The Value of Vesting Provisions

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Abstract

Employee stock options and other forms of deferred compensation typically include provisions stipulating forfeiture upon termination of employment prior to some date. This paper considers the impact of such vesting provisions on the value of the deferred claim. The answer depends on the full specification of the employment contract to the vesting horizon. In typical contracts, the value surrendered in the vesting provision is equal to the value of the claim, rendering the benefit worthless to the employee. Used in this way, vesting provisions destroy any incentive effects the benefit may have been intended to produce. They can also be inefficient, negating any savings of long-term contracting and potentially leading to underinvestment. We characterize optimal explicit and implicit contracts that might overcome these problems, and outline empirical implications that would distinguish the hypothesis that their efficient use is the exception rather than the rule.

1 Introduction

An employee assessing the value of a long-term compensation plan faces a difficult problem in quantifying the effect of vesting provisions upon the nominal value of promised benefits. Vesting provisions — the requirement that the employee remain

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with the firm until some future time $T$ in order to attain title to the benefit— are nearly universally attached to employee stock options, long-term share purchase plans, and pension rights as a way to promote loyalty. From the employee’s perspective, however, they clearly subtract value from the benefit by making the payoffs contingent on forgoing outside opportunities. The problem might then seem to require quantifying the rather intangible value of the option to pursue any of life’s future paths.

The difficulty of formulating a satisfactory approach to this issue has led to a dearth of useful research on vesting, this despite the widespread interest in all aspects of the compensation design problem. Recent important work has tackled several other challenging dimensions of valuing complex employment packages, including the negative effect of non-diversifiability of stock-based payoffs and the interaction between incentives and valuation,\(^1\) but has left the impact of vesting provisions largely out of the picture. This paper focusses exclusively on these provisions, essentially leaving incentive effects, diversification, tax, and credit risk issues unmodeled. Despite this, we offer some basic and surprisingly robust insight that further contributes to our ability to analyze realistic long-term contracts.

Our initial interest is in the employee’s valuation problem, taking the contract as given. From the firm’s perspective, on the other hand, understanding the valuation of the vesting provisions it selects is crucial to efficient compensation design. To the extent that such provisions purchase loyalty, the question is whether or not they are worth it. For every dollar by which they lessen the value of a package to the employee, a dollar must be paid somewhere else—in increased wages or other benefits—to retain her. This is a real economic cost, which may or may not exceed the actuarial or expected value of the savings. In addition, the firm needs to understand any modifications to the intended incentive structure which result from employee discounting of non-vested payoffs.

Our main contention is that, as typically used, vesting provisions may subtract value that is equal to the entire amount of the promised benefit. Deferred compensation can be literally worthless to the employee. Embedded in the wrong contract,

\(^1\)See the recent survey by Murphy (1999).
vesting provisions can destroy any incentive effects the benefit may have been intended to produce. They can also be economically inefficient, negating any renegotiation savings of long-term contracts and leading to underinvestment.

The intuition behind this claim is straightforward. Once an employee has accepted a contingent deferred benefit, it can be expropriated by the employer in future wage negotiations. Even if the employer has no intention to expropriate ex ante, the employee must anticipate the rational ex post action. Once she has done so, the firm has no reason not to take this action, and doing so will not surprise her. Hence, although motivational effects are not explicitly modeled, they will not generally alter the conclusion.

To overcome the problem, the firm must effectively commit itself not to act opportunistically in future wage negotiations. The simplest way to do that is to agree in advance to determine future compensation levels according to non-negotiable indices, such as output or outside wages. Where this is impossible due to contracting costs or non-verifiability, it may still be that implicit (or self-enforcing) agreements can effective constrain the firm in the required manner.

The empirical question then arises as to whether or not firms are using vesting provisions in conjunction with such efficient contracts. We deduce several testable implications of the contrary hypothesis, and argue that these are, in fact, consistent with a variety of evidence from the personnel literature. If there are strong implicit constraints binding firms to non-opportunistic behavior, they do not appear to be convincing to workers.

By connecting the valuation problem posed by vesting provisions to the entire course of future negotiations with the firm, this paper connects an active area of research in corporate finance to an important strand of the labor literature. Deferred compensation in general has previously been studied as a mechanism for eliciting effort (Lazear 1979, 1981), protecting human capital investment (Hashimoto 1981), insuring risk-averse workers (Holmstrom 1983), or deterring costly quits (Salop and Salop 1976). The empirical feature driving much of this work has been the observation
that firms pay deferred compensation in the form of efficiency wages and discretionary bonuses to older workers when there is no apparent obligation to do so. We complement this literature by analysing the effect of deferred compensation which is both explicitly promised and is not contingent on anything other than remaining with the firm. We articulate a new hold-up problem that comes through wage negotiations, even when the firm cannot legally expropriate benefits simply by firing workers before vesting occurs.

The outline of the paper is as follows. The next section formalizes the intuitive argument above in a simple setting which takes the contract as given. This makes clear what the necessary assumptions are, and suggests which contract forms can solve the problem. Section 3 extends the analysis by modeling the firm’s design problem when deferred compensation is necessary to retain workers. We are then able to characterize contracts – both implicit and explicit – that efficiently achieve the desired goals for both parties. The final section summarizes the positive, as well as the normative, implications of the analysis, and suggests important directions for future work.

2 Deferred Payoffs and Employment Contracts

Consider an opportunity for a firm $F$ to create a job for an employee $E$ under conditions which make the use of deferred compensation desirable. This section presents the basic argument of the paper concerning the valuation of a proposed deferred claim, without explicitly modeling those conditions. In addition, the parameters of the job opportunity – its productivity, cost, and duration – are all taken as exogenous. The viewpoint is that of the employee facing a given offer.

The following notation and assumptions will be used throughout.

A2.1 Both $E$ and $F$ are risk-neutral. The interest rate is zero, and there are no taxes.
A2.2 If hired, $E$’s productivity per unit time is $r$ and her reservation wage process is $w_t^e$. She may also be replaced at any time by an identical worker at wage $w_t^e$.

A2.3 $E$ has no wealth other than human capital. Negative wages or penalties against her are not enforceable.

A2.4 $F$’s contractual commitments are enforceable liabilities for which there is no risk of default.

A2.5 $E$’s effort at time $t$, $e_t$, is a binary variable which is observable and verifiable. If, having contracted to work at $t$, she fails to do so ($e_t = 0$), her employment may be terminated “for cause”, releasing $F$ from all liability for promised future payments. If $F$ terminates employment otherwise (when $e_t = 1$), it remains liable for any promised payments.

In this setting, deferred compensation is any payment $B$ promised contractually at time $t$ to be made at a later time $s$. All such payments are implicitly assumed non-vested until $s$, or, equivalently, the time index is taken to refer to the date at which $E$ assumes legal title to $B$. (This is inconsequential since the interest rate is zero.) Future salary entitlements, if guaranteed, are no different from pension rights in this regard. Note that, for the case of an option, $B$ would be random at the initial contracting time, and according to our notation, would denote the market value upon vesting.

As noted in the introduction, the paper essentially abstracts from all incentive issues in contracting, as seen from (A2.5). If $E$ works, she produces at rate $r$, if she does not, she produces zero, and $F$ will then replace her. Not working is thus tantamount to quitting for $E$.

Assumption (A2.5) also sharply distinguishes this type of exit from any other (not “for cause”) termination by $F$. We do not allow $F$ to unilaterally renege on non-vested benefits by this method. In reality, the situation may not be so clear, and may depend on the type of benefit. In many countries, statutory severance payments compensate workers for non-vested pensions lost. For stock option awards, the
“accelerated vesting” envisioned here is an increasingly common, though not obliga-
tory, feature of negotiated compensation packages. In any case, our assumption is
conservative: where it fails our conclusions will be even stronger.

Last, note that implicit in (A2.2) is a further strong simplification. We are going
to take productivity and wages as exogenous. Not only are they unrelated to effort,
but they are also unrelated to the course of employment. So neither $E$ nor $F$ need
take them into account when solving for their optimal actions. The firm can thus be
viewed as facing the opportunity of making (or continuing) a unit investment in a
very simple technology. And $E$ does not realize any increase in human capital from
working for $F$.

The next two subsections analyze two further specializations of this framework
which illustrate the main points from somewhat different angles. A third subsection
considers the robustness of the conclusions.

2.1 Certainty

Suppose at time zero an employee $E$ faces offers from firm $F$ to work on the interval
$[0, T]$. Assume the employee’s outside wage rate $w^*_t$ is known in advance to $E$ and $F$,
and that this is not more that the wage of an equivalent replacement worker. Finally,
assume that, if retained, $E$’s productivity, $r$, is also known and is greater than $w^*_t$.
Hence $F$ will want to retain $E$, and $E$ knows this.

We consider offers from $F$ of the form $(w, B)$ where $w = w_t$ is a guaranteed wage
rate (i.e. $w_t dt$ is paid over the interval $(t, t + dt]$) and $B$ is a non-vested lump sum to
be paid at $T$ conditional on participation throughout $[0, T]$. We wish to quantify the
value of such an offer to $E$ and, in particular, the marginal value of an extra dollar
in the non-vested component.

Define $V_t(w, B)$ to be this value. That is, $V_t$ is the expected future utility (or, here,
just total cash-flow) to $E$ on $[t, T]$ assuming optimal future policies by both parties.
Under our assumptions, these policies are simple: it is optimal for $F$ to retain $E$ (or it
suffices if $E$ believes this), hence $F$ can be expected to meet $E$’s reservation demand
at each point, and $E$ will not exit. The value of the offer, then, reflects not just the promised wage, but $E$’s anticipation of successful renegotiation. Note that (A2.4) - (A2.5) imply that, though $E$ may renegotiate benefits upwards, $F$ cannot lower any benefits previously promised. This leads us to the following characterization of $E$’s participation constraint.

**Lemma 1** In order for $F$ to ensure $E$’s participation for all $t \in [0,T]$, $E$ must be paid a wage rate $w_{t}^{\text{min}}$ such that

$$\int_{t}^{T} w_{u}^{\text{min}} du + B \geq \int_{t}^{T} w_{u}^{*} du$$

(2.1.1)

where $B$ is the promised terminal benefit. The realized wage rate, $\bar{w}$, will be the lowest wage satisfying this equation such that also $\bar{w}_{t} \geq w_{t}$, the originally promised wage.

Note: All proofs appear in the appendix.

Given this constraint, we can deduce the course of renegotiation of an initial package, and hence its time-zero value. Iterating backwards from time $T$, the lemma implies that $E$ cannot and will not seek to renegotiate at any $T$ for which the guaranteed wage, $w$, satisfies

$$\int_{t}^{T} w_{u} du + B > \int_{t}^{T} w_{u}^{*} du.$$ \hfill (2.1.2)

If we define $\tau^{+}$ as the highest $t$ for which this fails, then we conclude that $\bar{w}_{t} = w_{t}^{\text{min}} = w_{t}$ for all $t > \tau^{+}$. That is, the guaranteed wage is sufficient to ensure participation.

Continuing backwards, as soon as $t < \tau^{+}$, $E$ will negotiate $w$ upwards. In fact, the only way to satisfy (2.1.1) is then to negotiate $w$ all the way up to $w^{*}$, and it must remain at this level for all earlier $t$, unless at some point $w$ exceeds $w^{*}$. The entire solution path of the renegotiated wage $\bar{w}$, can thus be traced back to date zero in this manner. (The function is given explicitly in the appendix.) The situation is illustrated in Figure 1 which shows two possible offered wage curves $w$, and the renegotiated wage $\bar{w}$ for the same reservation wage $w^{*}$. Note that $\bar{w}$ takes on only the values $w$ or $w^{*}$. 

7
With the anticipation of future benefits given by the above recursive solution for \( \bar{w} \), we can immediately address the valuation question for a given package.

**Proposition 2.1** If \( E \) assumes \( F \) will ensure her participation on \([0, T]\), then her valuation of the package \((w, B)\) at \( t \) is

\[
V_t(w, B) = \int_t^T \bar{w}_u \, du + B
\]  

(2.1.3)

where \( \bar{w} \) is the renegotiated wage path defined as the minimal solution to the recursive equation (2.1.1) subject to \( \bar{w} > w \). For \( t < \tau^+ \) defined as

\[
\tau^+ \equiv \sup_t \{ B + \int_t^T w_u \, du < \int_t^T w^*_u \, du \}
\]

we have

\[
V_t(w, B) = V_t(w, 0).
\]

(2.1.4)

If, in addition, \( w \leq w^* \) on \([0, \tau^+]\), then we also have

\[
V_t(w, B) = V_t(w^*, 0) = V_t^{res}
\]

(2.1.5)

where \( V_t^{res} \equiv \int_t^T w^*_u \, du \) is the value of the offer \((0, 0)\).

The key point is the assertion (2.1.4). Unless deferred benefits are very large, their value in the compensation package is zero. Intuitively, once \( t \geq \tau^+ \), \( E \) will have no ability to negotiate wages above the minimum guaranteed, and \( F \) will not bestow them. At date \( \tau^+ \), then, total future benefits are exactly what they would be with no promises. Proceeding backwards, the prospect at earlier dates of receiving one’s reservation value in the future is likewise valueless. Indeed, barring the case that \( F \) has promised surplus wages \( w > w^* \) prior to \( \tau^+ \), the package has no anticipated net benefit at all. As a corollary of (2.1.4), of course, \( \partial V / \partial B = 0 \) before \( \tau^+ \): the marginal value of an extra non-vested dollar is zero.

The characteristic time \( \tau^+ \) of a given package may be thought of as the expropriation horizon. After \( \tau^+ \), the contract’s guaranteed total compensation exceeds \( V^{res} \).
In this case, where the firm cannot legally expropriate any of $B$, the value of the contract is $B + \int_0^T w \, du$ and the marginal value of a non-vested dollar is one.

As a practical matter, it is rare for employers to put employees in this position at the start of a contract (i.e. $\tau^+ < 0$ would be rare). But our analysis shows that not doing so renders the promise of the non-vested payment meaningless to the employee. In particular, its presence has no effect on the employee’s initial wage demand, which will be the market wage $w_0^*$. Nor, until the expropriation horizon is reached, will the benefit $B$ in any way influence $E$’s behavior.

The next subsection presents another example, which illustrates that the intuition here can apply equally well to the case of benefits and wages which are not known exactly in advance.

### 2.2 An Options Example

The following problem paraphrases a classic MBA case question

Employee $E$ has decided to work for firm $F$ and her starting salary has been agreed. In addition, she is offered a signing bonus of either $10,000 or 1000 at-the-money call options on $F$’s shares which expire in four years. However, these options only vest if $E$ stays with the company for three years. The current stock price is $40 and every year it will either move up or down by 25%. The stock pays no dividend, and can be costlessly sold short. Which bonus should she take?

Ignoring the vesting provision, the choice is simple. Using the binomial model of $F$’s share price, the options have a theoretical value of over $20,000 (using a zero risk free rate; more otherwise). How much of this does the vesting provision subtract?

To obtain title to the options $E$ must be employed by the firm through the end of year three. The key decision point for the problem comes at $T = 2$, the start of that year. Denote by $S_{2,i}$ the possible values of the share at that date, and let $C_{2,i}$ be the corresponding value of the options at each conditional upon $E$ staying with the firm. (See the diagram in Figure 2.) As in the last section, $E$ will rationally anticipate
being offered a wage at that point which marginally exceeds $w_2^* - C_{2,i}$, which will be accepted.

Thus, letting $v_3$ denote total expected compensation for year three, we have

$$v_3 = \sum_i E(v_3|S_2 = S_{2,i}) \Pr(S_2 = S_{2,i}) = \sum_i E(w_2^*|S_2 = S_{2,i}) \Pr(S_2 = S_{2,i}) = E(w_2^*).$$

Notice that this holds regardless of the joint distribution of $S_2$ and $w_2^*$. In particular it holds at each node of the tree at $T = 1$, the start of the previous year. In other words, the incremental value of the options is zero at each future node at that time. Hence salary negotiations at $T = 1$ (and, continuing backwards, at $T = 0$), are unaffected by the presence of the options. So $E$ will demand and receive her reservation wage at those dates. The total value of the contract — the expected sum of each future year’s compensation — is then just

$$\sum_{i=0}^4 E(w_i^*)$$

which is, of course, exactly what it would be without the options.

So the incremental value of the options is zero. Or, to put it another way, the value of the vesting provision must be precisely the negative of the theoretical value of the option. In the original problem, the two proposed packages do still differ in their year-one total value: one contains cash. Under this analysis, $E$ will accept the $10,000, or indeed any amount, up-front instead of the options.

The example concealed one subtlety: the analogue to the condition in the last section of the deferred benefit not being “very large”. If there were a chance at $T = 2$ that the non-negative wage constraint could bind, $F$ would not then be able to expropriate the entire value of the calls, leaving them with some residual worth at some nodes in the tree. For example if $w_2^* = 50$ at the top node in Figure 2, then even with a zero wage, the options do add 9.28 to $E$’s compensation in that state.

This does not alter the conclusion, however, because that residual value will still be (optimally) extracted at $T = 1$. To continue the calculation, at $S_1 = 125$ the contingent value of the 9.28 is now 4.64, and, assuming this is more than $w_1^*$, it will
be subtracted from $F$’s wage offer at that point. So, as before, the incremental value of the options at $T = 0$ is zero.

One could construct an example in which the non-negativity restriction continues to bind, even at the initial node, and hence the options would have positive value. But, as the example suggests, it requires extremely large options values, relative to outside wages, at the vesting date. Moreover, stochastically, such a construction is harder when, as seems likely, $E$’s value is positively correlated with $S$.2

Having noted the possibility of multi-step expropriation, a second important point to recognize is that this may, in fact, be the most feasible scheme for $F$ to implement, even when not necessitated by constraints. Our original formulation called for $F$ to lower $E$’s wages by up to $59,000 at the start of year three. Even if fully anticipated, such a move might be awkward in practice. However, reducing $w_2^*$ by half this amount, say, might still entail a nominal salary increase, and hence lead to fewer (unmodeled) behavioral side effects. $E$ would then attach residual value to the options at $T = 1$, and hence that remaining value could be subtracted from $w_1^*$. This is illustrated in the second panel of Figure 2, where we assume values 100, 125 and 150 for $w_0^*$, $w_1^*$, and $w_2^*$ respectively. The tree shows a potential salary schedule offering nominal increases with certainty. At each node, the difference between $w^*$ and the wage is subtracted from the remaining value of the call, which is then discounted back to the previous time-step, as usual. The numbers in square brackets show the residual value at each node after that difference has been subtracted.

Depicted this way – as a series of small hold-ups over the course of several years – the story told here might begin to sound less theoretical and, indeed, familiar to many readers.

The example has two main points. First, it shows the robustness of the basic argument of the last section to the inclusion of uncertainty – both in the value of the deferred benefit and in the level of future wages. Second, it illustrates the destruction

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2Technically, for the call not to be entirely expropriable there must be a node at the vesting date with value exceeding the cumulative reservation wage experienced by $E$ in attaining that node.
of incentive effects that vesting provisions can cause. In the MBA case, \( F \) is trying strongly to encourage \( E \) to take the options (by offering them well below “theoretical value”), presumably because of the expectation that owning them will enhance her productivity or team work. However our analysis implies not only that she places zero value on them initially, but also that they continue to have zero value at all future share prices. Thus – even if she were given the options for free – they would not have any effect whatever on \( E \)’s attitude toward the success of the enterprise.

2.3 Objections

The framework here does make some important assumptions, which, while not extreme by the usual standards of contract theory, do deserve scrutiny.

What will strike some as the least realistic aspect of the analysis is the (implicit) assumption that expropriating the value of a worker’s deferred benefits has no impact on productivity. Nor does it even predispose the worker to exit, as long as her outside wage is matched (or marginally exceeded). These are, in fact, separate assumptions. But they share a disregard for any reaction to perceived unfairness.

With respect to being able to retain the employee for her reservation wage, it should be pointed out that there is a second line of defence if this fails. We have assumed that \( w^* \) is not only \( E \)’s outside wage, but also the replacement wage. Hence if we imagine \( E \) quitting in disgust at each instance of expropriation, the conclusions are unaltered since she will be replaced by an identical \( E' \). (Costly replacement is considered in the next section.)

The issue of incentive compatibility is more complex. To understand it, it helps to distinguish between behavioral and game-theoretic critiques of the set up.

A richer game theory model would allow \( E \) to threaten to withhold effort in response to lower benefits. Recall that we essentially ruled out such threats (in (A2.5)) by stipulating that she could be fired “for cause”, relinquishing everything, if she failed to contribute her (binary) effort. If effort is continuously variable and affects productivity, then certainly \( F \)’s decision problem is altered. But the basic
hold-up opportunity is not. $F$ will now offer total compensation so as to maximize net yield of the production function; and the net value offered will still reflect whatever expropriation of deferred benefits are available in achieving this optimal level.

In behavioral terms, however, $E$ might still not choose her effort purely in response to total expected compensation. If $F$’s offers are perceived as exploitative, morale and productivity could suffer. How should this perception be modeled? What counts as exploitation? In our model $E$ is not given any reason to expect that $F$ will do anything other than act in a profit maximizing manner. Moreover we have allotted $E$ the right to continually renegotiate her wage upwards. So the grounds for resentment may not seem obvious.

Nonetheless, a positive depiction of real behavior could include a benchmark level, $w^f$, for $E$’s subjective fair wage rate, once a deferred benefit $B$ has been granted. For example, the benchmark could be the current level, so that any nominal cut in $w_t$ lowers output. In our setting, this could be incorporated as an incremental cost to $F$ proportional to $(w^f_t - w_t)$.

Our options example indicated that, even in this case, the main conclusion may be robust. Complete expropriation of $B$ can involve wage paths with small subtractions from $w^*$ over several periods, but with the net wage still increasing. This might make them pass as fair. The limit on the size of an expropriable benefit will, however, drop. Beyond this “fairness limit” the firm may still expropriate subject to a marginal benefit trade-off like the one in standard incentive models. This behavioral generalization will be explored in future work.

Similar in spirit to the fairness objection, another potential criticism concerns the firm’s ability to expropriate benefits without cost to its reputation. Presumably it anticipates trying to re-hire $E$ once the original contract expires, and likewise engages in hiring many other workers at all times. Might $F$ gain something in these other negotiations by refusing to exploit the opportunities in $E$’s contract?

Perhaps so. Clearly such implicit contracts do exist: all firms do not always behave opportunistically. We model such commitment devices formally in Section 3.2. For the moment, then, our supposition is simply that the strength of the implicit
constraints may not be apparent to all workers ex ante. To the extent that they are in doubt about the applicability of the reputation models to their case, they may indeed follow the valuation logic we have outlined.

To summarize this section, we have shown that an employee rationally anticipating the outcome of future wage negotiations will, prior to an intermediate horizon, attach zero value to any non-vested component of her wage package. Equivalently our results may be viewed as a theory of the marginal value of attaching vesting provisions to future payoffs. As such, it is noteworthy that the argument makes no assumption about individuals’ uncertainty about their future opportunities. That is, we do not need to quantify the “option value” surrendered to attain the deferred payoff. Indeed the analysis shows that vesting provisions achieve their full (negative) value even when the firm always pays full marginal wages, i.e. even when the opportunity cost of remaining with the firm until vesting is zero.

We explore in the next section the issue of optimal contract design in a setting where the employee’s perspective potentially matters to the firm, that is, where getting employees to accept and value deferred compensation could yield real benefits. Even before that analysis, the conclusions of this section clearly have relevance to workers, who – for whatever reason – do, in fact, face the situation envisioned here: deferred benefits plus repeated future salary negotiation.

3 Efficient Deferred Compensation

This section considers the compensation design problem in a setting where the choice of deferred provisions matters to the firm. Clearly many companies believe deferred compensation to be a valuable tool. The goal now is to re-examine the expropriation issue when, indeed, there is such a value, in order to delineate when and how the contract provisions can realize it. We consider first the fully contractable case, and then extend the model to the case where, due to incomplete contracting, the firm
must rely on implicit commitment. The framework used here is simple and standard.\textsuperscript{3} However, the inquiry enables us to offer a constructive solution to the problem posed in the last section.

### 3.1 Explicit Contracts

Among the reasons for using deferred compensation may be precisely to enable firms to expropriate employees’ earning power over time. When a firm can offer a potential worker experience and training that will substantially increase her transferrable human capital, it may be in a position to demand up-front that she relinquish the right to some of those future gains. One way of doing this might be to render a component of the incremental human capital effectively “firm-specific” by granting it in advance, with vesting provisions, and using this as leverage when outside wages rise.\textsuperscript{4}

Here we adopt a different view, in which the role of deferred compensation is not to implement such a transfer, but rather to achieve an efficiency gain. This is simpler in that it does not entail modeling the returns to investment in training (for both parties) and allows us to keep the reservation wage exogenous. It also enables us to highlight the unintentional and confounding effects of the expropriation threat upon the problem.

Our analysis is one in which bargaining is costly due to both asymmetric information and deadweight losses from turnover. In addition to the conditions (A2.1) - (A2.5) of the last section, the main assumptions are:

**A3.1** At time 0, the firm, \( F \), intends to employ a worker on \([0, T]\). The shortest possible contracting interval is \( \Delta \) and \( N = T/\Delta \). The current incumbent worker, \( E \), can be offered a contract of any duration \((\geq \Delta)\) promising payments at future dates.

\textsuperscript{3}Formally, the model below is very similar to the incentive theories of Lazear (1979,1981) and Akerlof and Katz (1989) with employees exploiting private information instead of shirking. Models with idiosyncratic outside opportunities are also used in Salep and Salep (1976), Ippolito (1994), and Oyer (2000).

\textsuperscript{4}The notion of firms using vesting provisions to protect specific human capital investment (training costs) goes back to Becker (1962).
Wages agreed for \([t, t + \delta]\) are paid at \(t + \delta\) contingent upon \(E\) working \((c = 1)\) on that interval.

**A3.2** At each \(t\), \(E\)'s one-period reservation wage \(w^*_t \Delta\) consists of two components: \(w^*_t \Delta + \delta_t\). The first is a non-stochastic component which is common knowledge. The second is an i.i.d. idiosyncratic term, known only to \(E\). With probability \(q \Delta\) it is zero, and with probability \(1 - q \Delta\) it is distributed uniformly on \([-a, a]\).

**A3.3** Exit is costless to \(E\) but imposes a fixed turnover cost \(c\) on \(F\). Replacement workers are always available with current reservation wage \(w^*_t \Delta\) and future opportunities identical (in distribution) to those of \(E\).

The model envisions an employer faced with the simultaneous problems of current workers having random outside opportunities and costly replacement. The outside opportunities are modeled as effectively arising at random exponential times and being of uniformly distributed magnitude. For concreteness, we may think of the scale, \(a\), as being on the order of a few months’ wages. These opportunities should be taken to represent any non-pecuniary transient shocks to reservation values, e.g. due to family demands or preference shifts. (Allowing negative opportunities does not drive any of our results, and just keeps the mean equal to \(w^r\). These might result from high short-term switching costs, for instance.) There are always interchangeable workers available without such an opportunity today, but hiring them involves deadweight losses, perhaps from search or training costs. The size of these might also amount to a significant fraction of a worker’s annual salary. Implicit in (A3.1) is the assumption that abandonment is a dominated option for \(F\) at all points. So even with only one period remaining until \(T\), the firm’s surplus is assumed to exceed \(c\).

It turns out that, if \(c\) is small or \(E\)'s opportunities are observable, then the deadweight losses would be either absent or unavoidable. However here there are more possibilities. We start by looking at single period bargaining.

**Proposition 3.1** For a single period contract, and for values of \(c\) in an interval \(I(q)\) containing \(a\), \(F\)'s optimal wage bid to \(E\) is \(w^*_t \Delta\). This produces expected cost
\[ w_i^* \Delta + eq \Delta/2 \text{ for } F, \text{ and expected benefit } w_i^* \Delta + aq \Delta/4 \text{ for } E. \] The interval \( I(q) \), which is given in the proof, is large unless \( q \) is close to one, and approaches all of \( \mathbb{R}^+ \) as \( q \Delta \downarrow 0 \).\(^5\)

The bargaining game results in \( E \) receiving some option value, since \( F \)'s offer removes the danger of a negative idiosyncratic shock. But the negotiation extracts a cost from \( F \). This makes the motivation for deferred compensation in a multiperiod setting clear. If \( E \)'s incentive to exploit her information could be countered, \( F \) could recover a constant cost per unit time. Intuitively, it ought to be sufficient to promise in advance to insure her against all future idiosyncratic shocks.

The general multiperiod bargaining problem involves consideration of the set of employment strategies – defined as sequences of contract offers – that \( F \) could undertake. \( E \)'s problem is then to determine the value of each offer – defined to be its expected lifetime benefit under optimal future negotiation. Likewise, \( F \)'s value function is the expected total cost of a particular employment strategy under the best response of (current and future) workers. Details of the game are presented in the appendix. The following proposition verifies that it is indeed optimal for the firm to avoid period-by-period bargaining.

**Proposition 3.2** Assume \( w_0^* > a/\Delta \). Then under optimal negotiation, at time zero \( F \) will tender a contract which, if accepted, would result in \( E \) optimally remaining with the firm without renegotiation until \( T \). If \( E \) does not accept it, a replacement worker will, who will likewise remain with the firm.

Multiperiod contracts having this retention property can be characterized purely by their promised payment schedule, as they involve no uncertain future re-contracting. Since \( E \) will never realize her idiosyncratic opportunities, and \( F \) will never incur replacement costs, once contracted, both parties' value function is just the total sum of these promises. We next characterize this optimal value explicitly.

---

\(^5\)Hereafter we assume \( c \in I(q) \).
Proposition 3.3 An optimal contract has value at $t = 0$ of
\[
\sum_{n=0}^{N-1} w_{i,n}^r \Delta
\]  
and at subsequent times $t_k > \Delta$ of not less than
\[
\gamma(t_k) \equiv \sum_{n=k}^{N-1} w_{i,n}^r \Delta + a.
\]

This proposition tells us two important things. First, it quantifies exactly how much $F$ saves relative to continual re-negotiation. As indicated in Proposition 3.2, at time 0, $E$ and $F$ will play a version of the one-period game where now the one period is the entire contract life $T$. This will entail a one-time expected cost of $cq\Delta/2$ to $F$. But since the game is never repeated, the long-term contract saves this amount per period. The attainability of these savings over the life of the contract could easily have efficiency implications. If the economics of the project in question were such that the expected deadweight loss from repeated bargaining were greater than the required profit, the construction of a renegotiation-proof contract could overcome an underinvestment problem and lead to the creation of a profitable job.

The second thing the proposition does is suggest how to do this construction. Referring to the continuation value in equation (3.1.7), there is a natural interpretation of each of the terms. The first term is simply the remaining (non-idiiosyncratic) reservation value of the worker, which, since it declines with time, suggests a contract specifying $w_i^r$ as the wage rate. The second term, $a$, cannot come from wages since it remains until $T$. Hence it must correspond to a non-vested, deferred component. (See Figure 3.) Intuitively, the presence of this term ensures that even if $E$ receives an idiosyncratic opportunity of $\delta = a$ at any future date, her expected benefit from staying still exceeds that from leaving to capture it. In effect, one single promised

\[\text{It might seem that } E \text{ could not be persuaded to forgo her per-period “information premium” of } cq\Delta/4. \text{ This would be right if each worker had the right to be some firm’s incumbent worker in all future periods. But we did not make that assumption. The point is discussed further in the appendix.}\]
payment insures $F$ against any number of potential turnover costs.\footnote{The assumption that $w_0^T > a/\Delta$ allows the optimal contract to be constructed through the worker effectively financing her own performance bond in the first period. If $\Delta$ is small enough, the financing must take place over multiple periods, resulting in some probability of early exit or renegotiation. This is essentially the problem identified by Akerlof and Katz (1989).}

In order for this to work, however, $E$ has to assess this promise at its nominal value at all points in the contract’s life. If at some intermediate $T'$, the reservation wage is not guaranteed, the result will be exactly as described in the last section. The terminal payment $a$ will become expropriable. The following result verifies that nothing in the framework here mitigates $F$’s incentive to carry out that expropriation.

**Proposition 3.4** Suppose $E$, the incumbent worker at time $T' = t_N$, has previously been guaranteed $B$ at $T > T'$ (but no other future wage). Let $V^{ret}_{T', T} = \sum_{n=N}^{N-1} w_t^R \Delta$. Then for any values of $a$ and $q$ and $c \in I(q)$, $F$ will optimally offer her a contract of value $v = \max[0, V^{ret}_{T', T} - B]$, and she will accept unless $\delta_{T'} > \max[0, v - V^{ret}_{T', T}]$. Hence the ex ante value to $E$ of the promise of $B$ is zero unless $B > V^{ret}_{T', T}$.

We can now complement the results of Section 2 by quantifying the consequences to the firm of such sub-optimal contracts.

**Proposition 3.5** A contract inducing participation on $[0, T']$ and promising $B > 0$ at $T > T'$ (but no other future wage) has expected cost to $F$ which is (i) at least that of an identical contract with $B = 0$, and (ii) at least $cq\Delta/2$ more than an optimal contract.

In other words, $F$ saves nothing on $[0, T]$ by promising $B$ and incurs the cost associated with a single renegotiation. Multiple renegotiations, e.g. via a succession of $N$ annual contracts, simply multiply this cost by $N$, with each period’s deferred promises being treated alike as worthless by the worker.

In this model, then, vesting provisions do indeed have a positive role, but only when used in tandem with guaranteed wages. This latter injunction may seem both unhelpful – in that future reservation wages are not actually known in advance – and
unrealistic – in apparently leaning heavily on the assumption that effort is always assured. Both points deserve comment.

First, if future reservation wages are stochastic, the analysis of this section can be repeated with current expectations of futures values substituted in the contracts for the values themselves. (The case where, in addition, these wages are costly to observe and/or verify, is considered below.) It is important to realize that the solution offered by the propositions above, does not require that wage guarantees be indexed to the market value \( w_t \). While we interpreted the expression (3.1.7) in terms of such a wage stream plus a deferred payment \( a \), the proposition only required that an optimal contract have value at least as much as this. Alternatively, one could pay any (presumably lower) wage \( w \) in conjunction with a deferred benefit of

\[
a + \sum_{n=0}^{N-1} (w_{t_n} - w_{t_n}) \Delta.
\]

And, again, that quantity may be replaced by its initial expectation, yielding an implementable optimal contract with no need to re-set wages.

This observation addresses also the incentive issue with our conclusions. It is worth stressing that we have not implied that a large fraction of compensation need be in guaranteed salary. In fact a wage of zero (which is certainly invulnerable to expropriation) can be perfectly compatible with an optimal contract. Back-loading compensation into the deferred component might also facilitate the inclusion of incentive-contingent terms. Conversely, where large incentives are desirable (for reasons not modeled here), as perhaps in CEO contracts, our analysis implies that it may be important not to combine these with high salaries.

### 3.2 Implicit Contracts

The preceding section follows a long history in the labor literature in identifying an optimal lifetime employment contract that looks very unlike what is observed in practice. The rarity of any type of long-term contracting is sometimes viewed as per se evidence of the efficacy of implicit commitment in an environment where it costly
or impossible to verify all the contingencies upon which the ideal agreement depends. We now ask whether, indeed, in our setting an implicit promise by the firm not to expropriate deferred benefits can substitute for an implicit one.

Recall that in the model the point of using deferred benefits is to deter employees from idiosyncratic exits, which are costly to the firm. In this context, it is the deferred benefit which is explicitly promised, and is contingent only upon the (verifiable) state of being employed on the vesting date. This may be contrasted with the standard implicit contracting models (c.f. Carmichael (1989)) which invoke deferred benefits to induce effort. There the deferred payoff is the implicit part of the compensation, and is contingent on the unverifiable provision of effort. These models are thus unsuited for our purpose as they abstract from the specific contractual features of interest here.

Nonetheless, the essential insights of the implicit contract approach can still be brought to bear on our formulation. We follow Bull (1987) and Baker, Gibbons, and Murphy (1994) in introducing reputation effects to restrain the firm in a multiperiod setting. We verify that an implicit contract with vesting provisions can in fact be sustained in the sense that a Nash equilibrium exists in which the employee treats the deferred promise at its face value. The result may or may not account for the widespread use of vesting provisions without explicit guarantees: the case in which non-vested benefits are always expropriated is an equilibrium as well.

The starting point for our implicit commitment model is the specification of some non-contractability which renders the optimal explicit contract impossible to implement. To this end, we add the following assumptions to (A3.1) - (A3.3) above.

**A3.4** The duration of the job is $N = 2$ periods. (We now normalize the time interval to be $\Delta = 1$.) The second period replacement wage $w_{t+1}$ for a worker hired at $t$ is observable but not verifiable, and takes on the values $\bar{w}$ or $\underline{w} < \bar{w}$ with probabilities $\pi$ and $1 - \pi$ respectively. These outcomes are independent of the occurrence of the (additional) idiosyncratic outside opportunity $\delta$. 

21
A3.5 At each \( t = 0, 1, \ldots \), the firm \( F \) requires \( K \) identical workers for identical two-period jobs. Workers cannot cooperate in wage bargaining. All wage offers are observed by all.

A3.6 The firm has fixed probability \( \gamma \) of being liquidated at the end of each period. All outstanding explicit liabilities are honored upon liquidation, but implicit ones are not. Each period’s survival is independent of all other stochastic quantities.

Aside from these assumptions, the situation is as in the previous section. The worker \( E \) may receive an idiosyncratic opportunity at the end of the first period, and it would be costly for \( F \) to bargain with her at that point. \( F \) would like to defer enough of the period-one reservation wage to bind \( E \) in period two, but \( E \) will anticipate expropriation unless all remuneration is guaranteed in advance. That is unattractive to \( F \) now because the guarantee cannot be made state-contingent. Renegotiation can only be forestalled by guaranteeing \( \bar{w} \), but this entails unnecessary cost with probability \( 1 - \pi \).\(^8\)

Formalizing this story requires two parameter assumptions, which are consistent with the conclusions of the last section.

A3.7 The following holds for each time-cohort of workers: \( a > c q / 2 \) and \( a < w^*_1 \). (Hereafter, except where needed, we will drop the notational reference to \( t \), the particular time cohort, and refer, e.g., to \( w^*_1 \) and \( w^*_2 \) as the first and second period replacement wages generically.)

To describe the infinite-horizon hiring game, we follow MacLeod and Malcolmson (1989) and consider valid strategies for \( E \) and \( F \) to be mappings from the set of observable actions to current policies. Let \( \mathcal{H} \) be the set of all wage and employment histories and let \( \mathcal{W}^2 \) be the set of all possible two-period wage offers by \( F \). Then \( E \)'s strategy, which we denote \( R \), is simply a rule mapping \( \mathcal{H} \times \mathcal{W}^2 \) to an accept/reject

---

\(^8\)If the worker’s productivity varies with the replacement wage, such a guarantee might mean locking in losses in the low state which would deter the firm from hiring initially. For present purposes, it is enough to just have the guaranteed contract be costly to the firm.
response. (As we have seen, $E$ has no interesting decisions at $t = 2$, and simply exits if the compensation offered is below $w^*_2$.) Since all workers are identical, we do not distinguish between the strategies of different time-cohorts.

For $F$, the actions at each time consist of two-period offers (which may specify nothing about the second period) to new workers and one-period offers to those hired last period. Denote this $S : \mathcal{H} \mapsto (W^2, W^1)$. We denote the components of the two period offers by the triplet $(w_1, e_2, i)$, where $w_1$ is the first period wage, $e_2$ is the explicit (minimum) amount guaranteed for the second period, and $i$ an implicit, non-enforceable promise of an amount which may depend on $w^*_2$ to be paid conditional on employment (and firm survival) in the second period. We now parameterize the expropriation decision by writing the firm’s second period wage offer as

$$w_2 = \max\{w^*_2 + \chi \cdot [w^*_1 - w_1] - e_2\} \quad (3.2.8)$$

where $\chi \in [0,1]$ is the decision variable. From our earlier analysis, we know that the firm’s optimal offer in the one-shot game with an incumbent worker is just $w^*_2$. So $\chi = 0$ corresponds to completely opportunistic behavior, whereas $\chi = 1$ corresponds to delivering total lifetime compensation of $w^*_1 + w^*_2$. This can be viewed as fully delivering on the implicit promise of

$$i = (w^*_1 + w^*_2) - (w_1 + e_2).$$

Hence $i$ is not a choice variable under this parameterization. The firm could, in principal, choose any random $\chi \in [0,1]$ which could also depend on the observed history. The firm’s actions at time $t$ may thus be summarized as $\{(w_1, e_2, i, \chi)\}$.

With these strategy sets, we now define the equilibria we are interested in.

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9 We have ruled out strategic liquidation by making death exogenous in (A3.4).

10 The only restriction this imposes is that the firm never over-pays in the second period. There will be equilibria where it does, but they will be weakly dominated.
Definition 3.1 A hiring equilibrium is a strategy pair $(R^*, S^*)$ such that both

(i) $E_t[(w_1) + (w_2)_{t+1} | R^*, S^*] \geq E_t[(w_1) + (w_2)_{t+1} | R', S^*] \forall R', t$

(ii) $E_t[\sum_{s=t}^{\infty}(w_1 + w_2)_s | R^*, S^*] \leq E_t[\sum_{s=t}^{\infty}(w_1 + w_2)_s | R^*, S'] \forall S', t$

where the second subscript denotes the time at which the offer is made, i.e. $(w_1 + w_2)_s$ is the sum of period-one offers to new workers at time $s$ and period-two offers to incumbents hired at $(s - 1)$.

The only aspect of the definition that perhaps should be clarified is that, in (ii), the optimality of $F$’s strategy is to be verified holding constant the strategy of all time-cohorts of workers, not just the immediate or preceding ones. Likewise, (i) requires optimality of $E$’s expected wages, holding fixed the firm’s policy towards herself and all other workers. We now establish that an implicit contract equilibrium can support the non-degenerate use of deferred compensation.

Proposition 3.6 Assume (A3.1) - (A3.7), and with that $w_2$ is given by (3.2.8).

(A) The following strategy pair is a hiring equilibrium

$R^* = \text{accept } w \in W^2 \text{ only if } \begin{cases} \{w_1 + e_2 \geq w^*_{1} \text{ and } \min_{s \leq t} \chi_s = 1\} \text{ or } \\
w_1 > w^*_{1} \text{ or } \\
w_1 + e_2 > w^*_{1} + Ew^*_{2} \end{cases}$

$S^* = \{(w^*_{1} - a, a, w^*_{2}), 1\} \ \forall t \text{ and } \forall h \in H_t$

provided that

$(w^*_{1})_t < (1 - \gamma)^{-1} \frac{cq}{2} \ \forall t. \ \ (3.2.9)$

(B) In this equilibrium, all workers stay with the firm for two periods, and the realized cost of each worker is $w^*_{1} + w^*_{2}$.

Here the firm always delivers on its implicit promises because otherwise no future workers believe them. It costs workers nothing to be skeptical, so their threat of punishment is credible. The scales tip however if (3.2.9) is violated. In that case,
there exists a better policy of using a good reputation to get workers to defer all time-one wages and then expropriating them at time-two. The right side of the expression is simply the expected cost of having to do period-by-period renegotiation with all future workers. Thus the condition reflects the general observation of Baker, Gibbons, and Murphy (1994) that it is not the total required amount of the implicit contract that dictates its feasibility, but rather its incremental savings over and above the explicit alternatives.

There are two further points about the equilibrium that are noteworthy. First, as long as the weaker condition \( a < (1 - \gamma)^{-1} cq/2 \) holds, then the firm’s implicit promise is still credible in any period in which it has offered \( w_1 = w_1^* - a \) since at that point the most it can make by expropriating is \( a \) per worker. The proposition just tells us that if (3.2.9) did not also hold it was suboptimal of the firm to have made this offer. But, conditional on having done so, it will keep its promise. In that sense, the policy \( S^* \), while not unique, will be the easiest for the firm to sustain.

Second, it might seem intuitively that, given a policy of not expropriating, the split of the period-two promise between implicit and explicit components is not identifiable. If reputation can sustain \( e_2 = x, i = y \), it should sustain, e.g., \( e_2 = 0, i = y + x \) as well. The reason that is not so here is the possible death of the firm after the first period. Since implicit promises have no standing in bankruptcy, it is necessary to guarantee each \( E \) her full period-one compensation to induce initial participation. This provides an explanation for the non-implicit character of deferred compensation, which is difficult to understand in the context of effort models.

Part (B) of the proposition verifies that the implicit contract equilibrium achieves the first-best outcome. Moreover the division between the worker and the firm is the same as in the fully-contractable case. Inefficient separations are avoided, and \( E \) achieves the lowest possible cost per worker, not just in expectation, but in every realization.

This may seem like a happy victory for implicit contracts. And, indeed, the result may prove illuminating from a positive standpoint. (That is the subject of the next section.) The difficulty is that the equilibrium is not very robust. In fact, the workers’
strategy is weakly dominated by the policy of valuing all implicit promises at zero. From a normative standpoint, even if a particular $E$ knows for certain that (3.2.9) holds, and even if she can take as given the policies of all future worker cohorts, it is still difficult to see why she should not simply adopt the valuation analysis described in Section 2, just in case.

4 Conclusions and Further Implications

Ultimately, it is an empirical question whether workers anticipate that their employers will use the leverage conveyed by vesting provisions to expropriate deferred benefits, or will instead feel bound by implicit constraints not to do so. The former hypothesis, however, represents a rather extreme view of much current employment practice. It suggests that the effort and cost of designing contracts containing these provisions is essentially wasted: no loyalty is purchased; and no incentives are aligned. It is certainly prudent, before accepting such a conclusion, to ask whether the evidence supports it. We close by considering various empirical implications of the argument, and connect these to previous findings in the literature. In addition, we draw attention to further tests that might help to distinguish the competing views.

The most straightforward implication of the anticipated-expropriation hypothesis is that employees would not value awards containing vesting provisions as much as expected. There is direct support for the assertion that employees discount the value of stock option awards, which almost always contain vesting provisions. As reported in Hall and Murphy (2000), when such grants are offered as explicit substitutes for cash bonuses, multiples of two or three times “theoretical” value are typically needed to induce participation. Discounting due to the lock-up effect (that is, the disutility of being forced to hold an inefficient portfolio) provides an alternative explanation. The two effects, while not exclusive, might be distinguished by examining small awards (relative to wealth) or awards by less risky companies where portfolio considerations are less likely to enter.
In fact, the expropriation story also makes the stronger prediction that, not just the value, but also changes in value of non-vested benefits are attenuated due to vesting provisions. In particular, for stock based awards, the changes with respect to share price (the “delta”) should be small.\textsuperscript{11} To the extent that this sensitivity is supposed to affect performance via induced incentives, the prediction would be that this behavioral response should be less than expected. Again, there is evidence suggesting that performance effects from stock-based awards are surprisingly low\textsuperscript{12}, although this has not been in any way linked to the presence of vesting provisions.

Beyond such contingent contracts, there are some predictions that apply to pensions and seniority benefits in general, a potentially much richer source of tests. Here the primary implication of the model is simply that workers should not lower their salary demands at all when non-vested benefits are included in their compensation. This anomaly has been widely remarked in the personnel literature (see Ippolito (1994)). Second, having already demanded other compensation, workers, once hired, would be no less likely to leave a job with such benefits. This too seems to be consistent with empirical work. Potential benefit losses seem to have little or no affect on quit rates.\textsuperscript{13} Since promoting loyalty is the point of vesting provisions in the first place, this would certainly seem evidence of contracting failure, whatever the cause.

The preceding implications concern anticipated expropriation. The model of Section 2 also suggests that later in the duration of a contract one should observe the effects of the actual expropriation itself. Specifically, salaries should be negatively related to non-vested awards and quit rates should be low once employees are past the expropriation horizon, \( \tau \), described in Proposition 2.1. Since identifying the precise date at which the indenture effect begins to bind presents problems, the most relevant data here are probably for workers near the vesting date of pensions, who are

\textsuperscript{11}The lock-in model of Hall and Murphy (2000) makes the opposite prediction for in-the-money options. The expropriation model would predict that executives would fail to hedge such options even if they could.

\textsuperscript{12}See the discussion in Predergast (1999).

\textsuperscript{13}Overall separation rates are lower for workers with pensions, but there is a countervailing sorting effect (Salop and Salop (1976)) under which workers with low probability of quitting tend to self-select for jobs with deferred compensation. This seems to account for the entire effect. See Allen, Clark, and McDermid (1993).
certainly beyond it. For them the model predicts reduced wages and other benefits compared, for example, to what they can command after having vested. Lazeat and Moore (1988) find a strong effect from the unvested component of cumulative future pensions in explaining separation frequencies. Overall, the model suggests that the relative magnitude of unvested benefits compared to future earnings may be a crucial variable in determining wage-tenure profiles.

Finally, we can view the results of Section 3 as predicting something about when – cross-sectionally – the expropriation story ought to be stronger. This, too, leads to a variety of testable hypotheses. Certainly where a large component of lifetime wages is, in fact, contractually guaranteed (perhaps in unionized sectors) or is explicitly linked to output, the effects above should be ameliorated. Moreover, implicit constraints will be more sustainable when workers know their firms have reputational benefits in the labor market and their actions in negotiations are observable. Condition (3.2.9) suggests that we are unlikely to find this for risky firms (whose planning horizon may be short) or firms for which replacement of workers is not costly. More generally, where the total benefits available for expropriation is large compared to the cost of writing explicit contracts, we should not expect workers to value non-vested benefits. Hence the marginal impact on loyalty and pay demands of these benefits should vary with this trade-off.

In conclusion, this paper has argued that vesting provisions are at least a first-order consideration in the valuation of long-term employment contracts. Much financial research has recently focussed on the compensation design problem, as it has become recognized that this is central to understanding the agency conflicts between owners and managers. Moreover the proliferation of large stock-based awards to CEOs and other top executives, often amounting to a significant fraction of firms’ liabilities, has heightened interest in the efficiency of current practice. Our analysis has suggested some new insights on these topics. However, as the discussion above indicates, senior

\footnote{Anecdotal evidence from the Israeli army also supports this view: job assignments apparently become notoriously unattractive for career officers in the years immediately prior to pension vesting. (Thanks to Oren Sussman for pointing this out.)}

28
managers may not actually be the employees for whom the issues raised here are most relevant. The pension benefits of ordinary workers – as well as, increasingly, their share options – are probably more vulnerable to the expropriation mechanism high-lighted here. Whether this matters or not in practice is an important topic for future research.
Figure 1: Wage Negotiations Under Certainty.

The figure illustrates two cases of the wage dynamics modeled in Section 2.1. At time 0 an employee whose reservation wage is \( w^* \) (solid line) is promised a guaranteed wage \( w \) (dotted line), and a non-vested lump sum equal to the shaded area \( B \). The wage rate that will be negotiated \textit{ex post} is \( \hat{w} \), the dashed line. The characteristic time \( \tau^+ \) is indicated on the horizontal axis for both cases.
The figure shows the tree for the options example of Section 2.2. Panel A gives the standard binomial call valuation. (The risk free rate is zero.) Panel B shows a potential salary schedule that can be offered to a worker whose reservation wage during the first three years is $w^*$ when the options do not vest until $T = 3$. The non-expropriated value of the call at each node is in square bracket below.
Figure 3: Optimal Contract Payments.

The figure shows an implementation of the optimal contract described by Propositions 3.2 and 3.3 for an employee with reservation wage rate $w^*$ (solid line). The dark bars show the payments promised for each of six periods. The amount $a$ is the maximum realization of the idiosyncratic one-period outside opportunity. The final payment includes the deferred benefit $a$ in excess of $w^*$.
Appendix

Proof of Proposition 2.1 and Lemma 1.

To ensure participation of \( E \) at time \( t \), it is necessary and sufficient that a proposed wage path \( w^\text{min} \) meet the condition

\[
\exists \tau \in (t, T]: \int_t^\tau w^\text{min}_u \, du + \int_\tau^T w^*_u \, du + B \cdot 1_{\{\tau=T\}} \geq \int_t^T w^*_u \, du \tag{A.10}
\]

since this is equivalent to the condition that accepting the offer is not dominated by the alternative of immediate exit. Now suppose the condition in the lemma fails, that is, for some \( i \in [0, T) \) we have

\[
\int_t^T w^\text{min}_u \, du + B < \int_t^T w^*_u \, du.
\]

Then define \( \sigma^+ \) to be the highest \( i \) for which this is true. Then for all \( s > \sigma^+ \) we must have

\[
\int_s^\sigma (w^\text{min}_u - w^*_u) \, du \leq 0
\]

since \( B + \int_s^\sigma (w^\text{min}_u - w^*_u) \, du \geq 0 \). And also there must be a finite interval \([\sigma^-\sigma^+]\) such that \( \int_{\sigma^-}^{\sigma^+} (w^\text{min}_u - w^*_u) \, du < 0 \). Together these two things imply that for no \( \tau > \sigma^+ \) will equation (A.10) hold. Hence exit is optimal at \( \sigma^- \). This proves the statement of necessity in the lemma. The given condition is also sufficient for participation, because it implies that \( \tau = T \) satisfies (A.10).

By assumption the firm will satisfy the participation constraint, and cannot lower previously promised wages. So its optimal strategy is then to simply minimize the total cost of doing so, which is the lemma’s characterization of the re-negotiated wage \( \tilde{w} \).

As the proof of the lemma shows, exit will be optimal for \( E \) at any point when \( w < w^* \) and the forward-looking surplus value defined by

\[
H(t) \equiv B + \int_t^T (\tilde{w}_u - w^*_u) \, du
\]
is zero. From this we can give an alternative characterization of the solution \( \bar{w} \) as

\[
\bar{w}_t = w_t \cdot 1_{\{H(t) > 0\}} + \min(w_t, w^*_t) \cdot 1_{\{H(t) \leq 0\}}.
\]  

(A.11)

Proposition 2.1 asserts first that, since \( E \) assumes \( F \) will follow this renegotiation strategy, her optimal strategy will be to never exit, and hence her valuation of the package is just the benefit received by staying.

Second, noting that the solution for \( \bar{w} \) yields the minimal value \( w \) for \( t \geq \tau^+ \) (where \( \tau^+ \) is defined in the proposition), we have

\[
B + \int_{\tau^+}^{T} \bar{w}_u \, du = \int_{\tau^+}^{T} w^*_u \, du \equiv V^\text{res}_{\tau^+,T}.
\]

And since \( V_t(w, B) \) is increasing in its arguments, this implies

\[
V_{\tau^+}(w, B) = V_{\tau^+}(w, 0) = V_{\tau^+}(0, 0) = V^\text{res}_{\tau^+,T}.
\]

The remaining assertions follow from the observation that, for any points \( t, s > t \), the value of the package \( (w, B) \) at \( t \) is the same as the value on \( [t, s] \) of the package \( (w, V_s(w, B)) \), i.e. the right to receive \( w \) until \( s \) and then be paid a lump-sum equal to the value of the remaining payments. (This can be deduced also from equation (A.11) above.)

Hence the two packages \( (w, B) \) and \( (w, 0) \), having equal value at \( s = \tau^+ \) and the same wage stream, must have the same value for all \( t < \tau^+ \) as well. From (A.11) we also see that, if \( w \leq w^* \) prior to \( \tau^+ \), both packages will have \( \bar{w} = w^* \) as well, implying

\[
V_t(w, B) = V_t(w, 0) = V^\text{res}_{t, T}
\]

as asserted.
**Proof of Proposition 3.1**

We solve for $F$‘s optimal bid $w^r + x$, under the assumption that $\delta$ is unknown to $F$ and that $E$ exits iff $x < \delta$.

Given the assumed distribution of $\delta$, $F$‘s expected cost from bidding $x$ (the additive term $w^r$ will be dropped for ease of notation) is

$$
\left\{ x \cdot q\Delta \cdot \left( \frac{x + a}{2a} \right) + c \cdot [(1 - q\Delta) + q\Delta \cdot \left( \frac{a - x}{2a} \right)] \right\} 1_{[x<0]} 
$$

$$
+ \left\{ x \cdot [(1 - q\Delta) + q\Delta \cdot \left( \frac{x + a}{2a} \right)] + cq\Delta \cdot \left( \frac{a - x}{2a} \right) \right\} 1_{[x\geq0]}.
$$

The first term is minimized at $x = (c - a)/2$ or, if this is positive, at $x \to 0$. The second term is minimized at

$$
x = \frac{c - a}{2} - \frac{(1 - q\Delta)}{q\Delta} a
$$

or 0 if that is negative. Assume that is the case, i.e.

$$
\frac{c - a}{2} < \frac{(1 - q\Delta)}{q\Delta} a \quad \text{or} \quad c < I^+ \equiv a \left( 1 + 2 \frac{(1 - q\Delta)}{q\Delta} \right). \quad (A.12)
$$

Then the optimal bid is zero unless both $(c - a)/2 < 0$ and the expected cost at $(c - a)/2$ is less than that at zero. At zero, this expectation is $cq\Delta/2$. At $(c - a)/2$, when this is negative, it is

$$
- \frac{q\Delta}{2a} \left( \frac{c - a}{2} \right)^2 + c[(1 - q\Delta) + q\Delta/2].
$$

Hence zero is the optimal bid whenever $c < I^+$ and

$$
\frac{cq\Delta}{2} < - \frac{q\Delta}{2a} \left( \frac{c - a}{2} \right)^2 + c[(1 - q\Delta) + q\Delta/2]
$$

or

$$
(c - a)^2 < 8ac \frac{(1 - q\Delta)}{q\Delta}. \quad (A.13)
$$
Writing this as an increasing quadratic in \( c \) that needs to be negative, we see the restriction always holds when \( c = a \), never holds at \( c = 0 \), and is minimized at \( c = a [1 + 4(1 - q\Delta) / q\Delta] > a \). Hence it has exactly one root \( I^- \) in \([0, a]\). This shows that \( w^r + 0 \) is the optimal bid for \( c \) in \([I^-, I^+]\), as claimed. (We also note from (A.12) and (A.13) that as \( q\Delta \to 0 \) both \( I^- \to 0 \) and \( I^+ \to \infty \).)

The expected cost to \( F \) at this bid was given above. The expected benefit to \( E \) is

\[
0 \cdot \Pr(\delta \leq 0) + E(\delta|\delta > 0) \cdot \Pr(\delta > 0)
\]

The second term works out to \( q\Delta \cdot (1/2) \cdot (a/2) \) since \( a \) is just conditionally uniform on \([0, a]\).

Although the model in the text envisions only take-it-or-leave-it offers by \( F \), it is straightforward to show that \( E \) cannot improve her expected value through multi-round bargaining. Since she faces no exit cost, she has no incentive to signal high realizations of \( \delta \), and hence cannot do so credibly. So the above results apply to the more general game as well.

**Multiperiod Bargaining**

In analyzing the multiperiod game, we start with a lemma which derives the appropriate participation constraint, and clarifies our assumptions about workers’ multiperiod opportunities.

**Lemma 2** A multiperiod contract whose value, if accepted, exceeds \( \sum_{n=k}^{N-1} w_{t_n} \Delta \) at \( t_k \) will induce participation by a replacement worker from \( t_k \) to \( t_k + \Delta \equiv t_{k+1} \).

A multiperiod contract whose value, if accepted, exceeds \( \sum_{n=k}^{N-1} w_{t_n} \Delta + \delta_{t_k} \) at \( t_k \) will induce participation by an incumbent from \( t_k \) to \( t_k + \Delta \equiv t_{k+1} \).

**Proof.** For notational simplicity, take the time interval \( \Delta \) to be unity. Define \( V_{t,T}^{res} \) to be the minimal value on \([t, T]\) of a contract inducing participation from \( t \) to \( t + 1 \), but not necessarily thereafter. Let \( V(\mathcal{E})_{t,T} \) be the value at \( t \) (under optimal renegotiation) of a contract \( \mathcal{E} \) which promises wages \( w_s^f \), possibly zero, for \( s \in [t, T] \).
Then the incremental benefit of accepting $\mathcal{E}$ at $t$ over the alternative of not doing so is

$$w_t^{\mathcal{E}} + \mathbb{E}_t[V(\mathcal{E})_{t+1:T} - V_{t+1:T}^{res}].$$

Hence $\mathcal{E}$ will be accepted by a replacement worker who has no time-$t$ idiosyncratic opportunity if this is greater than $w_t^{r}$. If not, that is

$$w_t^{r} + \mathbb{E}_t[V_{t+1:T}^{res}] > w_t^{\mathcal{E}} + \mathbb{E}_t[V(\mathcal{E})_{t+1:T}]$$

then the value of accepting is less than the value of the strategy of not accepting, and continuing to realize the single-period reservation value until $T$. Hence $\mathcal{E}$ will not be accepted. Thus, by definition,

$$V_{t,T}^{res} = w_t^{r} + \mathbb{E}_t[V_{t+1:T}^{res}]$$

and the first part of the lemma follows from iterating the expectation on the right forward.

The second statement in the lemma follows from the first by (A3.2) which stipulates that outside workers and incumbents differ only in the transient idiosyncratic opportunity $\delta_t$ of the latter at the current period. This establishes the lemma.

Note that our framework implies a difference, for example, between a single period contract and a two-period one having no guarantee for the second period ($w_2^{\mathcal{E}} = 0$). In a two-period contract, the worker will be an incumbent at the start of the second period and so will receive a bid of $w_2^{r}$ from $F$, which is $F$’s optimal action. This bid has an option value to the worker at time 2, which is not present in a one-period contract at time 1. In other words, our assumptions do not imply that workers are guaranteed to be incumbents for some future employer (equivalent to $F$) at future dates. If one wanted to assume they are, $w^{r}$ would simply be redefined accordingly.

A second distinction to note in the proof is between the time $t$ value to $E$ of contract $\mathcal{E}$ ex ante, and the value of the same contract conditional on accepting it at
$t$. The former includes the option value of being able to reject it at $t$. Formally the two are related by

$$V(\mathcal{E}|\text{accept } \mathcal{E})_{t,T} = w_t^\mathcal{E} + E_t[V(\mathcal{E})_{t+1,T}].$$

A corollary of the lemma is that a contract whose value, conditional on participation, exceeds $\sum_{n=k}^{N-1} w_{t_n}^r \Delta + \delta_t$ at each $t_k$ ensures participation by the same worker at all future dates. We call such an $\mathcal{E}$ a “no-exit” contract for short. These have the property that their value to both $E$ and $F$ is simply the sum of their future promised payments.

The next lemma shows the advantage of no-exit contracts to firms.

**Lemma 3** If there exist no-exit contracts over each interval $[t_j, t_k]$ whose expected cost to $F$ is $\sum_{n=j}^{k-1} w_{t_n}^r \Delta$, then the lowest expected cost employment strategy on $[0,T]$ is to offer such a contract at time 0.

**Proof.** The proof is by induction. Suppose the statement is true for intervals of the form $[t_{(N-M)}, T]$. (We again set $\Delta = 1$ for convenience.) Then on an $M + 1$-period interval, it suffices to consider the choice for $F$ between no-exit contracts and strategies ensuring participation on some two consecutive intervals. Any other strategy must include a sub-interval of length less than or equal to $M$ which does not ensure participation throughout, and hence is not optimal on that sub-interval by the induction hypothesis.

Now among no-exit contracts on $[t_{N-(M+1)}, T]$, one having value $\sum_{n=N-(M+1)}^{N-1} w_{t_n}^r = V_{t_{N-(M+1)}, T}^{\text{res}}$ is the best contract $F$ can achieve because any no-exit contract having lower expected cost has value to $E$ below the reservation wage, and will not be accepted.

Strategies having two consecutive no-exit sub-intervals, on the other hand, entail negotiating with the incumbent at the expiration of the first interval (call it $t_j$) or paying the turnover cost. By assumption, the optimal contract offer has expected cost $\sum_{n=j}^{N-1} w_{t_n}^r$, which is the reservation value of a replacement worker $V_{t_j}^{\text{res}}$. The incumbent’s reservation value is $V_{t_j}^{\text{res}} + \delta_j$. So the renegotiation game is identical to
the one-period game of Proposition 3.1, with \( V^r_{t_{j,T}} \) playing the role of the reservation wage. Hence the expected cost to \( F \) of this strategy is \( V^r_{t_{j,T}} + cq/2 \) on the second sub-interval, and the cost on the entire interval is at least \( V^r_{t_{N-(M+1),T}} + V^r_{t_{j,T}} + cq/2 \) which exceeds \( V^r_{t_{N-(M+1),T}} \), the cost of the single contract.

Since the induction hypothesis is trivially true for \( M = 1 \), the result follows.

**Proof of Propositions 3.2 and 3.3.**

We now prove Propositions 3.2 and 3.3 together by merely constructing a contract with the properties required by the last lemma.

As already noted, a sufficient condition for a contract \( \mathcal{E} \) to have the no-exit property is that

\[
V(\mathcal{E})_{t_k} \geq \sum_{n=k}^{N-1} w^r_{t_n} + \delta_k
\]

at each \( t_k \). Hence any contract whose promised future payments exceed \( \sum_{n=k}^{N-1} w^r_{t_n} + a \) accomplishes this since \( \delta \leq a \). The expected cost to \( F \) of such a contract is also the sum of the future payments, since these are certain once the contract is accepted.

Now assume \( w^r_0 - a \) is non-negative, and consider a contract whose promised payments at times \( \{t_n\}_{n=0}^{N-1} \) are given by the vector

\[
(w^r_{t_0} - a, w^r_{t_1}, w^r_{t_2}, \ldots, w^r_{t_{(N-1)}} + a).
\]

At all times greater than 0, this satisfies the sufficient condition for a no-exit contract. At time zero, the contract, if accepted, has value then of \( \sum_{n=0}^{N-1} w^r_{t_n} = V^r_{0,T} \) and so ensures the participation then of a replacement worker.

Hence the lemma implies offering this contract at time zero is \( F \)'s optimal strategy, which is the claim of Proposition 3.2. Proposition 3.3 gives the value function at each time of this contract. In addition, any contract having value less than \( \sum_{n=k}^{N-1} w^r_{t_n} + a \) at any \( t_k \) admits the possibility of optimal exit by the incumbent at that time, and hence is not a no-exit contract. So, in conjunction with Lemma 3, this shows that no
such contract can be optimal. This completes the proof.

Proof of Propositions 3.4 and 3.5.

To prove these propositions, we consider the renegotiation game at time $T'$, the date at which no future wage is guaranteed. Lemma 3 implies that $F$ will optimally propose only no-exit contracts on $[T', T]$. However now the value, $v$, of such an offer includes the benefit of staying and getting $B$ at $T$. So $E$’s response to an offer of value $v$ will be to accept and stay iff $v + B \geq V_{T', T}^ {res} + \delta_{T'}$. So the situation is analogous to the one-period game of Proposition 3.1, with $V_{T', T}^ {res} - B$ being $E$’s reservation value. By that proposition, $E$ will bid exactly this amount, or, in the present case, zero if that amount is negative, since negative wages are infeasible. The statements of Propositions 3.4 then follow immediately from the earlier proposition.

In computing the optimal expected cost to $F$ at time zero from including $B$, we distinguish two cases. If $B \leq V_{T', T}^ {res}$ (which includes the case of no deferred component, $B = 0$), then $F$’s offer at $T'$ is the same as the optimal one described by Proposition 3.3. However, the total expected cost at $T = 0$ of having to play the renegotiation game at $T'$ was shown (in the proof of Lemma 3) to be at least $cq/2$ greater than that of the optimal contract. In the second case, $B > V_{T', T}^ {res}$, $F$ is effectively forced to over-bid to $E$ at time $T'$ relative to the lowest expected cost strategy. Hence this must have incremental expected cost greater than $cq/2$, hence more than the expected cost of not having included $B$ at all. This establishes Proposition 3.5.

Proof of Proposition 3.6.

Holding $F$’s rule $S^*$ fixed, $E$’s rule results in receiving a total offer from $F$ of $w^*_1 + a$ for the second period. It is always optimal to accept, producing a lifetime benefit of $w^*_1 + w^*_2$. If the firm is liquidated after the first period, our assumption is that the explicit promise $a$ is still paid, and $E$ receives $w^*_2$ elsewhere, resulting in the same benefit. (This establishes part (B) of the proposition, *inter alia.*) An alternative
policy which rejects the time-zero offer entails accepting outside employment for \( w_{1}^{t} \) in the first period and \( w_{2}^{*} + \delta \), which is equal in expectation. So \( R^{*} \) is no worse.

For \( F \), we consider any alternative policy \( S' \) in which expropriation may occur at some \( t \) (i.e. \( P(\chi_t < 1) > 0 \)).

To simplify notationally, let us assume that all time-cohorts have the same replacement wage \( w_{1}^{t} \) in the first period, and have second-period wage \( w_{2}^{*} \) equal in distribution, and having mean \( w_{1}^{t} \).

Then we first note that, with \( R^{*} \) as given, \( S^{*} \) attains the objective value

\[
U_{t}(S^{*}) \equiv E_{t}\left[ \sum_{s=t}^{\infty} (w_{1} + w_{2})_{s} \right] = \frac{w_{1}^{t} + Ew_{2}^{*}}{1 - \gamma} = \frac{2w_{1}^{t}}{1 - \gamma}.
\]

Hence any optimal policy must have expected cost no greater than this.

Now let \( t \) be the first time expropriation is possible under \( S' \). Then if it occurs, all future workers demand their reservation wage initially and in the second period. From the last section, the firm’s optimal response in the renegotiation game has expected cost \( w_{2}^{*} + cq/2 \), which is achieved by offering \( w_{2}^{*} \) once it is known. (With \( w_{2}^{*} \) stochastic, \( F \) can do no better by guaranteeing any more at time one.) Hence, following expropriation, the firm will achieve

\[
U_{t+1}(S') = \frac{2w_{1}^{t} + cq/2}{1 - \gamma}.
\]

Since this is the result for any \( \chi_{t} < 1 \), conditional on expropriation, the firm will set \( \chi_{t} = 1 \), making its time-\( t \) wage expected cost \( (w_{1} + w_{2}^{*})_{t} + cq/2 \), and

\[
U_{t}(S') = (w_{1} + w_{2}^{*})_{t} + cq/2 + \gamma \frac{2w_{1}^{t} + cq/2}{1 - \gamma}.
\]

If, instead, \( F \) does not expropriate at \( t \), it must pay \( (w_{1} + w_{2}^{*})_{t} + (w_{1}^{t} + w_{1})_{t-1} \) at \( t \), and can then attain a subsequent cost which does not exceed \( \gamma \frac{2w_{1}^{t}}{1 - \gamma} \). Hence a
necessary condition for optimality of $S'$ is that
\[
\frac{cq/2}{1-\gamma} \leq (w^*_t + w_1)_{t-1}.
\]

Since $w_1 > 0$, condition (3.2.9) rules this out for all $t$. Hence, no optimal policy can involve any probability of expropriation.

Policies not involving expropriation all set $w_2 = (w^*_2)_t + (w^*_1 + w_1)_{t-1}$ by definition. Hence they all achieve the same expected cost $U_t(S^*)$ given above. This shows that $S^*$ is optimal, which completes the proof.
References


