d_0, s_0, \gamma, s)\), for \(i = l, h\), are:

\[
\mathcal{VC}_{i}^{CD} = \frac{1}{\lambda_i} [1 - e^{-\lambda_i [d_0 - \gamma(\hat{s} - s_0)]} (1 - \hat{s})],
\]

\[
\mathcal{E}_{i}^{CD} = \frac{1}{\lambda_i} e^{-\lambda_i [d_0 - \gamma(\hat{s} - s_0)]} (1 - \hat{s}),
\]

where \(\hat{s} \in [s_0, s]\) is the venture capitalist’s post-conversion equity stake. Denoting by \(\hat{d} \in [d, d_0]\) the post-conversion debt, we have \(\hat{d} = d_0 - \gamma(\hat{s} - s_0)\). If the venture capitalist converts the entire amount of convertible debt, then \(\hat{s} = s\) and \(\hat{d} = d\).

**The joint distribution of signal and output**

At time 1, a signal, \(x\), about the state of nature is realized and observed by the entrepreneur and the venture capitalist. The signal has a continuous distribution on the interval \([0, 1]\). Let \(\alpha_h(x)\) denote the joint density function of \(x\) and \(h\), and \(\alpha_l(x)\) the joint density function of \(x\) and \(l\). Let \(q_h(x) = \frac{\alpha_h(x)}{\alpha_h(x) + \alpha_l(x)}\) denote the probability of \(h\) conditional on the signal realization \(x\), where the denominator denotes the marginal density of \(x\). For clarity of exposition, we concentrate on a special case where the joint density of signal and output is linear:

\[
\alpha_h(x) = x, \quad \alpha_l(x) = 1 - x.
\]

The intuition for \(\alpha_h(x)\) being an increasing function of \(x\) is that in state \(h\) high realizations of \(x\) are more likely than low realizations of \(x\) (and analogously for \(\alpha_l(x)\) being a decreasing function). The conditional probability of state \(h\) is

\[
q_h(x) = \frac{x}{x + 1 - x} = x.
\]

The signal \(x\) is strictly informative in the sense that the interim probability, \(q_h(x)\), is strictly increasing in \(x\); see Figure 1.

**Interim payoffs with debt-equity and convertible debt financing**

Consider the debt-equity contract, \((d, s)\). Interim payoffs (after the realization of the signal \(x\)) are averages of the ex-post payoffs in equation (3), weighted by the interim conditional
probabilities $q_h(x)$ and $1 - q_h(x)$, minus $I_2$ for the venture capitalist.\footnote{Notice that, at time 1, $I_1$ is sunk.}

\[
E[\mathcal{VC}]_{x}^{DE} = \left[1 - q_h(x)\right] \frac{1}{\lambda_l} \left[1 - e^{-\lambda_l d(1 - s)}\right] + q_h(x) \frac{1}{\lambda_h} \left[1 - e^{-\lambda_h d(1 - s)}\right] - I_2,
\]

\[
E[\mathcal{E}]_{x}^{DE} = \left[1 - q_h(x)\right] \frac{1}{\lambda_l} e^{-\lambda_l d(1 - s)} + q_h(x) \frac{1}{\lambda_h} e^{-\lambda_h d(1 - s)}.
\]

For the convertible debt contract, $(d_0, s_0, \gamma, s)$, interim payoffs are similarly obtained as averages of the ex-post payoffs in equation (4):

\[
E[\mathcal{VC}]_{x}^{CD} = \left[1 - q_h(x)\right] \frac{1}{\lambda_l} \left[1 - e^{-\lambda_l [d_0 - \gamma (\hat{s} - s_0)](1 - \hat{s})}\right] + q_h(x) \frac{1}{\lambda_h} \left[1 - e^{-\lambda_h [d_0 - \gamma (\hat{s} - s_0)](1 - \hat{s})}\right] - I_2,
\]

\[
E[\mathcal{E}]_{x}^{CD} = \left[1 - q_h(x)\right] \frac{1}{\lambda_l} e^{-\lambda_l [d_0 - \gamma (\hat{s} - s_0)](1 - \hat{s})} + q_h(x) \frac{1}{\lambda_h} e^{-\lambda_h [d_0 - \gamma (\hat{s} - s_0)](1 - \hat{s})},
\]

where $\hat{s}$ is the post-conversion equity held by the venture capitalist. If he converts the entire amount of convertible debt, then $\hat{s} = s$, while if he converts nothing, $\hat{s} = s_0$.

**The refinancing decision and ex-ante payoffs**

The entrepreneur never wants to liquidate the project once it has started since he provides no financing and (because of limited liability) always obtains a positive payoff as long as $s < 1$.\footnote{Moreover, we can imagine that any investment by the entrepreneur in human capital is firm-specific and, hence, sunk.}

The driving force of our model is the conflict of interest between the entrepreneur and the venture capitalist. The former always wants to proceed with the project, while the latter wants to refinance the project only if the interim news regarding the probability of success are sufficiently favorable. To compute ex-ante payoffs, this refinancing/liquidation decision must be taken into account.

We calculate the maximal ex-ante surplus of the project, defined as the ex-ante surplus given efficient liquidation, as a weighted average of the interim surplus over all signal realizations, $x$,.
for which refinancing is efficient, minus the time 0 investment, \( I_1 \). The interim surplus is the sum of the interim payoffs of the entrepreneur and the venture capitalist—e.g., the sum of the expressions in equations (7) and (8)—and is equal to \([1 - q_h(x)] \frac{1}{\lambda_l} + q_h(x) \frac{1}{\lambda_h} - I_2\). Denote by \( x' \) the smallest realization of the signal that makes the interim surplus positive (\( x' \) is the socially efficient refinancing/liquidation cut-off), determined by

\[
[1 - q_h(x')] \frac{1}{\lambda_l} + q_h(x') \frac{1}{\lambda_h} - I_2 = 0. \tag{11}
\]

Recalling that \( q_h(x) = x \), we can solve for \( x' \) in terms of the model’s parameters:

\[
x' = \frac{I_2 - 1/\lambda_l}{1/\lambda_h - 1/\lambda_l}. \tag{12}
\]

The maximal ex-ante surplus of the project can then be calculated:\(^{14}\)

\[
\int_{x'}^{1} \left\{ [1 - q_h(x)] \frac{1}{\lambda_l} + q_h(x) \frac{1}{\lambda_h} - I_2 \right\} [\alpha_l(x) + \alpha_h(x)] \, dx - I_1
\]
\[
= \int_{x'}^{1} \left\{ \alpha_l(x) \frac{1}{\lambda_l} + \alpha_h(x) \frac{1}{\lambda_h} - I_2 [\alpha_l(x) + \alpha_h(x)] \right\} \, dx - I_1 \tag{13}
\]
\[
= \frac{(1-x')^2}{2} \frac{1}{\lambda_l} + \frac{(1-x')^2}{2} \frac{1}{\lambda_h} - (1-x') I_2 - I_1 > 0.
\]

We assume that the ex-ante surplus is strictly positive. Since \( x' \) is expressed in terms of the model’s parameters (equation (12)), this assumption constitutes a condition on the parameters of the model.\(^{15}\)

The assumption that the maximal ex-ante surplus is strictly positive implies that, without signal manipulation, the project is worth financing in stages, thanks to the information revealed by the signal. Therefore, debt-equity and convertible debt contracts exist so that, with stage financing but without signal manipulation, both the entrepreneur and the venture capitalist are willing to participate in the venture. Given such contracts, the refinancing/liquidation cut-off value of \( x \) will be determined by the interim payoff of the venture capitalist and will be denoted \( x_0 \), which is defined as the smallest realization of the signal that makes the interim payoff of the venture capitalist positive. This cut-off value and the resulting ex-ante payoffs of the venture

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\(^{14}\)The first line of (13) is the sum of the interim surplus conditional on \( x \), over all realizations of \( x \) for which refinancing is socially efficient (i.e., \( x \geq x' \)), weighted by the probability of observing \( x \). The second line follows from the definition of \( q_h(x) \). Substituting for \( \alpha_l(x) \) and \( \alpha_h(x) \) using (5), and integrating, yields the third line.

\(^{15}\)Tedious calculations, using (12), yield \( \left\{ \frac{1}{2} \left[ \left( \frac{1}{\lambda_l} \right)^2 - I_2^2 \right] - \left( \frac{1}{\lambda_l} - I_2 \right) \right\} / \left( \frac{1}{\lambda_h} - \frac{1}{\lambda_l} \right) - I_1 > 0. \)
capitalist and the entrepreneur will be introduced as the need arises.

2.2 Signal Manipulation

To reduce the chances that the project will be liquidated, the entrepreneur can manipulate the distribution of the signal in period 0. We require that after any form of signal manipulation, the unconditional distribution of profits is unchanged. More specifically, this means that the ex-ante probabilities of the good and bad states, \( h \) and \( l \), continue to be \( 1/2 \). Namely, signal manipulation affects the distribution of the signal \( x \) without affecting the true performance of the project.

We model signal manipulation (window dressing) as a rightward shift of probability mass which reduces the information content of low values of the signal, but before doing so, we explain why only this type of signal manipulation captures the essential feature of stage financing that we want to study—the conflict of interest between the entrepreneur and the venture capitalist regarding liquidation of the project.

We begin by establishing

**Result 1** The entrepreneur will not jam the signal in the commonly used sense of “adding noise” to it.

We follow the literature and model signal jamming as a transfer of mass of size \( \epsilon \) from the center to each of the tails of the marginal distributions \( \alpha_h(x) \) and \( \alpha_l(x) \) in a mean preserving manner, as illustrated in panel (a) of Figure 2. Suppose that in period 0 the entrepreneur can jam the signal in this way. Given any refinancing/liquidation cut-off value of \( x \), denoted \( \bar{x} \), the entrepreneur does not want to shift mass from the right of \( \bar{x} \) to its left since this would increase the probability of liquidation. Moreover, the mass that is shifted from the center to the right tail of the distribution does not affect the probability of liquidation nor the expected return of the project. Therefore, jamming the signal involves only drawbacks and no benefits for the entrepreneur. It follows that: (a) the venture capitalist believes that the signal is not jammed; (b) this belief pins down a refinancing/liquidation cut-off value of \( x \); and (c) the entrepreneur does not jam the signal.

We now establish

**Result 2** The entrepreneur and the venture capitalist both benefit from the precise opposite of signal jamming—shifting mass from the tails to the center.
This type of signal manipulation is illustrated in panel (b) of Figure 2. Given any beliefs of the venture capitalist (that pin down a refinancing/liquidation cut-off value, \( \bar{x}' \)) the entrepreneur wants to shift mass of size \( \epsilon \) from the tails to the center since this reduces the probability of liquidation. The venture capitalist benefits from such manipulation, and would not discourage it. The intuition is as follows. On the one hand, the probability of liquidating a bad project decreases since mass is shifted in state \( h \) from the left of \( \bar{x}' \) to its right. On the other hand, the probability of liquidating a good project also decreases since mass is shifted also in state \( l \) from the left of \( \bar{x}' \) to its right. The latter effect dominates because, for small values of \( x \), \( \alpha_h(x) < \alpha_l(x) \), so the impact of a shift of mass of given size is higher in state \( h \). In terms of the conditional probability, \( q_l \), this means that the information content of the signal increases on the interval \([x_1, x_2] \) since a signal realization in this interval indicates with greater certainty that the state of nature is \( l \) (namely, \( q_h \) declines on this interval). Thus, the decision rule used by the venture capitalist—liquidate if and only if \( x < \bar{x}' \)—is based on better information, increasing the value of the option to abandon.

It follows that both kinds of signal manipulation do not constitute a conflict of interest: signal jamming is undesirable to both the venture capitalist and the entrepreneur, whereas the opposite of signal jamming is desirable to both. Since we want to model the tension described in Sahlman (1988), we focus on a type of signal manipulation that constitutes a conflict of interest between the entrepreneur and the venture capitalist. To capture this conflict, it is essential that mass is shifted from the left of the refinancing/liquidation cut-off to its right, and that more mass is shifted in state \( l \) than in state \( h \). (We elaborate on this point in Section 7 where other forms of signal manipulation are discussed.) We will now present one type of signal manipulation, as displayed in Figure 2b, increases both \( \int_{x'}^1 \alpha_l(x)dx \) and \( \int_{\bar{x}'}^1 \alpha_h(x)dx \) by the same amount, \( \epsilon \). The effect on the venture capitalist’s ex-ante payoff thus depends on the sign and magnitude of \( A \equiv \int_{x'}^1 \alpha_l(x)dx \) and \( B \equiv \int_{\bar{x}'}^1 \alpha_h(x)dx \). Since \( \bar{x}' \) is determined by the equation \( \frac{\alpha_l(x)}{\alpha_l(x)+\alpha_h(x)} A + \frac{\alpha_h(x)}{\alpha_l(x)+\alpha_h(x)} B = 0 \), we have \( A < 0 \) and \( B > 0 \). By the definition of \( \alpha_l(x) \) in equation (5), and since \( \bar{x}' < 1/2 \), we have \( \alpha_l(\bar{x}') > \alpha_h(\bar{x}') \) implying \( B > |A| \). Hence, the venture capitalist’s ex-ante payoff increases as a result of signal manipulation.

\[ \text{Signal manipulation, as displayed in Figure 2b, increases both } \int_{x'}^1 \alpha_l(x)dx \text{ and } \int_{\bar{x}'}^1 \alpha_h(x)dx \text{ by the same amount, } \epsilon. \]

\[ \text{The effect on the venture capitalist’s ex-ante payoff thus depends on the sign and magnitude of } A \equiv \int_{x'}^1 \alpha_l(x)dx \text{ and } B \equiv \int_{\bar{x}'}^1 \alpha_h(x)dx. \]

\[ \text{Since } \bar{x}' \text{ is determined by the equation } \frac{\alpha_l(x)}{\alpha_l(x)+\alpha_h(x)} A + \frac{\alpha_h(x)}{\alpha_l(x)+\alpha_h(x)} B = 0, \text{ we have } A < 0 \text{ and } B > 0. \]

\[ \text{By the definition of } \alpha_l(x) \text{ in equation (5), and since } \bar{x}' < 1/2, \text{ we have } \alpha_l(\bar{x}') > \alpha_h(\bar{x}'). \]

\[ \text{Hence, the venture capitalist’s ex-ante payoff increases as a result of signal manipulation.} \]

\[ \text{We implicitly assume that any signal manipulation that is feasible and desirable to both parties is already incorporated in the primitive joint distribution of signal and output.} \]

---

\[ {\text{The proof is as follows. We assume that } \bar{x}' \text{ is smaller than } 1/2 \text{ (otherwise shifting mass from the left of } \bar{x}' \text{ to the center of the interval } [0, 1] \text{ is meaningless). Consider a sharing rule, e.g., a debt-equity contract, } (d, s). \text{ The venture capitalist’s ex-ante payoff, prior to signal manipulation, is} \]

\[ \int_{x'}^1 \left\{ [1 - q_h(x)] \frac{1}{\lambda_h} [1 - e^{-\lambda_h d(1 - s)}] + q_h(x) \frac{1}{\lambda_h} [1 - e^{-\lambda_h b d(1 - s)}] - I_2 \right\} \alpha_l(x) dx - I_1 \]

\[ = \left\{ [1 - e^{-\lambda_h d(1 - s)}] - I_2 \right\} \int_{x'}^1 \alpha_l(x) dx + \left\{ [1 - e^{-\lambda_h b d(1 - s)}] - I_2 \right\} \int_{\bar{x}'}^1 \alpha_h(x) dx - I_1. \]

Signal manipulation, as displayed in Figure 2b, increases both \( \int_{x'}^1 \alpha_l(x)dx \) and \( \int_{\bar{x}'}^1 \alpha_h(x)dx \) by the same amount, \( \epsilon \).
manipulation that fits this description.

**Window dressing**

The main concern of the entrepreneur in our model is to avoid liquidation of the project, and he has a strong incentive to engage in signal manipulation that shifts probability mass from the left of the refinancing/liquidation cut-off value of \( x \) to its right. Since the entrepreneur does not know, at time 0, whether the project is good or bad (whether the state of nature is \( l \) or \( h \)), it is unlikely that he can shift mass to the right only in one state of nature. Rather, we assume that he can engage in activities that give an appearance of better short-term performance in general by shifting probability mass from left to right for various values of \( x \) and in both states of nature. We turn to a specific model that captures the essential economic forces at work.

Suppose that at time 0, the entrepreneur can affect the distribution of the signal \( x \), reducing the probability of observing low realizations of \( x \) and increasing the probability of observing high realizations of \( x \) without affecting the probability of states \( h \) and \( l \) which remains \( 1/2 \). Let us call such activity window dressing and denote it \( a = 1 \), while \( a = 0 \) denotes no window dressing. The venture capitalist cannot observe whether the entrepreneur engages in window dressing.

Denote \( \alpha_h(x, 0) \) the joint density function of \( x \) and \( h \) when \( a = 0 \), and similarly for \( \alpha_l(x, 0) \).

Denote \( q_h(x, 0) = \frac{\alpha_h(x, 0)}{\alpha_h(x, 0) + \alpha_l(x, 0)} \) the probability of state \( h \) conditional on the signal realization \( x \) when \( a = 0 \). This is just different notation for the expressions in equations (5) and (6). Thus, for \( x \) on the interval \([0, 1]\):

\[
\alpha_h(x, 0) = x, \quad \alpha_l(x, 0) = 1 - x, \quad q_h(x, 0) = \frac{x}{x + 1 - x} = x, \quad (14)
\]

i.e., the signal is strictly informative, and \( \int_0^1 \alpha_h(x, 0)dx = \int_0^1 \alpha_h(x, 1)dx = 1/2 \).

We express the rightward shift of mass in terms of a parameter \( k \). Window dressing involves: (a) transferring mass in state \( l \) from the interval \([0, k]\) to the interval \((k, (1 + k)/2)\); and (b) transferring mass in state \( h \) from the interval \((k, (1 + k)/2] \) to the interval \([((1 + k)/2, 1), (18)\]

\[\text{In other words, such signal manipulation gives the appearance of better performance at time 1 without actually affecting the “true” performance.}\]

\[\text{For simplicity, we assume that the entrepreneur can window dress costlessly. In Section 6 we introduce both a direct cost of window dressing (born by the entrepreneur) and an indirect cost—window dressing comes at the expense of effort devoted to the improvement of long-run performance. The results do not change in a meaningful way.}\]
illustrated in Figure 3. The logic is that as a result of window dressing, poor projects (state \( l \)) are less likely to exhibit low interim performance, but good projects (state \( h \)) are more likely to exhibit high interim performance. Thus,

\[
\alpha_h(x, 1) = \begin{cases} 
  x & x \leq k \\
  k & k < x < \frac{1+k}{2} \\
  1 & x \geq \frac{1+k}{2},
\end{cases}
\]

(15)

\[
\alpha_l(x, 1) = \begin{cases} 
  1 - k & x \leq k \\
  1 - x + \frac{k^2}{1-k} & k < x < \frac{1+k}{2} \\
  1 - x & x \geq \frac{1+k}{2},
\end{cases}
\]

implying

\[
q_h(x, 1) = \begin{cases} 
  \frac{x}{x+1-k} & x \leq k \\
  \frac{k}{x+1-x+k^2/(1-k)} & k < x \leq \frac{1+k}{2} \\
  \frac{1}{x} & x > \frac{1+k}{2}.
\end{cases}
\]

(16)

Window dressing increases the conditional (interim) probability of state \( h \) on the intervals \((0, k)\) and \((1 + k)/2, 1)\)—on these intervals \( q_h(x, 1) > q_h(x, 0) \). Notice that \( q_h(x, 1) \) is strictly informative (strictly increasing in \( x \)) everywhere except at \( x = k \). This is due to a specific feature of the joint distribution of signal and output that we are using, in particular to the discontinuity of \( \alpha_l(x, 1) \) at \( x = k \). It plays no role in the analysis nor does it drive any of the results, and is assumed only in order to simplify calculations.\(^{20}\)

We will now show that such window dressing reduces the ex-ante value of the project. An immediate consequence is that the venture capitalist wants to prevent window dressing. Define \( x'_{a=1} \) as the socially efficient refinancing/liquidation cut-off value when the signal is manipulated. For simplicity of exposition, we restrict attention to the case where \( x'_{a=1} \) is unique, as illustrated in panel (c) of Figure 3. A sufficient condition for this is \( x' < k' \), where \( x' \) is given in (12) in terms of the model’s parameters, and \( k' \) can be derived as shown in the figure. From the figure, it is also evident that \( x'_{a=1} \) is smaller than \( x' \). Uniqueness of \( x'_{a=1} \) is convenient but by no means necessary. If it were not unique, the liquidation/refinancing decision would be non-monotone in \( x \). The analysis would go through but would be cumbersome.\(^{21}\)

\(^{20}\)In Section 7 we study another specification of the joint distribution of signal and output where both \( q_h(x, 0) \) and \( q_h(x, 1) \) are strictly informative.

\(^{21}\)Moreover, the possibility of non-uniqueness of the refinancing/liquidation cut-off for \( a = 1 \) arises only because
**Result 3** Window dressing reduces the maximal ex-ante surplus.

The intuition is that for low realizations of $x$ the information content of the signal decreases as a result of window dressing, since a low signal realization indicates with less certainty that the state of nature is $l$. The refinancing/liquidation decision is thus based on lower quality information, reducing the value of the option to abandon. By a similar logic, for any sharing rule (and its corresponding refinancing/liquidation cut-off value of $x$), window dressing reduces the payoff of the venture capitalist since he is the one investing the additional money on the basis of less accurate information.

**Proof of Result 3.** The maximal ex-ante total surplus when $a = 0$ is

$$
\int_{x'}^{1} \left\{ [1 - q_h(x, 0)] \frac{1}{\lambda_l} + q_h(x, 0) \frac{1}{\lambda_h} - I_2 \right\} \left[ \alpha_l(x, 0) + \alpha_h(x, 0) \right] dx - I_1,
$$

and we will now show that it is larger than the maximal ex-ante total surplus when $a = 1$. The expression in (17) is larger than

$$
\int_{x'_{a=1}}^{1} \left\{ [1 - q_h(x, 0)] \frac{1}{\lambda_l} + q_h(x, 0) \frac{1}{\lambda_h} - I_2 \right\} \left[ \alpha_l(x, 0) + \alpha_h(x, 0) \right] dx - I_1,
$$

since the payoff in (18) is derived using a socially sub-optimal liquidation cut-off.\(^{22}\) In turn, the expression in (18) is larger than the ex-ante total surplus when $a = 1$,

$$
\int_{x'_{a=1}}^{1} \left\{ [1 - q_h(x, 1)] \frac{1}{\lambda_l} + q_h(x, 1) \frac{1}{\lambda_h} - I_2 \right\} \left[ \alpha_l(x, 1) + \alpha_h(x, 1) \right] dx - I_1,
$$

which can be seen as follows. If the integrals in (18) and (19) were taken on the entire interval $[0, 1]$, they would both be equal (since window dressing does not affect the project’s true performance). Therefore, to show that (18) is larger than (19), it is sufficient to show that the integrand in (18) integrated on $[0, x'_{a=1}]$ is smaller than the integrand in (19) integrated on the same interval. Using the definitions of $\alpha_h(x, 0)$, $\alpha_h(x, 1)$, $q_h(x, 0)$, and $q_h(x, 1)$ (equations (14), (15), and (16)), this is equivalent to showing that

$$
\left( \frac{1}{\lambda_l} - I_2 \right) \int_{0}^{x'_{a=1}} [(1 - x) - (1 - k)] dx < 0,\)

\(^{23}\)

Since on the interval $[0, x'_{a=1}]$, $x < k$, we have $\int_{0}^{x'_{a=1}} [(1 - x) - (1 - k)] dx > 0$, and from equation (11) we have $\frac{1}{\lambda_l} - I_2 < 0$, establishing the result. \(\square\)

\(^{22}\)More specifically, on the interval $[x'_{a=1}, x']$ the integrand is negative, by definition of $x'$.

\(^{23}\)Notice that on the interval $[0, x'_{a=1}]$ window dressing does not affect $\alpha_h$.\(^{16}\)
To underline the damage caused by window dressing, we assume that the decline in the total ex-ante surplus as a result of window dressing is sufficiently pronounced so that the expression in (19) is strictly negative. This condition can be written as

$$\int_{a=1}^{1} \left\{ \alpha_l(x, 1) \frac{1}{\lambda_l} + \alpha_h(x, 1) \frac{1}{\lambda_h} - I_2 [\alpha_l(x, 1) + \alpha_h(x, 1)] \right\} dx - I_1 < 0, \quad (20)$$

implying that the project is not worth financing. An implication is that when it is known by the parties that $a = 1$, there is no contract such that the ex-ante payoff of the venture capitalist is positive (since he bears all the costs and what he can obtain has an upper bound equal to the ex-ante surplus, which is negative).

Summarizing, window dressing reduces the probability that the project will be liquidated but, at the same time, also reduces the benefit of introducing stage financing (by reducing the venture capitalist’s value of the option to abandon), rendering the project nonviable. Next, we will show that, nevertheless, once debt-equity financing has been obtained, the entrepreneur will always want to window dress the distribution of the signal.

2.3 Analysis

Given the contract that is in effect, the entrepreneur decides at time 0 whether to window dress. We begin by showing that, for any straight debt-equity contract, the entrepreneur will do so. We will then show that there is a convertible debt contract such that the entrepreneur will not window dress and the project will be financed.

Window dressing with debt-equity financing

We establish

**Proposition 1** If the project is financed with a debt-equity contract, $(d, s)$, the entrepreneur will always window dress.

The proof and the intuition are one and the same: given any beliefs of the venture capitalist, that pin down a refinancing/liquidation cut-off value, $x_0$, the entrepreneur wants to shift mass from the left of $x_0$ to its right since this reduces the probability of liquidation. More formally, consider an arbitrary debt-equity contract, $(d, s)$, and its corresponding liquidation cut-off value
The effect of window dressing on the probability of refinancing varies according to whether $x_0$ is smaller than $k$, larger than $(1 + k)/2$, or between the two. Consider Figure 3. If $x_0 \leq k$, the rightward shift of mass does not affect the probability of refinancing in state $h$, but increases the probability of refinancing in state $l$. If $k < x_0 \leq (1 + k)/2$, the probability of refinancing in both states increases, and if $x_0 > (1 + k)/2$, only the probability of refinancing state $h$ increases. In all three cases, the payoff of the entrepreneur increases by deviating to $a = 1$ and, therefore, $a = 0$ cannot be part of an equilibrium.

Using the same logic, if (for some reason) the project is financed at time 0 with a straight debt-equity contract then, in equilibrium, $a = 1$: the belief of the venture capitalist that $a = 1$ determines a refinancing/liquidation cut-off and, given these beliefs, the entrepreneur’s payoff is strictly larger when $a = 1$ than when $a = 0$ since the probability of refinancing is higher. Anticipating this, the venture capitalist will not provide financing at time 0.

Summarizing, the implication of Proposition 1 is that the project will not be financed with a straight debt-equity contract because straight debt-equity fails to provide the incentives for no window dressing and, as a consequence, the project is not profitable ex-ante.

**The role of convertible debt in preventing window dressing**

We will now show that with an appropriately designed financing scheme that combines equity, straight debt, and convertible debt the project will be financed and the entrepreneur will not engage in window dressing at the ex-ante stage. The result is summarized in:

**Proposition 2** There is a convertible debt contract, $(d_0, s_0, \gamma, s)$, such that the entrepreneur does not window dress and the project is financed.\(^\text{25}\)

A detailed proof is provided in Appendix 1. We provide here the main steps of the argument and their economic significance. The proof consists of constructing a convertible debt contract, $(d_0, s_0, \gamma, s)$, and finding two cut-off values of the signal, $x_0$ and $x_1$, satisfying $0 < x_0 < k < x_1 < 1$, such that in equilibrium: the venture capitalist (a) provides $I_1$ at time 0; (b) liquidates the project after observing $x < x_0$, but not otherwise; (c) does not convert any debt after observing $x > x_1$.

\(^{24}\)Since the venture capitalist obtains only part of the total ex-ante surplus, $x_0$ is greater than $x'$, the socially efficient refinancing/liquidation cut-off (the inequality is strict since we are assuming throughout $s < 1$).

\(^{25}\)Typically, there will be many convertible debt contracts that induce $a = 0$. The choice among them will depend, among other things, on the relative bargaining power of entrepreneur and venture capitalist.
\(x \in [x_0, x_1]\); (d) converts the maximal amount of debt allowed, \(d_0 - d\), after observing \(x \geq x_1\) (where \(d = d_0 - \gamma(s - s_0)\)); and the entrepreneur chooses \(a = 0\).

The conversion ratio, \(\gamma\), and the venture capitalist’s post-conversion share of the enterprise, \(s\), play a key role in the construction of the contract. If \(\gamma\) is low, converting debt into equity is cheap and the venture capitalist will always convert debt if he decides to refinance the project. If \(\gamma\) is high, the venture capitalist will never convert debt. In both cases, the option to convert does not constitute a threat that prevents window dressing. A conversion ratio, \(\gamma\), that is not too high nor too low must be selected to provide incentives for the entrepreneur. The attributes “low” and “high” depend on \(s\), the venture capitalist’s post-conversion share of the enterprise. The higher is \(s\), the smaller is the entrepreneur’s post-conversion share of the enterprise (i.e., the bigger is the “punishment” for window dressing), and the larger \(\gamma\) can be and still be effective. The \(\gamma\) and \(s\) that jointly prevent window dressing are such that when the venture capitalist chooses to convert debt, conversion takes place at a convenient rate for the venture capitalist and hurts the entrepreneur (in other words, the venture capitalist buys underpriced equity). The loss to the entrepreneur more than offsets the advantage from the fact that liquidation is less likely to take place.

We begin by fixing an arbitrary value for \(s_0\). This illustrates that the result in Proposition 2 does not rely on “razor’s edge” arguments, and that there is considerable leeway in constructing convertible debt contracts that prevent window dressing. We want to choose the other parameters of the contract and the two cut-off values of \(x\), so that the venture capitalist will not convert any debt after observing \(x \in [x_0, x_1]\), but will convert all the convertible debt after observing \(x \geq x_1\). To exploit the model’s “natural structure,” we design the contract so that \(x_1 = (1 + k)/2\). Recalling that \(q(x, 0) = x\), and using the expressions for the venture capitalist’s interim payoffs (equations (7) and (9)), we choose \(d_0\), \(\gamma\), and \(s\) such that the following condition is satisfied:

\[
\frac{1-k}{2} \frac{1}{\lambda_l} \left[1 - e^{-\lambda_l d_0 (1 - s_0)} \right] + \frac{1+k}{2} \frac{1}{\lambda_h} \left[1 - e^{-\lambda_h d_0 (1 - s_0)} \right] = \frac{1-k}{2} \frac{1}{\lambda_l} \left[1 - e^{-\lambda_l [d_0 - \gamma(s - s_0)] (1 - s)} \right] + \frac{1+k}{2} \frac{1}{\lambda_h} \left[1 - e^{-\lambda_h [d_0 - \gamma(s - s_0)] (1 - s)} \right].
\] (21)

Condition (21) states that the signal realization \(x = (1 + k)/2\) renders the venture capitalist indifferent between not converting debt and converting all the convertible debt. Later in the proof, we will show that, for any signal realization, the venture capitalist’s interim payoff is
maximized either by converting no debt or by converting all the convertible debt. Therefore, if $x > (1 + k)/2$, the venture capitalist converts the entire amount of convertible debt, and if $x < (1 + k)/2$ he converts none. From (21)—see Appendix 1 for details—we obtain

$$e^{\lambda_1 \gamma(s-s_0)} \frac{1 - s}{1 - s_0} = \frac{1 - k}{2} \frac{1}{\lambda_t} e^{-\lambda_h [d_0 - \gamma(s-s_0)]} e^{-(\lambda_1 - \lambda_h) d_0} + \frac{1 + k}{2} \frac{1}{\lambda_t} e^{-\lambda_h [d_0 - \gamma(s-s_0)]} e^{-(\lambda_1 - \lambda_h) d_0}.$$

Condition (22) defines an implicit relation between $\gamma$, $s$, and $d_0$. Throughout the proof, we require that this equality is satisfied. In particular, any argument involving a change in $\gamma$ and $d_0$ entails a corresponding change in $s$ so that (22) is satisfied and, consequently, the debt conversion cut-off value remains $x_1 = (1 + k)/2$.

Next, we impose a condition that ensures that the entrepreneur prefers not to manipulate the signal. That is, given the convertible debt contract and the cut-off values, $x_0$ and $x_1 = (1+k)/2$, his ex-ante payoff with $a = 0$ is larger than his ex-ante payoff with $a = 1$ (given that the venture capitalist believes that $a = 0$). Spelling out this condition explicitly is tedious (see Appendix 1), but after manipulation it boils down to the following inequality:

$$e^{\lambda_1 \gamma(s-s_0)} \frac{1 - s}{1 - s_0} \leq 1 + \frac{1}{2} \frac{x_0(x_0 - 2k)}{x_0(1-k)^2} \frac{\lambda_h}{\lambda_t} e^{-(\lambda_1 - \lambda_h) d_0}.$$

To construct a convertible debt contract such that both (22) and (23) hold, we increase $\gamma$ and $d_0$ so that (a) for each value of $\gamma$ and $d_0$, we choose $s$ that satisfies (22); and (b) the post-conversion debt, $d_0 - \gamma(s-s_0)$, remains positive and finite.26 (In Appendix 1, we show formally how this is accomplished.) The driving economic force in this construction is the change in $\gamma$ and the corresponding change in $s$, as discussed above.

We then show that as $\gamma$ and $d_0$ increase, the right hand side of (23) approaches 1 (taking into account that $x_0$ changes as $\gamma$ and $d_0$ increase). We further show that as $\gamma$ and $d_0$ increase, the right hand side of (22) approaches a number strictly between 0 and 1, and so must the left hand side in order to preserve the equality. Since the left hand side of (22) is equal to the left hand side of (23), and because the right hand side of (23) approaches 1, the inequality in (23) is eventually satisfied. Therefore, there are values of $\gamma$, $d_0$, $s$, and $x_0$—combined with the value $s_0$ that was chosen in the beginning of the proof—that ensure that both (22) and (23) hold.

26This condition guarantees that we are considering “pure” convertible debt. In Section 4, we introduce the use of warrants and this condition is no longer required.
Next, we verify that given such a contract, the venture capitalist’s interim payoff, as a function of his post-conversion equity stake, achieves a unique interior minimum on the interval $(s_0, s)$. This implies that the venture capitalist’s interim payoff is maximized either at $s_0$—“no debt conversion,” or at $s$—“full debt conversion” which implies that the formulation of conditions (22) and (23) is consistent with optimization by the venture capitalist. Finally, we turn to the venture capitalist’s ex-ante participation constraint, and show that it is satisfied.\footnote{Notice that $x_0$ can be chosen to be arbitrarily close to $x'$, the socially efficient cut-off value of $x$. The argument is essentially as follows. The refinancing/liquidation cut-off, $x_0$, is determined by

$$
[1 - q(x_0)] \frac{1}{\lambda_l} [1 - e^{-\lambda_l d_0 (1 - s_0)}] + q(x_0) \frac{1}{\lambda_h} [1 - e^{-\lambda_h d_0 (1 - s_0)}] - I_2 = 0,
$$

where $q(x) = x$; this equation is the analogue of equation (11) that determines $x'$, the socially efficient cut-off value of $x$. As $d_0 \to \infty$, $1 - e^{-\lambda d_0 (1 - s_0)} \to 1$ and, therefore, using (11), $x_0 \to x'$.}

**Economic intuition for Proposition 2**

To gain more intuition for this result, we present in Figure 4 the interim payoffs of both venture capitalist and entrepreneur as a function of the signal realization, $x$. Panel (a) looks at the case with no window dressing ($a = 0$). The dashed line represents the payoffs for straight debt-equity financing, $(d_0, s_0)$, and the solid line for convertible debt financing, $(d_0, s_0, \gamma, s)$. The upward kink in the venture capitalist’s payoff and the corresponding downward kink in the entrepreneur’s payoff at $x = (1 + k)/2$ are clearly visible. These kinks illustrate the benefit to the venture capitalist from converting debt at a favorable conversion ratio and the corresponding damage to the entrepreneur. The ex-ante payoff for each party is computed as a weighted average—over the signal realizations, $x$—of the displayed interim payoff (minus $\int_{x_0}^1 I_2 [\alpha_l(x, 0) + \alpha_h(x, 0)] \, dx + I_1$ for the venture capitalist). The shaded area illustrates the damage to the entrepreneur—from debt conversion by the venture capitalist. (Had the venture capitalist not converted, the interim payoffs would be those represented by the dashed lines on the entire interval $[x_0, 1]$.) Note that, from an ex-ante point of view, the shaded area serves only as an illustration for this damage because the actual ex-ante payoff is computed as an average of the interim payoffs, where the weight given to signal realization $x$ is $\alpha_l(x, 0) + \alpha_h(x, 0)$. Therefore the actual damage will be somewhat different.

Panel (b) similarly displays the interim payoffs if there is window dressing ($a = 1$) when the venture capitalist believes that there is no window dressing (thus, the refinancing/liquidation cut off, $x_0$, is the same as in panel (a)). The decision of the entrepreneur whether to window
dress consists of comparing the ex-ante payoff with convertible debt financing as derived from his interim payoff in the top-right graph \((a = 0)\), to the ex-ante payoff with convertible debt financing as derived from his interim payoff in the bottom-right graph \((a = 1)\). For this comparison, the most relevant intervals are \([0, x_0]\) and \([(1 + k)/2, 1]\). On \([0, x_0]\), the interim payoff of the entrepreneur is zero but for \(a = 1\) it occurs with lower probability. This is the force that operates in favor of window dressing from the entrepreneur’s perspective. The main force against window dressing from his perspective is the damage from debt conversion by the venture capitalist, as illustrated by the shaded areas on the interval \([(1 + k)/2, 1]\). Ex-ante, the entrepreneur does not know for sure in which interval the signal will be realized. But he does know that window dressing increases the likelihood that the interim payoff will be in the interval where he is damaged (in other words, window dressing increases the weights given to the payoffs on the interval \([(1 + k)/2, 1]\)). An appropriately designed convertible debt contract, and especially the choice of \(\gamma\) and \(s\), ensure that the latter force dominates in terms of the entrepreneur’s ex-ante payoff.\(^{28}\)

In the proof of Proposition 2, we make use of the fact that, as \(\gamma\) increases, \(s\) approaches 1 and \(\gamma(1 - s)\) approaches zero. We reiterate the intuition. Changing \(\gamma\) and, at the same time, \(s\) at the appropriate rate changes the slopes of the curves and achieves the correct balance between terms of conversion that are not too favorable to the venture capitalist, yet constitute a sufficient threat for the entrepreneur in that the purchased equity is underpriced.

While \(\gamma\) and \(s\) are chosen in order to provide the right incentives, \(d_0\) and \(s_0\) are chosen so that the individual rationality constraints of both parties are satisfied. The individual rationality constraint of the entrepreneur is trivially satisfied: due to limited liability, and since he does not invest his own money, his payoff cannot be negative. Therefore, we must choose \(d_0\) and \(s_0\) to guarantee a sufficiently high (ex-ante) payoff to the venture capitalist. In other words, \(d_0\) and \(s_0\) only affect the division of the surplus between the entrepreneur and the venture capitalist. That is why we have degrees of freedom in the choice of \(d_0\) and \(s_0\): the actual choice will depend on how the parties want to split the surplus.\(^{29}\)

\(^{28}\)For an exhaustive analysis, we should also look at what happens on the interval \([x_0, (1 + k)/2]\). This discussion was only meant to provide the intuition for the result, but the proof of Proposition 2 takes into account all the effects.

\(^{29}\)Both \(d_0\) and \(s_0\) can be used to adjust the division of the surplus between the entrepreneur and the venture capitalist but in the proof of Proposition 2 (see Step 9 in Appendix 1), we fix \(s_0\) and vary \(d_0\) to show that the venture capitalist’s participation constraint is satisfied. Varying only \(d_0\) is sufficient for this purpose because the project’s output \((\pi_i, i = l, h)\) is unbounded. Then, for fixed \(s_0\), the share of the surplus that goes to the venture capitalist increases monotonically with \(d_0\) (and approaches unity as \(d_0\) approaches infinity).
Allowing for renegotiation with convertible debt financing

Conditional on a decision by the venture capitalist to refinance (i.e., conditional on \( x \geq x_0 \)), there will be no renegotiation of the terms of the convertible debt contract even if such renegotiation is allowed. The reason is that there is no mutual gain at time 1 from changing the terms of the contract (the conversion ratio, \( \gamma \), the pre-conversion equity stake of the venture capitalist, \( s_0 \), and the amount of debt that can be converted, \( d_0 \)) since the entrepreneur and the venture capitalist are both risk neutral, and any such change will benefit one party at the expense of the other.

Renegotiation can arise only if there is an inefficiency that induces the parties to change the contract in a Pareto improving manner. This arises if the project is liquidated when instead there is a positive surplus from continuation. In other words, there is renegotiation after signal realizations such that \( x' \leq x < x_0 \), where \( x' \) is the socially efficient cut-off value of \( x \).

Thus, there is no renegotiation only if \( x_0 = x' \) which can happen only if \( s_0 = 1 \) (i.e., the venture capitalist internalizes all the social surplus). But this is incongruent with a basic assumption of our model—that the interim payoff to the entrepreneur cannot be exactly zero for any realization of \( x \), otherwise he cannot be induced not to window dress.\(^{30}\) Therefore, although an appropriately designed convertible debt contract ensures that financing is provided at time 0, inefficient liquidation at time 1 after some realizations of \( x \) is inevitable.\(^{31}\)

In the proof of Proposition 2 we showed that the convertible debt contract can be constructed so that \( x_0 \) is arbitrarily close to \( x' \).\(^{32}\) In other words, inefficient liquidation and the ensuing renegotiation can be made to occur with an arbitrarily small probability. This implies that renegotiation can be rendered arbitrarily negligible in terms of payoffs. Since none of the arguments depend on strict indifference and mixed strategies, this further implies that the possibility of renegotiation at time 1 does not affect the feasibility of the project with convertible debt financing.\(^{33}\)

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\(^{30}\) This feature was captured by the assumption \( s < 1 \) (and hence, \( s_0 < 1 \)) and was further justified on the grounds that if the entrepreneur owns no shares, there will be no incentive to exert long-term effort on his part. (See section 6 for an analysis of the model with long-term effort.)

\(^{31}\) Note that this inefficiency would happen with any contract that does not render the venture capitalist the sole owner of the project’s equity. This well understood type of inefficiency has nothing to do with the phenomenon we are studying—preventing window dressing with a convertible security. This inefficiency does not arise only when the signal is verifiable, as we will show in section 5, since in that case the liquidation decision and the allocation of the surplus among the parties are not linked.

\(^{32}\) See footnote 27.

\(^{33}\) This also implies that a convertible debt contract can approximate first best. The inefficiency due to the fact
The deadline for conversion

We assumed that the deadline for conversion precedes time 2, when there is full realization of uncertainty. We will now show that indeed it is not optimal for the parties to set the deadline for conversion at time 2 (or later). The argument is simple: if the deadline for conversion were at time 2, the venture capitalist would wait until time 2, when the uncertainty regarding the state of nature is resolved, to decide whether to convert debt and how much to convert. This decision would be independent of the realization of the signal (that is observed at time 1). But then the logic behind Proposition 1 applies: since the threat of debt conversion at a favorable conversion ratio in response to a good signal is no longer present, the entrepreneur has an incentive to window dress in order to reduce the probability of liquidation. As a consequence, the project will not be financed. The parties thus have a common interest to set the deadline for debt conversion strictly before time 2.

3 Allowing for renegotiation of the debt-equity contract

In the previous sections we argued that a convertible debt contract is superior to a straight debt-equity contract since it prevents window dressing by the entrepreneur. We assumed that the debt-equity contract could not be renegotiated at the interim stage, after the signal is observed by the parties. In this section, we explicitly consider this possibility, and argue that there is a role for convertible debt even when debt-equity contracts can be renegotiated. Renegotiation can take place in two ways. First, we allow the parties to spontaneously renegotiate the contract: they will choose to renegotiate whenever there is space for a Pareto improvement. In this case, the form of renegotiation which will take place (timing, sequence of offers, bargaining power, etc.) is not specified in the contract. Next, we consider the possibility of explicitly designing the renegotiation in the initial contract.

Spontaneous renegotiation of the debt-equity contract

Intuitively speaking, for renegotiation to prevent window dressing, its outcome must be such that the entrepreneur’s share of the surplus after “medium” realizations of \( x \) is large relative

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that \( x_0 > x' \) has nothing to do with the use of convertible debt and would remain with any type of financing as long as the signal is non-verifiable. Therefore, if the signal is non-verifiable, convertible debt achieves constrained efficiency (where the constraint is determined by the need to provide incentives to the entrepreneur to undertake long-term effort) and is therefore optimal.
to his share after “high” realizations of $x$. This payoff profile serves as an incentive not to shift mass from left to right. (This is the same intuition for why convertible debt prevents window dressing, as illustrated in Figure 4.) We argue that spontaneous renegotiation of the debt-equity contract is unlikely to achieve such a payoff profile.

Consider a debt-equity contract, $(d, s)$, and let $x_0$ be the corresponding refinancing/liquidation cut-off that makes the venture capitalist’s payoff zero. Suppose, first, that $x_0 < 1$. Then, after a signal realization $x \geq x_0$ the project is continued, there is no renegotiation, and the payoffs to the venture capitalist and the entrepreneur are those determined by the debt-equity contract, $(d, s)$. After $x' \leq x < x_0$ (where $x'$ is the socially efficient refinancing/liquidation cut-off), inefficient liquidation would occur, and there is renegotiation. For renegotiation to result in refinancing, it must ensure the venture capitalist more than he would obtain without renegotiation (since with $(d, s)$ his expected payoff is negative by definition of $x_0$). Thus, for $x' \leq x < x_0$ the entrepreneur’s share of the surplus must be smaller than his share with the contract $(d, s)$, which is inconsistent with the need to ensure him a high share of the surplus after “medium” realizations of $x$ (as explained above), and the resulting profile of his interim payoffs does not provide incentives to refrain from window dressing.

Suppose that $x_0 = 1$, i.e. the debt-equity contract, $(d, s)$, is chosen so that the venture capitalist’s payoff is so low that he never wants to refinance. Then, there is renegotiation after any signal realization $x \geq x'$.

Is it plausible that the resulting payoff profile to the entrepreneur will prevent window dressing? For this to happen, the entrepreneur’s bargaining power must decrease with $x$, so that his share of the surplus decreases with $x$, despite the fact that the expected size of the total “pie” increases with $x$. Otherwise, he will have an incentive to window dress.

It is implausible that, in reality, spontaneous renegotiation has these properties. At least it is reasonable to assume that in many situations the parties cannot rely on spontaneous renegotiation such that the relative bargaining power changes endogenously as a function of $x$ in this particular manner. In such situations, the entrepreneur will manipulate the signal even when he anticipates renegotiation. The only remaining possibility is for the parties to rely on some sort of renegotiation design, to which we now turn.

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34In this case, the debt-equity contract, $(d, s)$, serves only as a trigger for renegotiation, and the sharing rule that is actually used is entirely determined through renegotiation.
Designed renegotiation of the debt-equity contract

Aghion, Dewatripont, and Rey (1994) argue that in situations with “under-investment,” efficiency can be achieved by explicitly specifying in the initial contract the rules of the renegotiation. Since such renegotiation design can take many forms, they focus on “simple” renegotiation. Renegotiation is said to be simple if it is characterized by only two rules: (i) a default allocation in the event that renegotiation breaks down or fails to occur; and (ii) all the bargaining power is given to one party.

In our model, window dressing cannot be prevented if all the bargaining power is given to one party. If the venture capitalist has all the bargaining power, the entrepreneur’s payoff will remain constant (at the default). If the entrepreneur has all the bargaining power, his share of the surplus will rise with $x$, since the expected size of the total “pie” increases with $x$ (and the venture capitalist’s payoff is constant at the default). In neither case the resulting profile of interim payoffs to the entrepreneur prevents window dressing.

Therefore, “simple” renegotiation as defined by Aghion, Dewatripont, and Rey will not prevent window dressing. We can, however, extend their definition, adapting it to the context of our model, in a way that prevents window dressing. The point is that this “simple” renegotiation is isomorphic to a convertible debt contract.

This can be seen as follows. We will call renegotiation “simple” if (i) it specifies a default sharing rule (a debt-equity contract) that determines the allocation of the surplus in the event that renegotiation breaks down or fails to occur; (ii) it specifies a menu of debt-equity contracts from which the parties must choose at time 1 if successful renegotiation takes place; and (iii) the right to choose from this menu is allocated to one party ex-ante (at time 0). Requirements (i) and (iii) correspond to requirements (i) and (ii) in Aghion, Dewatripont, and Rey’s “simple” renegotiation. In addition, we introduce (ii) which has the effect of limiting the bargaining power allocated in (iii), since the party with the right to choose the sharing rule cannot choose any such rule, but only one that is specified in the menu.

Let the default debt-equity contract in (i) be $(d_0, s_0)$, the menu of contracts in (ii) be $[(d_0, s_0), (d, s)]$, and give the venture capitalist the right to choose a contract from this menu after $x$ is realized. This designed renegotiation is equivalent to a convertible debt contract where $\gamma$ is such that $d = d_0 - \gamma(s - s_0)$. The venture capitalist’s right to choose a contract

\[35\] This menu is the simplest possible, since there are only two choices.
from the menu in (ii) is equivalent to his choice whether not to convert debt or convert all the convertible debt. Finally, (iii) does not give all the bargaining power to the venture capitalist since he is constrained by the menu of contracts in (ii), namely, by the conversion ratio, $\gamma$. We stressed in previous sections that the debt conversion ratio plays a key role in providing the right incentives. If it is too low, converting debt into equity is cheap and the venture capitalist will always convert debt if he decides to refinance the project. If it is too high, the venture capitalist will never convert debt. In both cases, the option to convert debt does not constitute a threat that prevents window dressing. Thus, convertible debt financing that incorporates an option to convert debt at an appropriately chosen conversion ratio can be interpreted as “simple” renegotiation in the spirit of Aghion, Dewatripont, and Rey.

4 Convertible debt cum warrants

The model can accommodate the use of warrants. Suppose that all the initial debt, $d_0$, is convertible at the exercise price $\gamma$ dollars of debt per share and, in addition, the entrepreneur issues to the venture capitalist warrants that can be converted to equity at the same rate, $\gamma$, provided that all the debt has been converted. If all the debt is converted and some of the warrants are exercised, then $d = d_0 - \gamma(\hat{s} - s_0) < 0$, namely, the venture capitalist receives $\hat{s} - s_0$ shares in exchange for all the initial debt, $d_0$, and further pays $|\hat{d}|$ dollars.$^{36}$

The ex-post payoff to the venture capitalist from this contract, for $i = l, h$, is:

$$\int_{s_0}^{\infty} \hat{s} \pi_i e^{-\lambda_i \pi_i} d\pi_i + \hat{d} - \hat{s} \hat{d} = \hat{s} \frac{1}{\lambda_i} + (1 - \hat{s}) \hat{d}. \quad (24)$$

Consider the first line of (24). The first term is the venture capitalist’s share of the profits, the second term—which is negative—is what he pays when exercising the warrants, and the third

$^{36}$The debt conversion ratio and the exercise price of warrants are both $\gamma$, which is a modeling simplification. This formulation implies that $\gamma$ dollars of risky debt and $\gamma$ dollars in cash buy the same amount of shares. Allowing for different conversion ratios would not substantially alter the analysis in this paper, but would render it more cumbersome. Moreover, since in our model a unique $\gamma$ is sufficient to prevent window dressing, there is no reason to introduce a more complex contract. If additional inefficiencies (or more complex manipulations of the signal) were introduced, then allowing $\gamma$ to change would provide additional degrees of freedom to design the contract. A more general model of convertible debt cum warrants financing would have to address interesting issues such as the right mix between convertible debt and warrants and by how much we should expect the conversion ratios to differ.
term is his own share of this payment. Thus, the entrepreneur’s ex-post payoff is

\[(1 - \hat{s}) \frac{1}{\lambda_i} - (1 - \hat{s}) \hat{d}. \] 

(25)

The remainder of the analysis is basically the same. Proposition 1—with debt-equity financing the entrepreneur will window dress—and its proof are identical. Proposition 2 is also virtually unchanged:

**Proposition 3** There is a convertible debt cum warrants contract such that the entrepreneur does not window dress and the project is financed.

The proof is extremely similar but not identical, due to the somewhat different form of the ex-post payoffs and is provided in Appendix 2. (Allowing for warrants actually provides more degrees of freedom in designing a contract that prevents window dressing.)

## 5 Verifiable signal

In many circumstances, interim performance can be measured objectively and in a verifiable manner. In such cases, the initial contract does not need to allocate to one party the option to convert debt. Instead, an automatic conversion clause can be specified, contingent on a prespecified threshold of a verifiable signal. A security with such an automatic conversion clause is equivalent to a contract which specifies different debt-equity combinations contingent on the realization of a verifiable signal. In this section, we analyze such contracts and compare them to the contracts that can be written when signals are nonverifiable.

An efficient contingent contract must ensure that the project is liquidated only when it is socially optimal to do so (i.e., when the expected surplus from continuation is negative). This requirement is easily satisfied: the contract at time 0 should specify that the project will be liquidated if and only if \( x < x' \), where \( x' \) is the smallest realization of the signal that makes the interim total surplus positive; see equation (11).

An efficient contingent contract must also ensure that the entrepreneur does not window dress. From Proposition 1 we know that this cannot be achieved by specifying a debt-equity sharing rule that does not vary as a function of the interim performance signal. The sharing rule must change so that the entrepreneur “loses” if the signal is too favorable rendering \( a = 0 \)

\[^{37}\text{Notice that } (1 - \hat{s}) \hat{d} < 0.\]
optimal from his perspective. When the signal is verifiable, it is straightforward to render the sharing rule contingent on the realization of the signal since this can be specified explicitly in the contract. The “simplest” manner to do so is to formulate a contingent debt-equity contract with two debt-equity sharing rules, \((d_0,s_0)\) and \((d,s)\), and a cut-off value of the signal that triggers the switching from one sharing rule to the other.

To show that such a contract exists, we can mimic the convertible debt contract constructed in Proposition 2 as follows: (a) set the initial debt-equity allocation equal to the pre-conversion debt-equity allocation, \((d_0,s_0)\); (b) set the refinancing/liquidation cut-off equal to \(x'\); (c) set the trigger for switching to the alternative debt-equity allocation equal to the debt conversion trigger, \(x_1 = (1+k)/2\); and (d) set the alternative debt-equity allocation equal to the post-conversion allocation (when all the convertible debt is converted), \((d,s)\).

Contingent debt-equity financing exhibits an improvement with respect to convertible debt financing: socially efficient liquidation. This is because efficient liquidation can be specified in the contract regardless of the sharing rule.

The analogy between contingent debt-equity and convertible debt financing highlights a fundamental role of the debt conversion option in convertible debt financing, which is to adjust the debt-equity structure to new information that is revealed during the life-time of the project. This feature bears resemblance to provisions allowing the provider of financing to take control of the project at an interim stage in response to nonverifiable information, a feature that is very common in incomplete contracts environments. In our model, convertible debt financing allows the provider of financing to alter the financial structure of the project (changing the output allocation rule) at an interim stage in response to nonverifiable information. If some of the information that is expected to arise is verifiable and some is not, convertible debt financing may be preferred to contingent debt-equity financing since convertible debt prevents window dressing whether the signal is verifiable or not (at the cost of inefficient liquidation) whereas contingent debt-equity financing prevents window dressing only if information is verifiable.\(^{38}\)

That could explain why convertible securities used in venture capital financing typically leave the venture capitalist with the option to convert even when there is an automatic conversion clause.

The study by Kaplan and Strömberg (2001) is consistent with this reasoning. In their sample of venture capital financing contracts, automatic conversion is contingent only on an

\(^{38}\)Moreover, as mentioned, the cost of inefficient liquidation can be made arbitrarily small; see footnote 27.
initial public offering (IPO) taking place and satisfying certain conditions (e.g., regarding the issue price, proceeds, or market capitalization), but never on other verifiable interim information such as sales revenue, profitability, or market share. This suggests that while the “final” outcome (the occurrence and terms of an IPO) is a well-defined verifiable signal, pre-IPO performance signals are not. Therefore, it may be better to leave the venture capitalist with the option to convert debt into equity at pre-specified terms.

6 Long-term effort

The literature has often focused on the “long-term effort” of entrepreneurs (and managers), namely, on their contribution to profits in the long-run. For expositional reasons, our analysis abstracted from this feature but the basic short-run versus long-run trade-off is implicit in our model, as we will now demonstrate. One may imagine that the entrepreneur, in order to undertake short-term effort (signal manipulation), would reduce long-term effort. This shift of resources away from long-term effort constitutes an additional cost of short-termism. We now incorporate in the model such a choice for the entrepreneur in the simplest possible way, since the purpose of this section is simply to show that the introduction of long-term effort does not affect our result.

Suppose the entrepreneur can make two types of effort: short-term effort, $a = 0, 1$, which is exactly the signal manipulation described in previous sections, and long-term effort, $\ell = 0, 1$. We assume that both $a$ and $\ell$ involve a cost $c$ that is born by the entrepreneur, and that the entrepreneur can only exert one of the two types of effort (or none). For example, one can think of $c$ as being high enough so that the entrepreneur will never choose to exert both types of effort.\footnote{This is just a simplification that allows us to focus on a reduced number of possibilities and it is not critical for the result. In general, the entrepreneur will always window dress (if $\psi$ is not too high) and whether he will exert or not the long-term effort will depend on the cost $c$.}

If $a = 0$ and $\ell = 1$ (the entrepreneur exerts long-term effort and does not manipulate the signal), the joint distribution of signal and output is the one described in previous sections when $a = 0$. If, instead, $\ell = 0$, then, whether $a = 0$ or $a = 1$, the ex-ante probability of the good state of nature, $h$, is reduced by $\psi$, and the ex-ante probability of state $l$ increases by $\psi$. A simple way to introduce this change is to assume that the mass $\psi$ is subtracted from $\alpha_h$ and added to $\alpha_l$ on the central interval $[k, \frac{1+k}{2}]$, reducing both $q(x, 0)$ and $q(x, 1)$ on this interval. To render the trade-off between short- and long-term effort interesting, we assume that, from
the perspective of joint profit maximization, $\ell = 1$ is optimal (total expected benefits exceed the cost):

$$\psi \left( \frac{1}{\lambda_h} - \frac{1}{\lambda_l} \right) \geq c. \tag{26}$$

The entrepreneur will exert long-term effort only if his payoff increases by more than the cost of exerting the effort:

$$\psi (1 - s) \left( \frac{1}{\lambda_h} e^{-\lambda_h d} - \frac{1}{\lambda_l} e^{-\lambda_l d} \right) \geq c. \tag{27}$$

It is evident that if $a = 0$, a necessary condition for (27) to hold is $s < 1$. Thus, if (26) is satisfied, the venture capitalist will want to induce the entrepreneur to exert long-term effort, and will insist on a debt-equity contract with $s < 1$ that satisfies (27). The need to induce the entrepreneur to exert long-term effort thus provides the justification for our assumption in previous sections that in any debt-equity contract, the venture capitalist does not own one hundred percent of the project’s equity.\(^\text{40}\)

The analysis of this version of the model is almost analogous to that of previous sections, so to economize on space we will only sketch the main arguments. If $a = 0$ and $\ell = 1$, the socially efficient liquidation cut-off value is the same as $x'$ in previous sections, but is larger if $a = 0$ and $\ell = 0$ since the overall prospects of the project are less favorable. We use the following self-explanatory notation: $x'_{a=0,\ell=0} > x'_{a=1,\ell=1}$. Similarly, if $a = 1$ and $\ell = 0$, then $x'_{a=1,\ell=0} > x'_{a=1,\ell=1}$.

Condition (13) is preserved: without window dressing ($a = 0$) and with long-term effort ($\ell = 1$) the project will be financed. Moreover, we assume that with window dressing ($a = 1$) but without long-term effort ($\ell = 0$) the project will not be financed:\(^\text{41}\)

$$\int_0^1 \left\{ [1 - q_h] \frac{1}{\lambda_l} + q_h \frac{1}{\lambda_h} - I_2 \right\} \{ \alpha_h + \alpha_l \} \, dx < 0. \tag{28}$$

Conditions (13) and (28) together imply that the absence of signal manipulation is necessary for the project to be financed.

We obtain a result that is analogous to Proposition 1: with debt-equity financing only, the entrepreneur always exerts the short term effort and not the long term effort ($a = 1$ and $\ell = 0$), provided $\psi$ is low enough. We now sketch the proof. Given a contract $(d, s)$, with corresponding

\(^{40}\)An alternative to equity participation could be a wage contingent on long-term performance. The same problems of “window dressing” would then arise.

\(^{41}\)$q_h$, $\alpha_h$, and $\alpha_l$ denote $q_h(x, a = 1, \ell = 0)$, $\alpha_h(x, a = 1, \ell = 0)$, and $\alpha_l(x, a = 1, \ell = 0)$. 

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cut-off value for the venture capitalist, $x''_{a=0,\ell=1}$, if in equilibrium $a = 0$ and $\ell = 1$, the ex-ante payoff of the entrepreneur is

$$E[\mathcal{E}]_{a=0,\ell=1} = \frac{(1 - x''_{a=0,\ell=1})^2}{2} \frac{1}{\lambda_l} e^{-\lambda_id(1 - s)} + \frac{1 - (x''_{a=0,\ell=1})^2}{2} \frac{1}{\lambda_h} e^{-\lambda_hd(1 - s)} - c. \quad (29)$$

If (27) is satisfied, deviating to $a = \ell = 0$ is not worthwhile, but by deviating to $a = 1$ and $\ell = 0$ the entrepreneur obtains the payoff\(^{42}\)

$$E[\mathcal{E}]_{a=1,\ell=0} = \left[(1 - k)(k - x''_{a=0,\ell=1} + 1) - 1/2 + k^2 + \psi\right] \frac{1}{\lambda_l} e^{-\lambda_id(1 - s)}$$

$$+ \left[(1 - (x''_{a=0,\ell=1})^2)/2 - \psi\right] \frac{1}{\lambda_h} e^{-\lambda_hd(1 - s)} - c,$$

which is higher if $\psi$ is small enough. This is quite intuitive: if long-term effort is very important then window dressing may not be worthwhile, but if it is not, the entrepreneur will window dress reducing long-term profits. By assumption (28), the project will not be financed at time 0.

Convertible debt solves this problem in exactly the same manner as in previous sections. The conditions ensuring that the venture capitalist refinances the project without converting debt after $x_0$, and converts all the convertible debt after $x_1$, remain the same. The condition ensuring that in equilibrium $a = 1$ and $\ell = 0$ is

$$e^{\lambda_h \gamma(s - s_0)} \frac{1 - s}{1 - s_0} \leq 1 + \frac{1}{8} x_0 (x_0 - 2k) \frac{\lambda_h}{(1 - k)^2} e^{-(\lambda_l - \lambda_h)d_0} + \frac{\psi \left(1 - \frac{\lambda_h}{\lambda_l} e^{-(\lambda_l - \lambda_h)d_0}\right)}{1/8 (1 - k)^2}. \quad (31)$$

This condition is more easily satisfied than (23) because of the additional last term in the numerator, which is positive. This term represents the loss in expected profits due to the fact that the entrepreneur does not exert long-term effort. The intuition is clear: we have added an additional cost to manipulating a signal (the reduction in long-term profits) which makes it easier to induce the entrepreneur not to deviate from the desired behavior.

\(^{42}\)We write here the payoff only for the case in which $x''_{a=0,\ell=1} < k$. The other cases can be derived in a similar way.
7 Other specifications of the joint distribution of signal and output

Until now we focused on one particular way in which window dressing can affect the joint distribution of signal and output, the one presented in Figure 3. In this section we discuss the robustness of that particular form of signal manipulation. In particular, we show that, although some specific features were introduced in order to simplify the exposition, signal manipulation as presented in Figure 3 captures well the phenomenon of window dressing.

In Results 1 and 2, we showed that two (a priori reasonable) forms of signal manipulation do not capture the essence of window dressing. We reiterate that in these cases the problem is not whether our suggested convertible contract alleviates the conflict of interest between entrepreneur and venture capitalist, but rather that there is no such conflict to begin with.43

In the case described in Result 1 we introduced the possibility of signal jamming which consists of adding noise to the distribution. We showed that the venture capitalist need not worry about this type of signal manipulation because the entrepreneur never wants to engage in such manipulation. Signal jamming is a good representation of other economic phenomena, but it does not capture situations where the entrepreneur wants to pass a minimum performance threshold. In the case described in Result 2 we considered the opposite phenomenon—moving a given amount of mass from the tails to the center of the distribution—and showed that such manipulation is beneficial to both entrepreneur and venture capitalist. Again, since no conflict of interest arises, this type of signal manipulation does not capture situations where the entrepreneur wants to pass a minimum performance threshold.

The proof of Result 2 relies on the fact that, for low realizations of \( x \), the amount of mass that is shifted from left to right (\( \epsilon \)), relative to the density of the signal, is larger in state \( h \) than in state \( l \). This can be generalized: any type of signal manipulation that has this property does not create conflict of interest between entrepreneur and venture capitalist regarding the project’s liquidation because it improves the quality of information on which the decision to liquidate is based.

It follows that in order to create a conflict of interest between entrepreneur and venture capitalist regarding the project’s liquidation it is necessary that mass is shifted from left to

43 According to Sahlman (1990), conflict of interest between entrepreneurs and venture capitalists regarding refinancing decisions is a central reason for window dressing by entrepreneurs.
right and that the amount of mass shifted (relative to the density of the signal) is smaller in state $h$ than in state $l$. The case displayed in Figure 3 is an extreme case where no mass is shifted from left to right in state $h$. This feature is not crucial for our main result—it is simply easier to treat. The same result would go through with an intermediate situation, as displayed in Figure 5, where mass on the interval $[0, k]$ is shifted to the right in both states. (It can be shown that $q_h$ changes as presented in the figure.) In this case, signal manipulation hurts the venture capitalist and is a good representation of window dressing. Our analysis and main result would go through but we prefer to focus on the case displayed in Figure 3 (a limit case) because it saves the need to verify algebraically the conditions on parameters ensuring that signal manipulation is actually harmful to the venture capitalist.

The discussion so far established that to capture the essence of window dressing mass should be moved from the left of the refinancing/liquidation cut-off value of $x$ to its right so that the information content of low values of $x$ decreases. It still remains to be determined where to the right this mass should be moved. In Figure 3, no mass is moved to the interval $[(1 + k)/2, 1]$ in state $l$. Again, this is an extreme case that simplifies the computations, but may be attenuated so that some mass is moved to the interval $[(1 + k)/2, 1]$ in both states. As long as $q_h$ does not decrease for high values of $x$, our result goes through.

If, instead, $q_h$ decreases for high values of $x$, convertible debt may still be effective in preventing window dressing as long as the decrease in $q_h$ is not substantial (which depends on parameter values). Notice that for $q_h$ to decrease substantially for high values of $x$, it is necessary that more mass (relative to the density of the signal) is moved to high values of $x$ in state $l$ than in state $h$. The economic interpretation is quite intuitive: if the entrepreneur succeeds in increasing substantially the performance of “bad” projects without increasing by too much the performance of “good” projects, the venture capitalist is hurt when he converts securities into equity following a good signal realization, and there is no threat that prevents window dressing. While we cannot exclude this possibility a priori, we believe that it is not very realistic since it should be easier to improve the interim performance of a project that is fundamentally good than of a project that is fundamentally poor.

Finally, we present a case that is similar in all respects to the case presented in Figure 3 with the only difference that when $a = 1$, the signal is informative everywhere. This case illustrates that the discontinuity in $q_h(x, 1)$ at $x = k$ is not instrumental for our main result. Consider Figure 6. This type of window dressing represents a situation where, when the project is “bad,”
the probability of observing different values of the signal is constant on a large interval (from 0 to 2/3). In other words, the entrepreneur succeeds in manipulating the short-term performance so that, when the project is “bad,” poor interim performance is as likely as medium performance. The analysis is very similar to that of previous sections and is omitted. We can show that a convertible debt contract can be constructed such that in equilibrium the venture capitalist liquidates if $x < \frac{1}{3}$, refinances but does not convert any debt after observing $\frac{1}{3} \leq x < \frac{2}{3}$, refinances and converts all the convertible debt after observing $x \geq \frac{2}{3}$, and the entrepreneur does not window dress.\textsuperscript{44}

8 Concluding remarks

We have illustrated that convertible securities dominate a simple mix of debt and equity in stage financing situations. When the venture capitalist retains the option to abandon the project if in the medium term he receives bad news, the entrepreneur has an incentive to engage in “window dressing,” or short-termism, and bias the process that generates these interim news in order to reduce the probability that the project is liquidated. Convertible debt reduces the incentive to engage in such short-termism through the threat of debt conversion.

The fundamental role of the conversion option is to adjust the ownership (debt-equity) structure to new information. This feature resembles provisions allowing the provider of financing to alter the financial structure of an existing contract at some interim stage in response to nonverifiable information, a feature that is very common in incomplete contracts environments. The need to commit in advance to the terms of debt conversion is in line with the logic of Aghion, Dewatripont, and Rey (1994) who show that by specifying in the contract the bargaining procedure to be used in future renegotiation, an efficient outcome can be achieved.

Venture capital convertible debt contracts sometimes incorporate, in addition to the voluntary debt conversion, automatic conversion agreements.\textsuperscript{45} In such situations, where the signal that triggers conversion is verifiable, there is no gain from endowing the financier with the power to unilaterally alter the debt-equity structure by converting debt. In situations where signals

\textsuperscript{44}It is possible to generalize this example so that the cut-off values of $x$ are parameters (rather than the specific values $1/3$ and $2/3$).

\textsuperscript{45}Gompers (1996, Table 4) documents the use of such agreements, but Kaplan and Strömberg (2001) report that, in their sample of venture capital financing contracts, automatic conversion is contingent only on an initial public offering (IPO) taking place at certain conditions, but never on other verifiable interim information such as sales revenue, profitability, or market share.
are not verifiable, authority to unilaterally change the terms of the contract is crucial. Our model captures well the contingent financial structure aspect that is relevant for both types of situations, and is especially suitable for settings where interim signals are not commonly and immediately verifiable.

We conclude by discussing some of the implications and limitations of our 3-period set-up. In our model, if the venture capitalist does not convert at time 1, he loses the option to do so forever (so, in fact, this is as if the convertible debt has been bought back by the entrepreneur at face value). We can think of a set-up with more than three periods where window dressing can occur at various stages where additional financing is needed. The venture capitalist can decide not to convert debt at a given stage but still keeping the option to do so in a future financing round. In such a multi-stage set-up, the possibility of issuing new securities at interim stages would naturally arise. These are important extensions that should be studied in future work.
Appendix 1: Proof of Proposition 2

Step 1. Fix $s_0 < 1$. The value of $s_0$ will not change throughout the proof.\(^46\)

Step 2. Given $s_0$, we impose the following condition on $\gamma$, $s$, and $d_0$:

\[
\frac{1-k}{2} \frac{1}{\lambda_1} \left[ 1 - e^{-\lambda_1 d_0} (1 - s_0) \right] + \frac{1+k}{2} \frac{1}{\lambda_2} \left[ 1 - e^{-\lambda_2 d_0} (1 - s_0) \right] = \\
\frac{1-k}{2} \frac{1}{\lambda_1} \left[ 1 - e^{-\lambda_1 [d_0 - \gamma(s - s_0)]} (1 - s) \right] + \frac{1+k}{2} \frac{1}{\lambda_2} \left[ 1 - e^{-\lambda_2 [d_0 - \gamma(s - s_0)]} (1 - s) \right].
\]

(32)

This is precisely condition (21) in the main text that states that the signal realization $x = (1 + k)/2$ renders the venture capitalist indifferent between not converting debt and converting the entire amount of convertible debt. In step 8, we will show that, for any signal realization, the venture capitalist’s interim payoff is maximized either by converting no debt or by converting the full amount of convertible debt available. Therefore, if $x \geq (1 + k)/2$, the venture capitalist converts the entire amount of debt, and if $x < (1 + k)/2$ he converts no debt.

Dividing both sides of (32) by $(1 - s_0)$, solving for $(1 - s)/(1 - s_0)$, and dividing the numerator and denominator of the right hand side by $e^{-\lambda_d d_0}$ yields

\[
\frac{1 - s}{1 - s_0} = \frac{\frac{1-k}{2} \frac{1}{\lambda_1} e^{-(\lambda_1 - \lambda_2) d_0} - \frac{1+k}{2} \frac{1}{\lambda_2} e^{-(\lambda_1 - \lambda_2) d_0}}{\frac{1-k}{2} \frac{1}{\lambda_1} e^{-(\lambda_1 - \lambda_2) d_0} - \frac{1+k}{2} \frac{1}{\lambda_2} e^{-(\lambda_1 - \lambda_2) d_0}}.
\]

(33)

This equation defines an implicit relation between $\gamma$, $s$, and $d_0$. Throughout the proof, we will require that this equality is satisfied. In particular, any argument involving a change in $\gamma$ and $d_0$ entails a corresponding change in $s$ so that (33) is satisfied and, consequently, the debt conversion cut-off value remains $x_1 = (1 + k)/2$.

Step 3. We establish that along any sequence such that $\gamma \to \infty$, $d_0 \to \infty$, and (33) holds, $s \to 1$. This is true because as $d_0 \to \infty$, the first term in the numerator of the right hand side of (33) approaches zero (since $\lambda_1 > \lambda_2$) and, therefore, the numerator approaches the constant $\frac{1+k}{2} \frac{1}{\lambda_2}$, while the denominator approaches infinity.\(^47\)

Step 4. To construct a convertible debt contract that prevents window dressing, we increase $\gamma$ and $d_0$ so that: (a) $\gamma \to \infty$; (b) $d_0 \to \infty$; (c) condition (33) is satisfied along the sequence; and (d) the expression $d_0 - \gamma(s - s_0)$ approaches a finite non-negative number.\(^48\)

---

\(^46\) As mentioned in the main text, this illustrates that the result in Proposition 2 does not rely on “razor’s edge” arguments, and that there is considerable leeway in constructing convertible debt contracts that prevent window dressing.

\(^47\) The second term in the denominator approaches infinity as $\gamma \to \infty$, and the first term is bounded below by zero.

\(^48\) This is easily accomplished. Consider a sequence such that $\gamma \to \infty$, $d_0 \to \infty$, and (33) holds. Since along the sequence, $s \to 1$, the limit of $d_0 - \gamma(s - s_0)$ is the same as the limit of $d_0 - \gamma(1 - s_0)$. There are many sequences...
Step 5. We establish that if $\gamma$ and $d_0$ increase as in step 4, then $\gamma(1-s) \to 0$, a fact that will be used later in the proof. To see this, multiply both sides of (33) by $e^{\lambda_k \gamma(s-s_0)}$ and then multiply the numerator and denominator of the right hand side by $e^{-\lambda_k d_0}$, to obtain

\[
e^{\lambda_k \gamma(s-s_0)} \frac{1-s}{1-s_0} = \frac{\frac{1-k}{2} \frac{1}{\lambda_k} e^{-\lambda_k d_0 - \gamma(s-s_0)}}{\frac{1-k}{2} \frac{1}{\lambda_k} e^{-\lambda_k d_0 - \gamma(s-s_0)} + \frac{1+k}{2} \frac{1}{\lambda_k} e^{-\lambda_k d_0 - \gamma(s-s_0)}} + \frac{1+k}{2} \frac{1}{\lambda_k} e^{-\lambda_k d_0 - \gamma(s-s_0)}
\]

which is condition (22) in the main text. First, we establish that the right hand side approaches a finite limit as $\gamma \to \infty$ and $d_0 \to \infty$. By step 4, $d_0 - \gamma(s-s_0)$ approaches a finite limit and, therefore, both terms in the denominator of the right hand side as well as the second term in the numerator on the right hand side approach a finite limit. By contrast, the first term in the numerator on the right hand side approaches zero. Thus, denoting $z$ the (strictly positive finite) limit of $d_0 - \gamma(s-s_0)$, the right hand side of (34) approaches the finite limit

\[
0 < \frac{\frac{1-k}{2} \frac{1}{\lambda_k} e^{-\lambda_k z}}{\frac{1-k}{2} \frac{1}{\lambda_k} e^{-\lambda_k z} + \frac{1+k}{2} \frac{1}{\lambda_k} e^{-\lambda_k z}} < 1.
\]

To preserve the equality, so must the left hand side of (34) implying that $s \to 1$ at the same rate at which $e^{\lambda_k \gamma(s-s_0)} \to \infty$ and, therefore, $s \to 1$ at a faster rate than that at which $\gamma \to \infty$. Thus, $\gamma(1-s) \to 0$ as $\gamma \to \infty$.

Step 6. Denote $x_0$ the liquidation/refinancing cut-off value. This cut-off value changes when $d_0$ changes. The entrepreneur's ex-ante payoff with $a = 0$ is greater than with $a = 1$ (when the venture capitalist believes that $a = 0$) if the following condition is satisfied:

\[
\begin{align*}
\int_{x_0}^{1+k} (1-x) \frac{1}{\lambda_k} e^{-\lambda_k d_0} (1-s_0) dx &+ \int_{x_0}^{1+k} x \frac{1}{\lambda_k} e^{-\lambda_k d_0} (1-s_0) dx \\
\int_{1+k}^1 (1-x) \frac{1}{\lambda_k} e^{-\lambda_k [d_0 - \gamma(s-s_0)]} (1-s) dx &+ \int_{1+k}^1 x \frac{1}{\lambda_k} e^{-\lambda_k [d_0 - \gamma(s-s_0)]} (1-s) dx \\
\geq & \int_k^{1+k} (1-k) \frac{1}{\lambda_k} e^{-\lambda_k d_0} (1-s_0) dx + \int_k^{1+k} x \frac{1}{\lambda_k} e^{-\lambda_k d_0} (1-s_0) dx \\
\int_k^{1+k} \left(1 - x + \frac{k^2}{1-k^2} \right) \frac{1}{\lambda_k} e^{-\lambda_k d_0} (1-s_0) dx &+ \int_k^{1+k} k \frac{1}{\lambda_k} e^{-\lambda_k d_0} (1-s_0) dx \\
\int_{1+k}^1 (1-x) \frac{1}{\lambda_k} e^{-\lambda_k [d_0 - \gamma(s-s_0)]} (1-s) dx &+ \int_{1+k}^1 \frac{1}{\lambda_k} e^{-\lambda_k [d_0 - \gamma(s-s_0)]} (1-s) dx,
\end{align*}
\]

of $d_0$ and $\gamma$ such that this expression approaches a finite limit (e.g., if $d_0(n) = n$ and $\gamma(n) = (n-2)/(1-s_0)$, then $d_0(n) - \gamma(n)(1-s_0) = n - \frac{n^2}{1-s_0} (1-s_0) = 2$).
which, after manipulation, yields
\[
e^{\lambda_h \gamma(s-s_0)} \frac{1 - s}{1 - s_0} \leq 1 + \frac{x_0(x_0 - 2k)}{h} \frac{\lambda_h}{\lambda_l} e^{-(\lambda_l - \lambda_h)d_0},
\]
(37)
which is equation (23) in the main text.

As \( \gamma \to \infty \) and \( d_0 \to \infty \), the second term on the right hand side of (37) approaches 0 (notice that \( x_0 \) changes along the sequence but is bounded so it does not affect the limit of the right hand side). Therefore, the right hand side approaches 1. From step 5, the left hand side approaches a limit that is strictly smaller than 1 (the left hand side of (37) is also the left hand side of (34)), implying that far enough along the sequence the inequality (37) is (strictly) satisfied, namely, the entrepreneur does not gain by deviating from \( a = 0 \) to \( a = 1 \)

**Step 7.** We now establish two simple auxiliary results that will be used in the next step. In step 4, we showed that as \( \gamma \to \infty, \gamma(1-s) \to 0 \). An implication of this fact is that for \( \gamma \) large enough, \( \frac{1}{\lambda_h} - \gamma(1-s) > \frac{1}{\lambda_l} > \frac{1}{\lambda_h} \). In addition, for \( \gamma \) large enough, \( \frac{1}{\lambda_l} - \gamma(1-s_0) < \frac{1}{\lambda_l} - \gamma(1-s_0) < 0 \) (since \( s_0 \) is fixed and \( \gamma \to \infty \)).

**Step 8.** The venture capitalist decides how much debt to convert, thus determining his post-ownership equity stake, \( \hat{s} \in [s_0, \bar{s}] \). We will now show that after any signal realization, \( x \), the venture capitalist will convert no debt (\( \hat{s} = s_0 \)) or will convert all the convertible debt (\( \hat{s} = \bar{s} \)).

The venture capitalist’s interim payoff as a function of \( \hat{s} \) and \( x \) is:
\[
[1 - q_h(x, a)] \frac{1}{\lambda_l} [1 - e^{-\lambda_l[d_0 - \gamma(\hat{s} - s_0)](1 - \hat{s})}] + q_h(x, a) \frac{1}{\lambda_h} [1 - e^{-\lambda_h[d_0 - \gamma(\hat{s} - s_0)](1 - \hat{s})}] - I_2.
\]
The first derivative of the venture capitalist’s interim payoff with respect to \( \hat{s} \) is
\[
[1 - q_h(x, a)] \frac{1}{\lambda_l} \lambda_l e^{-\lambda_l[d_0 - \gamma(\hat{s} - s_0)]} \left[ \frac{2}{\lambda_l} - \gamma(1 - \hat{s}) \right] + q_h(x, a) \frac{1}{\lambda_h} \lambda_h e^{-\lambda_h[d_0 - \gamma(\hat{s} - s_0)]} \left[ \frac{2}{\lambda_h} - \gamma(1 - \hat{s}) \right],
\]
(38)
and the second derivative of this payoff with respect to \( \hat{s} \) is
\[
[1 - q_h(x, a)] \frac{1}{\lambda_l} \lambda_l^2 \gamma e^{-\lambda_l[d_0 - \gamma(\hat{s} - s_0)]} \left[ \frac{2}{\lambda_l} - \gamma(1 - \hat{s}) \right] + q_h(x, a) \frac{1}{\lambda_h} \lambda_h^2 \gamma e^{-\lambda_h[d_0 - \gamma(\hat{s} - s_0)]} \left[ \frac{2}{\lambda_h} - \gamma(1 - \hat{s}) \right].
\]
(39)

By step 7, we know that for \( \gamma \) large enough, \( \frac{1}{\lambda_l} - \gamma(1-s_0) < 0 \) and \( \frac{1}{\lambda_l} - \gamma(1-s) > 0 \), implying that there exists \( \bar{s} \in (s_0, \bar{s}) \) for which \( \frac{1}{\lambda_l} - \gamma(1-s) = 0 \). Analogously, there exists \( \underline{s} \in (s_0, s) \) for which \( \frac{1}{\lambda_l} - \gamma(1-s) = 0 \). Moreover, \( \underline{s} < \bar{s} \) (since \( \lambda_l > \lambda_h \)). Consequently, \( \frac{1}{\lambda_l} - \gamma(1-s) \) evaluated at \( \underline{s} \) is strictly negative and \( \frac{1}{\lambda_l} - \gamma(1-s) \) evaluated at \( \bar{s} \) is strictly positive. It follows that the first derivative of the venture capitalist’s interim payoff is strictly negative to the left of \( \underline{s} \), strictly positive to the right of \( \bar{s} \),

\[\text{This is the same expression as in equation (9).}\]
and zero at a value $s^* \in (\bar{s}, \bar{s})$.

An analogous set of arguments applies to the second derivative of the venture capitalist’s interim payoff, which is strictly negative to the left of $s'$, strictly positive to the right of $\bar{s}'$, zero at a value $s^{**} \in (s', \bar{s}')$, where $s'$ and $\bar{s}'$ satisfy $\frac{2}{\lambda_l} - \gamma (1 - s') = 0$ and $\frac{2}{\lambda_h} - \gamma (1 - \bar{s}') = 0$.

From $\frac{2}{\lambda_l} - \gamma (1 - s') = 0$ and $\frac{1}{\lambda_h} - \gamma (1 - \bar{s}') = 0$ (see assumption (1)), we obtain $\bar{s}' < s$. Therefore, the interval $(s', \bar{s}')$ lies to the left of the interval $(s, \bar{s})$, with no overlap. Thus, when the venture capitalist’s interim payoff function becomes flat, at $s^* \in (s, \bar{s})$, the second derivative of this function is strictly positive.

It follows that the interim payoff of the venture capitalist, as a function of his post-conversion equity stake, achieves a unique interior minimum on the interval $(s_0, s)$. Therefore, the function is maximized either at $s_0$—“no debt conversion,” or at $s$—“full debt conversion,” and the formulation of conditions (32) and (37) is consistent with optimization by the venture capitalist.

Step 9. We now turn to the venture capitalist’s ex-ante participation constraint. To show that this constraint is satisfied, it is sufficient to show that his payoff from the debt-equity contract $(d_0, s_0)$ is positive, since the option to convert can only increase this payoff. We will show that as $\gamma \to \infty$ and $d_0 \to \infty$, this payoff becomes strictly positive, ensuring that the venture capitalist will provide financing at time 0.

Consider the ex-ante payoff of the venture capitalist from a debt-equity contract $(d_0, s_0)$, when $a = 0$ and with the refinancing/liquidation cut-off $x_0$. This payoff is computed in an analogous way to the total ex-ante surplus in (13), and is given by

$$\int_{x_0}^{1} \left[ (1 - x) \frac{1}{\lambda_l} [1 - e^{-\lambda_l d_0} (1 - s_0)] + x \frac{1}{\lambda_h} [1 - e^{-\lambda_h d_0} (1 - s_0)] - I_2 \right] dx - I_1 , \tag{40}$$

where $x_0$ is determined by

$$[1 - q(x_0, 0)] \frac{1}{\lambda_l} [1 - e^{-\lambda_l d_0} (1 - s_0)] + q(x_0, 0) \frac{1}{\lambda_h} [1 - e^{-\lambda_h d_0} (1 - s_0)] - I_2 = 0. \tag{41}$$

As $d_0 \to \infty$, $1 - e^{-\lambda d_0} (1 - s_0) \to 1$. Therefore, by equations (41) and (11), $x_0$ approaches $x'$ (the socially efficient cut-off value of $x$). By the strict inequality in (13)—namely, by the assumption that the total ex-ante surplus with $a = 0$ and efficient liquidation is strictly positive)—far enough along the sequence, the expression in (40) is strictly positive. □

Appendix 2: Proof of Proposition 3

Step 1. Fix $s_0 < 1$. The value of $s_0$ will not change throughout the proof.

Step 2. Choose $d_0$ and a corresponding refinancing/liquidation cut-off, $x_0$—the value of $x$ that makes the
venture capitalist indifferent between liquidating and refinancing given the debt-equity contract \((d_0, s_0)\)—so that: (1) \(x_0 < k\); and (2) without window dressing, the ex-ante payoff of the venture capitalist is strictly positive. The values of \(d_0\) and \(x_0\) will not change throughout the proof. To show that this can be done, consider the venture capitalist’s ex-ante payoff from a debt-equity contract \((d, s)\), when \(a = 0\), and with the refinancing/liquidation cut-off, \(x_0\),\(^{51}\)

\[
\int_{x_0}^{1} \left[ (1 - x) \frac{1}{\lambda_l} [1 - e^{-\gamma d}(1 - s)] + x \frac{1}{\lambda_h} [1 - e^{-\gamma d}(1 - s)] - I_2 \right] dx - I_1
\]

where \(x_0\) is determined by

\[
[1 - x_0] \frac{1}{\lambda_l} [1 - e^{-\gamma d}(1 - s)] + x \frac{1}{\lambda_h} [1 - e^{-\gamma d}(1 - s)] - I_2 = 0.
\]

Let \(s = s_0\) (step 1). As \(d\) increases, \(1 - e^{-\gamma d}(1 - s)\) approaches unity. From equations \((42), (43), (13),\) and \((11)\) it follows that \(x_0\) approaches \(x'\) which, by assumption, is smaller than \(k\). Thus, there is a value \(d_0'\) that is large enough so that, for all \(d_0 \geq d_0'\), the corresponding cut-off \(x_0\) is smaller than \(k\). Moreover, by the assumption that the total ex-ante surplus with \(a = 0\) and efficient liquidation is strictly positive—see \((13)\)—the expression in \((42)\) is strictly positive.

**Step 3.** Given \(s_0\) and \(d_0\), we impose the following condition on \(\gamma\) and \(s\):

\[
\frac{1-k}{2} \frac{1}{\lambda_l} [1 - e^{-\gamma (d_0 s_0)}] + \frac{1+k}{2} \frac{1}{\lambda_h} [1 - e^{-\gamma d_0 s_0}] = \frac{1-k}{2} \left( s \frac{1}{\lambda_l} + (1-s) [d_0 - \gamma (s - s_0)] \right) + \frac{1+k}{2} \left( s \frac{1}{\lambda_h} + (1-s) [d_0 - \gamma (s - s_0)] \right).
\]

This condition states that the signal realization \(x = (1 + k)/2\) renders the venture capitalist indifferent between not converting debt, and converting the entire amount of convertible debt and exercising all the warrants.\(^{52}\) Therefore, if \(x \geq (1 + k)/2\), the venture capitalist converts the entire amount of debt and exercises all the warrants, and if \(x < (1 + k)/2\) he converts no debt and exercises no warrants.

\(^{50}\)In the proof of Proposition 2, only \(s_0\) was fixed in advance and, as \(\gamma\) increased, \(d_0\) increased along with it to ensure that the post-conversion debt, \(d_0 - \gamma (s - s_0)\), remains positive. As \(d_0\) increased, \(x_0\) had to be adjusted to render the venture capitalist indifferent between refinancing and liquidating. Here, there is no requirement that \(d_0 - \gamma (s - s_0)\) remain positive, allowing us to fix \(s_0, d_0\), and a corresponding \(x_0\) at the outset. This highlights that convertible debt cum warrants financing provides more degrees of freedom in the design of the contract, making it “easier” to prevent window dressing.

\(^{51}\)These equations are analogous to \((40)\) and \((41)\) which were written for a specific contract \((d_0, s_0)\), whereas \((42)\) and \((43)\) are written for a generic contract \((d, s)\).

\(^{52}\)The interim payoff on the right hand side of \((44)\) uses the ex-post payoffs in \((24)\) evaluated at \(s = \hat{s}\) and \(d = \hat{d}\) with \(d = d_0 - \gamma (s - s_0)\). As in the proof of Proposition 2, we will show that, for any signal realization, the venture capitalist’s interim payoff is maximized either by converting no debt or by converting the full amount of convertible debt and exercising all the warrants; see step 7.
Manipulation yields
\[
\frac{1 - s}{1 - s_0} = \frac{\frac{1-k}{\lambda_1} e^{-\lambda_1 d_0} + \frac{1+k}{\lambda_0} e^{-\lambda_0 d_0}}{\frac{1-k}{\lambda_1} + \frac{1+k}{\lambda_0} - d_0 + \gamma(s - s_0)},
\]
which defines an implicit relation between \(\gamma, s, s_0,\) and \(d_0\). Throughout the proof, we will require that this equality be satisfied. In particular, any argument involving a change in \(\gamma\) (given \(s_0\) and \(d_0\) entails a change in \(s\) so that (33) is satisfied and, consequently, the debt conversion cut-off value remains \((1+k)/2\).

**Step 4.** Along any sequence such that \(\gamma \to \infty\) and (45) holds, \(s \to 1\). This is because all the terms on the right hand side of (45) are constant except \(\gamma\) so the right hand side approaches zero as \(\gamma \to \infty\).

**Step 5.** Next, we establish that \(\gamma(1-s)\) approaches a strictly positive finite limit. This can be seen by multiplying both sides of (45) by \(\gamma(s-s_0)\), obtaining
\[
\gamma(s-s_0) \frac{1 - s}{1 - s_0} = \gamma(s-s_0) \left( \frac{\frac{1-k}{\lambda_1} e^{-\lambda_1 d_0} + \frac{1+k}{\lambda_0} e^{-\lambda_0 d_0}}{\frac{1-k}{\lambda_1} + \frac{1+k}{\lambda_0} - d_0 + \gamma(s - s_0)} \right).
\]
As \(\gamma \to \infty\), the right hand side approaches the strictly positive finite limit \(\frac{\frac{1-k}{\lambda_1} e^{-\lambda_1 d_0} + \frac{1+k}{\lambda_0} e^{-\lambda_0 d_0}}{\frac{1-k}{\lambda_1} + \frac{1+k}{\lambda_0} - d_0} \equiv X\), and so must the left hand side. Taking the limit of the left hand side, we have \(\lim_{\gamma \to \infty} \gamma(s-s_0) \frac{1 - s}{1 - s_0} = \lim_{\gamma \to \infty} \gamma(1-s)\lim_{\gamma \to \infty} \frac{s-s_0}{s-1} = X\). As \(\gamma \to \infty\), \(s \to 1\), implying \(\lim_{\gamma \to \infty} \frac{s-1}{s-1} = 1\) and, therefore, \(\lim_{\gamma \to \infty} \gamma(1-s) = X\) which is strictly positive and finite. (This means that \(s \to 1\) at the same rate as \(\gamma \to \infty\).)

**Step 6.** The entrepreneur’s ex-ante payoff with \(a = 0\) is greater than with \(a = 1\) (when the venture capitalist believes that \(a = 0\)) if the following condition is satisfied:
\[
\frac{1}{x_0} \int_{x_0}^{1-k} (1-x) \frac{1}{\lambda_1} e^{-\lambda_1 d_0} (1-s_0) \, dx + \int_{x_0}^{1-k} x \frac{1}{\lambda_0} e^{-\lambda_0 d_0} (1-s) \, dx
\]
\[
+ \int_{x_0}^{1-k} (1-x) \int_{x_0}^{1-k} (1-x) \left[ (1-s) \frac{1}{\lambda_1} - (1-s) \right] \, dx + \int_{x_0}^{1-k} x \left[ (1-s) \frac{1}{\lambda_1} - (1-s) \right] \, dx
\]
\[
\geq \frac{1}{k} \int_{x_0}^{1-k} (1-k) \frac{1}{\lambda_1} e^{-\lambda_1 d_0} (1-s_0) \, dx + \int_{x_0}^{1-k} x \frac{1}{\lambda_0} e^{-\lambda_0 d_0} (1-s_0) \, dx
\]
\[
+ \int_{x_0}^{1-k} (1-x) \left[ (1-s) \frac{1}{\lambda_1} - (1-s) \right] \, dx + \int_{x_0}^{1-k} x \left[ (1-s) \frac{1}{\lambda_1} - (1-s) \right] \, dx,
\]
which, after manipulation and multiplication both sides by \(\gamma(s-s_0)\), yields
\[
\gamma(s-s_0) \frac{1-s}{1-s_0} \leq \frac{\gamma(s-s_0)}{\frac{1}{2} x_0(x_0-2k) \frac{1}{\lambda_1} e^{-\lambda_1 d_0} + \frac{1}{8} (1-k)^2 \frac{1}{\lambda_0} e^{-\lambda_0 d_0}}.
\]
From step 5, it follows that, as \( \gamma \to \infty \), the left hand side of (48) approaches \( \frac{1+k}{2} \frac{1}{\lambda} e^{-\lambda d_0} + \frac{1+k}{2} \frac{1}{\lambda} e^{-\lambda d_0} \) (in (46) and (48) the left hand side is the same). The right hand side of (48) approaches \( \frac{2}{k} \left[ \frac{1}{(1-k)^2} \right] \frac{1}{\lambda} e^{-\lambda d_0} + \frac{1}{\lambda} e^{-\lambda d_0} \). It is easily verified that, in the limit, the right hand side exceeds the left hand side if \( d_0 \geq \frac{1}{\lambda - \lambda_0} \log \left[ \frac{1+k}{2} \frac{1}{\lambda} e^{-\lambda d_0} + \frac{1+k}{2} \frac{1}{\lambda} e^{-\lambda d_0} \right] \equiv d_0' \). For all \( d_0 \geq d_0' \), the entrepreneur will not window dress.

**Step 7.** We will now show that after any signal realization, the venture capitalist’s interim payoff is maximized either by converting no debt or by converting the full amount of convertible debt and exercising all the warrants.

Denote \( s^* \) the value of \( \hat{s} \) such that all the convertible debt is converted but no warrants are exercised, that is, \( d_0 - \gamma (s^* - s_0) = 0 \), or

\[
\hat{s}^* = s_0 + \frac{d_0}{\gamma}.
\]

(49)

If debt is converted but no warrants are exercised, then \( d_0 - \gamma (\hat{s} - s_0) \geq 0 \) (\( \hat{s} \leq s^* \)), and the venture capitalist’s interim payoff is\(^{53}\)

\[
f(\hat{s}) = (1 - x) \frac{1}{\lambda} \left[ 1 - e^{-\lambda \gamma (\hat{s} - s_0)} (1 - \hat{s}) \right] + x \frac{1}{\lambda h} \left[ 1 - e^{-\lambda h \gamma (\hat{s} - s_0)} (1 - \hat{s}) \right] - I_2.
\]

If the full amount of convertible debt is converted and warrants are exercised, then \( d_0 - \gamma (\hat{s} - s_0) < 0 \) (\( \hat{s} > s^* \)), and the venture capitalist’s interim payoff is\(^{54}\)

\[
g(\hat{s}) = (1 - x) \left\{ \hat{s} \frac{1}{\lambda h} + (1 - \hat{s}) \left[ d_0 - \gamma (\hat{s} - s_0) \right] \right\} + x \left\{ \hat{s} \frac{1}{\lambda h} + (1 - \hat{s}) [d_0 - \gamma (\hat{s} - s_0)] \right\}.
\]

(50)

We need the following auxiliary facts: (1) \( f(s^*) = g(s^*) \), namely, the venture capitalist’s interim payoff is continuous in \( \hat{s} \) on the interval \([s_0, 1] \);\(^{55}\) (2) \( g(\hat{s}) \) is a parabola that achieves its minimum at \( \hat{s} = \left\{ \gamma (1 + s_0) + d_0 - \left( (1 - x) \frac{1}{\lambda h} + x \frac{1}{\lambda h} \right) \right\} / 2 \gamma \),\(^{56}\) implying that, as \( \gamma \to \infty \), \( \hat{s} \to (1 + s_0) / 2 \); (4) as \( \gamma \to \infty \), \( s^* \to s_0 \); see (49); (5) for \( \gamma \) large enough, \( f'(\hat{s}) < 0 \) in the interval \([s_0, \hat{s}] \) (where \( \hat{s} < 1 \)).\(^{57}\)

Fact (5) implies that, for \( \gamma \) large enough, the venture capitalist’s interim payoff is strictly decreasing in \( \hat{s} \) as he begins converting debt. All the debt is converted when \( \hat{s} = s^* \). By fact (4), for \( \gamma \) large enough, \( s^* < (1 + s_0) / 2 \). Fact (2) establishes that \( g(\hat{s}) \) is a parabola that achieves its minimum at \( \hat{s} = (1 + s_0) / 2 \) which means that the venture capitalist’s interim payoff decreases further as he begins to exercise warrants,\(^{58}\) and begins to rise if enough warrants are exercised, at \( \hat{s} = (1 + s_0) / 2 \). \( \hat{s} \)From that point on, the payoff keeps increasing as more warrants are exercised until \( \hat{s} \) reaches its maximal value, \( s \).

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\(^{53}\)This is the same expression as in equation (9).

\(^{54}\)This expression builds on equation (24).

\(^{55}\)This is obtained by simple substitution.

\(^{56}\)This is easily seen by collecting terms: \( g(\hat{s}) = d_0 + \left( (1 - x) \frac{1}{\lambda h} + x \frac{1}{\lambda h} - \gamma (1 - s_0) \right) \hat{s} + \gamma \hat{s}^2 \).

\(^{57}\)This fact was established Appendix 1, step 8. \( \text{(There, equation (38) is the expression for } f'(\hat{s}).) \)

\(^{58}\)In fact, the interim payoff is continuous at \( s^* \)—fact (1)—but the derivative of the payoff is not.
Thus, the venture capitalist’s interim payoff is maximized either at $\hat{s} = s_0$ or at $\hat{s} = s$.

**Step 8.** It remains to be shown that the venture capitalist’s ex-ante participation constraint is satisfied. As in the proof of Proposition 2 (see step 9), it is sufficient to show that his payoff from the debt-equity contract $(d_0, s_0)$ is positive, since the option to convert debt and exercise warrants can only increase this payoff. The payoff is the same as in equation (40). As $d_0 \to \infty$, this payoff becomes strictly positive. Thus, there is a value $d''_0$ that is large enough so that, for all $d_0 \geq d''_0$, the venture capitalist’s ex-ante participation constraint is satisfied.

**Step 9.** We design the contract so that $d_0 \geq \max(d'_0, d''_0, d''_0)$ (see steps 2, 6, and 8).

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59 The determination of $x_0$ is the same as in equations (41) and (43).

60 In step 2, $d_0$ was chosen so that $x_0 < k$; in step 6, $d_0$ was chosen so that the entrepreneur will not window dress; and in step 8, $d_0$ was chosen so that the venture capitalist’s ex-ante participation constraint is satisfied.
References


Figure 1: The joint distribution of signal and output
Figure 2: (a) Signal jamming; (b) The opposite of signal jamming
Figure 3: Window dressing: (a) the shift of mass in states \( l \) and \( h \); (b) the effect on the conditional probability \( q_h \); the figure on the right is an enlargement of the figure on the left, on the interval \( x \in [0, k] \).
(a): $a = 0$

(b): $a = 1$

Figure 4: Interim payoffs as a function of the signal realization
Figure 5: An alternative form of signal manipulation that is desirable to both venture capitalist and entrepreneur
Figure 6: An alternative form of signal manipulation with a strictly informative signal